

ON THE DESIGN OF OVERHANGS INSTALLED ABOVE SOUTH-FACING WINDOWS

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ABSTRACT

In passive solar systems, large south facing glazed windows designed for winter heating are often troublesome in summer. One effective means of reducing summer solar energy gain is the installation of overhangs. This paper presents a simple mathematical model for determining the incident solar energy on a vertical rectangular window with a horizontal overhang at any solar time. Examples at 31° north latitude were studied to determine the effect of the overhang projection on incident solar energy. The results of this study enable the choice of the suitable overhang projections, which permit the maximum solar energy to be collected by the window in heating months and intercept the majority of solar radiation in cooling months.

Keywords: Passive solar heated building, Overhang, Window, Shaded area, Sunlit area.

INTRODUCTION

Buildings consume a large portion of the total energy in the world. Energy conservation and use of renewable sources of energy in buildings can go a long way in extending the life of fossil fuels and cleaning the environment. Passive solar heated building is a good contribution to energy conservation. In such system solar radiation is admitted directly into the building through large south-facing glazed windows designed for winter heating. However, they are often troublesome in summer cooling. The most effective way to reduce the solar load on passive windows is to intercept direct radiation from the sun before it reaches the glass by installation of overhangs or awnings. As these devices may partially shade the windows in winter heating periods, it is important to adopt a well balanced design from the standpoint of both heating and cooling requirements.

The design of shading overhangs for vertical windows is often done from relatively simple considerations such as the solar noon shadow at summer and winter solstice. Utzinger and Klein [1] presented a graphical method for estimating monthly average radiation on shaded vertical surfaces, while Jone [2] and Sharp [3] offered an analytical method for the same configuration. They based their analysis on the assumption that the overhang extends laterally far past the sides of the window, so that it

can be approximated as being infinitely long and overhang side effects can be neglected. The calculation procedures that were developed in the above mentioned papers are comprehensive and as a result are complex.

Recently, some investigators [4-8] have studied the shading caused by collectors on one another. The analyses given in these researches are very complex and not reasonable to be used in case of window-overhang-arrangement.

McCluney [9] developed a simpler algorithm for calculating the sunlit fraction of a window shaded by an awning or overhang of arbitrary size. This algorithm has the limitation that it can not handle cases for which the shadow of the awning or overhang crosses the window. This is valid only for the case of awning with sidewalls, which are attached to the wall above the top of the window.

A general simple method is presented here which allows the determination of the shaded area covered by an overhang of a width equal to the window width. Herein the effect of overhang sides are taken into consideration. This method is then extended to calculate the amount of clear day solar radiation received by the window surface. Based on the results obtained using this developed method, the overhang can be sized.

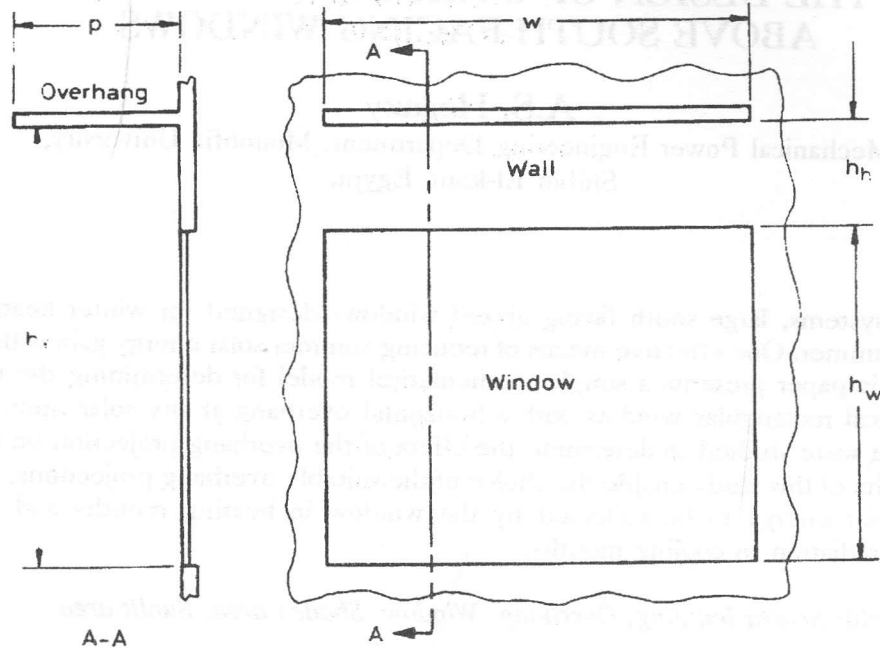


Figure 1. Representation of window, overhang and wall.

WINDOW AREA LIGHTED BY SUN RAYS

Figure (1) depicts a schematic diagram of a vertical rectangular glazed window and a horizontal parallel overhang. The window has a height h_w and a width w . An overhang is installed at an offset height h_h above the top of the window. The overhang has a horizontal projection p and its width is equal to the window width. The total height of the window and wall (facade height) is h_f .

For the sake of determining the window area lighted by sun rays, Figure (2) shows the geometrical relationship of the incident sun rays and the window-overhang arrangement. The window plane A faces the south. The x, y, z coordinated system is set at the lower left-hand corner (point o) of the window so that the x -axis coincides with the south direction.

The direction of the sun rays is represented by the unit vector \vec{s} . This vector can be explained in components as:

$$\vec{s} = (x_s, y_s, z_s) = (-\cos\alpha \cos\gamma, -\cos\alpha \sin\gamma, -\sin\alpha) \quad (1)$$

where α and γ are the altitude and azimuth angles of the sun rays respectively.

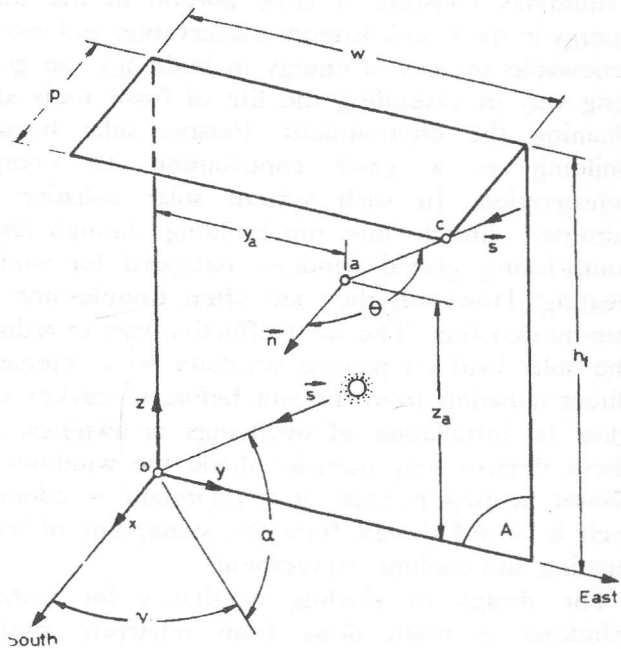


Figure 2. Geometry of incident sun rays and window-wall-overhang arrangement.

The parametric equation of the incident sun ray passing through the point c of the right-hand corner of the overhang is represented as:

$$\vec{c} + t\vec{s} = (x_c, y_c, z_c) + t(x_s, y_s, z_s) \quad (2)$$

where t is a parameter

The coordinates of the point c may be obtained from Figure (2) as:

$$\vec{c} = (x_c, y_c, z_c) = (p, w, h_f) \quad (3)$$

The equation of the window plane A can be represented by:

$$(\vec{r} - \vec{o}) \cdot \vec{n} = 0 \quad (4)$$

The point r is an arbitrary point lying on the A -plane. The point o is the origin of the x, y, z coordinate system (lower left-hand corner of the window) and its coordinates are given by:

$$\vec{o} = (x_o, y_o, z_o) = (0, 0, 0) \quad (5)$$

\vec{n} is a unit vector normal to the A -plane. It is easy to see by examining Figure (2) that \vec{n} can be given by:

$$\vec{n} = (x_n, y_n, z_n) = (1, 0, 0) \quad (6)$$

The sun ray passing through the point c will meet the A -plane at point a . Given Eqns. (2) and (4), it follows for the coordinates of point a

$$\vec{a} = (x_a, y_a, z_a) = \vec{c} + t\vec{s} \quad (7)$$

where t is given by:

$$t = \frac{(\vec{o} - \vec{c}) \cdot \vec{n}}{\vec{s} \cdot \vec{n}} \quad (8)$$

From eqns. (7) and (8) the y - and z -coordinates y_a and z_a of the point a can be determined.

The objective of the following analysis is to evaluate the area of the window which is sunlit, $A_1 + A_2$ in Figure (3). All rays passing through the points of the frontal overhang edge meet the plane A of the window at points having the same z -coordinate z_a of the point a and smaller y -coordinate than y_a , i.e. at line segment aa_1 in Figure (3). Consequently a rectangular area A_1 of width y_a and height z_a is impinged by the incident sun rays

passing under the frontal overhang edge. It follows from Figure (3) for the area A_1 that:

$$A_1 = y_a z_a \quad (9)$$

Besides, all incident sun rays passing throughout the eastern lateral overhang edge meet the A -plane at line segment aa_2 . All sun rays passing under the eastern edge impinge the A -plane in the trapezoidal area confined by the line aa_2 and the y -axis.

Excluding the wall area from the trapezoidal area gives the area A_2 in Figure (3)

$$A_2 = \left(\frac{w - y_a}{2} \right) \left(z_a + h_f - \frac{h^2}{h_f - z_a} \right) \quad (10)$$

The sunlit fraction F is defined as the ratio of the sunlit area of the window glass and the facade area (window area plus wall area above the window).

$$F = \frac{A_1 + A_2}{w h_f} \quad (11)$$

Accordingly, and with the help of Eqns. (9) and (10), it follows that

$$F = \frac{z_a y_a + \left(\frac{w - y_a}{2} \right) \left(z_a + h_f - \frac{h^2}{h_f - z_a} \right)}{w h_f} \quad (12)$$

CALCULATION OF INCIDENT SOLAR RADIATION ON WINDOW SURFACE

Having determined the window area lighted by sun rays, it is now possible to calculate the amount of beam solar energy received by the window surface. Let the intensity of the beam solar radiation on normal surface be I_b . The areas A_1 and A_2 receive beam solar power

$$Q_b = (A_1 + A_2) I_b \cos \theta \quad (13)$$

from which, in consideration of Eqn. (11) it follows that

$$Q_b = F w h_f I_b \cos \theta \quad (14)$$

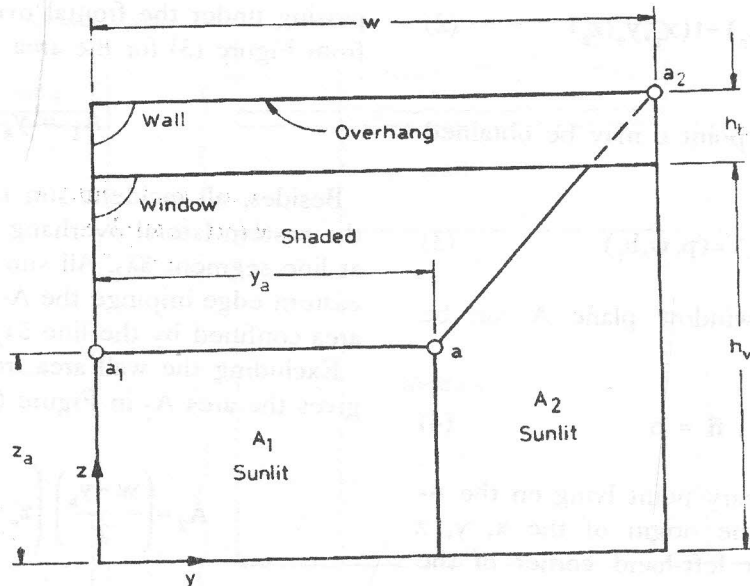


Figure 3. Demonstration of sunlit window area A_1+A_2 .

θ is the angle between incident sun rays and the window surface (A-Plane); i.e. the intersection angle of the unit vectors \bar{n} and \bar{s} , s. Figure (1). Accordingly, the angle θ can be calculated from

$$\cos\theta = -\bar{n} \cdot \bar{s} = -x_n x_s - y_n y_s - z_n z_s \quad (15)$$

From which and by help of Eqns. (1) and (6) it follows that:

$$\cos\theta = \cos\alpha \cos\gamma \quad (16)$$

In many cases it is important to determine the effect of overhang on intercepting diffuse sky radiation. For this purpose one describes the sky radiation by the altitude and azimuth angle α and γ respectively (see Figure (3)). In this case α extends from 0 to $\pi/2$ and γ has the range from $-\pi/2$ to $\pi/2$. If the radiance of the sky from a direction (α, γ) is I_d and if F is the sunlit fraction of the window for radiation coming from this direction, F can be determined by using Eqn. (12), then the irradiance Q_d reaching the window from diffuse sky radiation can be calculated by:

$$Q_d = w h_f \int_{2\pi} F I_d \cos\theta d\omega \quad (17)$$

where θ is the angle of incidence of the radiation

coming from the direction (α, γ) on the window plane and $d\omega$ is an infinitesimal element of a solid angle in the direction (α, γ) . The solid angle $d\omega$ may be given by:

$$d\omega = \cos\alpha d\alpha d\gamma \quad (18)$$

from which and by aid of Eqn. (16), Eqn. (17) becomes

$$Q_d = w h_f \int_{-\pi/2}^{\pi/2} \int_0^{\pi/2} F I_d \cos^2\alpha \cos\gamma d\alpha d\gamma \quad (19)$$

For an isotropic (uniform sky) I_d is constant and can be removed from the integrals. Consequently Eqn. (19) can be rewritten as

$$Q_d = w h_f I_d F_d \quad (20)$$

where F_d is the diffuse sky radiation shape factor which is given by:

$$F_d = \int_{-\pi/2}^{\pi/2} \int_0^{\pi/2} F \cos^2\alpha \cos\gamma d\alpha d\gamma \quad (21)$$

For determining F_d , the integration in Eqn. (21) was carried out numerically.

Table 1. Predicted beam solar radiation fluxes on normal surface (W/m^2).

Solar time hour	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept	Oct.	Nov.	Dec.
5					16.6	34.5	18.2					
6				66.4	130.3	157.2	132.0	66.9				
7		35.0	114.1	223.4	299.9	326.9	300.7	222.7	114.5	31.5		
8	76.7	165.2	284.4	406.6	479.1	502.1	478.6	404.1	283.3	158.0	73.7	50.0
9	202.7	319.8	453.2	575.2	640.3	658.8	638.5	570.9	450.3	309.8	917.6	159.9
10	320.1	420.2	589.3	708.3	766.6	781.3	763.7	702.6	584.8	438.5	313.8	269.7
11	398.9	535.2	676.6	793.1	846.8	859.1	843.3	786.4	671.1	522.3	391.8	344.7
12	426.3	564.5	706.6	822.2	874.3	885.7	870.5	815.2	700.7	551.3	419.0	371.0

Table 2. Predicted diffuse sky radiation fluxes on window surface without overhang (W/m^2).

Solar time hour	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept	Oct.	Nov.	Dec.
5					35.5	23.5	18					
6				42.5	45	33.5	27.5	25				
7		46	51.5	67.5	62.5	52.5	48	45	42	42.5		
8	58	67.5	74.5	94.5	82.5	67.5	69.5	62.5	57.5	58	63	62.5
9	73	81.5	96	114.5	103	90	97.5	89	78	80	72.5	73.5
10	91	98	118.5	133	124	109.5	107	97.5	95.5	93	87.5	89.0
11	96.5	113.5	128	147	140	116	111.5	111.5	108	102	97	94
12	102	116	133.5	159	150	127.5	125	121.5	114	107	100.5	97

PRESENTATION OF THE RESULTS

To implement the preceding analysis a computer program was constructed to study the effect of the overhang on both the incident beam solar energy and diffuse sky radiation received by a passive window. The following results were obtained for sites located at 31° north latitude, e.g. Alexandria in Egypt. The fluxes of beam and sky radiation at these sites were calculated using a clear day computer program developed based on the analyses presented in Duffie and Beckman [10] and Howell, Bannerot, and Vliet [11]. These two programs were run for time steps of 10 minutes from sunrise to solar noon.

Tables (1) and (2) list the predicted fluxes of beam radiation on a normal surface and sky radiation on a

vertical surface respectively for each hour on the 22nd of each month for the sites of study. Figure (4) depicts the window sunlit area for a window overhang arrangement having a facade height ratio h_f/w of 0.75, a projection ratio p/h_f of 0.3 and a window height ratio h_w/h_f of 0.8. The dimensionless z-coordinates z/h_f of the window is plotted versus the dimensionless y-coordinate y/w for solar times of 10, 11, and 12 hrs. on the 22nd of January, April and July. The area above $z/h_f = 0.80$ is wall area. The area above each curve is shaded. The area below each curve is sunlit area. The ratio of the sunlit window area to the total facade area (window and wall) is 0.80, 0.39 and 0.20 at 10 hr for January, April and July respectively. This ratio becomes 0.78, 0.27, and 0.00 at solar noon for the above given months.

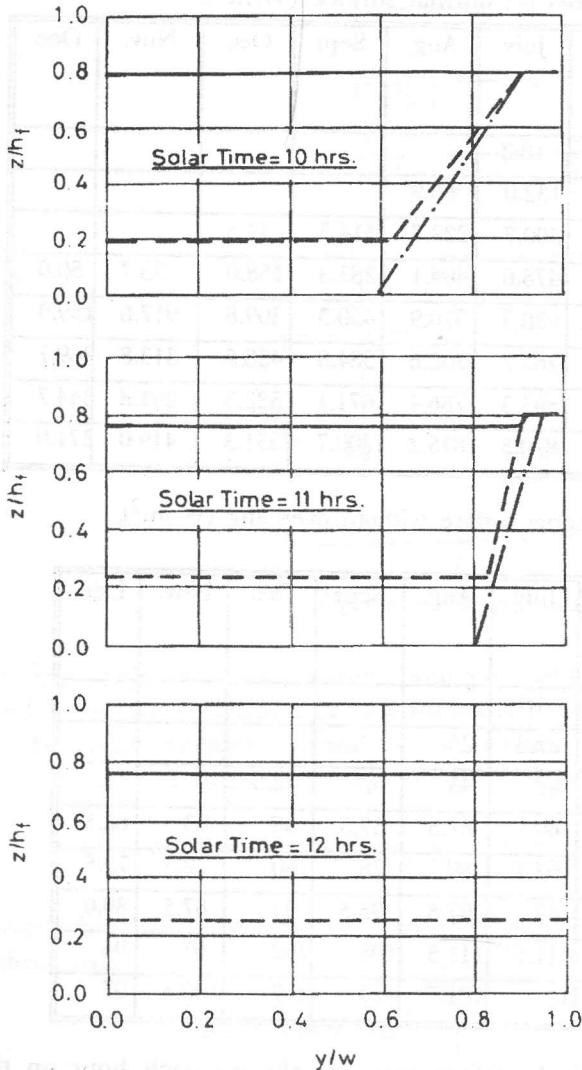


Figure 4. Sunlit and shaded window area for $p/h_f=0.30$, $h_w/h_f=0.80$, $h_f/w=0.75$ and 31° north latitude.

22 January - - - 22 April - - - 22 July

Figure (4) reveals that the sunlit area has a rectangular shape at solar noon irrespective of the month of the year. This rectangle covers the whole window width and its height equals z_a obtained from Eq. (7) at solar noon. The ratio z_a/h_f (ratio of sunlit height to facade height) at solar noon is plotted in Figure (5) as a function of the day of the year for different values of projection height ratio p/h_f (0.1, 0.2,..... 1.0). It is seen in Figure (5) that the ratio z_a/h_f decreases as the day number increases until it reaches its minimum on the 22nd of June when the sun is at its highest position. After this day the ratio

z_a/h_f increases until it reaches its maximum value on the 22nd of December when the sun is at its lowest position (smallest solar altitude angle). The rate of decrease or increase in the ratio z_a/h_f with the day of the year is smaller in the winter heating months (lower solar altitude angles) November, December, January, and February. In contrast, the rate of change in z_a/h_f is much greater in summer cooling months (higher solar altitude angles) May, June, July and August. The ratio z_a/h_f decreases as the projection height ratio p/h_f increases. This decrease is much steeper in months of higher altitude angles than those of lower altitude angles. For a ratio p/h_f of 0.26 the window is not impinged by any sun ray at solar noon on 22nd of June. For ratios of p/h_f greater than 0.26 the window is not subjected to any sun ray at solar noon for a certain period centered about the 22nd of June. This period is 1.1 months, 2.9 months, and 4.0 months for $p/h_f = 0.3, 0.4,$ and 0.5 respectively.

Examining Figures (4) and (5), one can conclude that the part of the sunlit area at any solar time above the height of the sunlit area at solar noon of the 22nd of December is negligibly small.

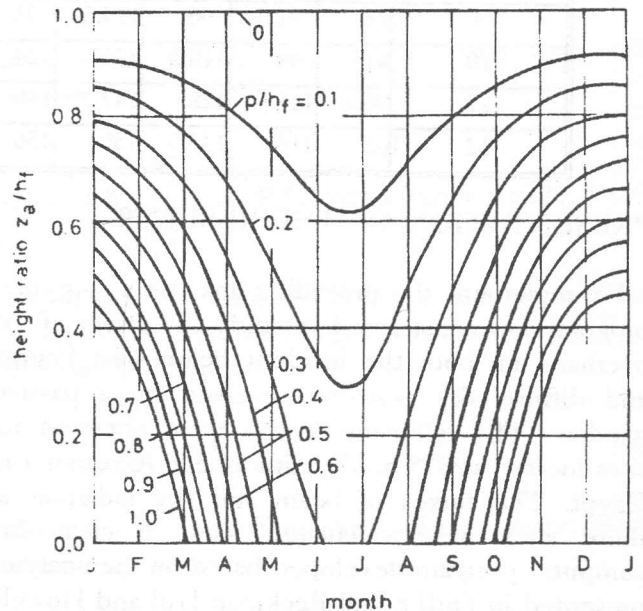


Figure 5. Effect of overhang projection on irradiated facade height at solar noon for 31° north latitude.

Therefore, if the purpose of the window is to collect solar energy during winter, there is no need

to make the window height greater than z_a at solar noon on December 22nd. In the following the window's height is taken equal to the height of the sunlit area at solar noon of the 22nd of December. In Figure (6) the window height ratio h_w/h_f is plotted versus the projection height ratio p/h_f . It is clear from this figure that h_w/h_f decreases linearly as p/h_f is increased.

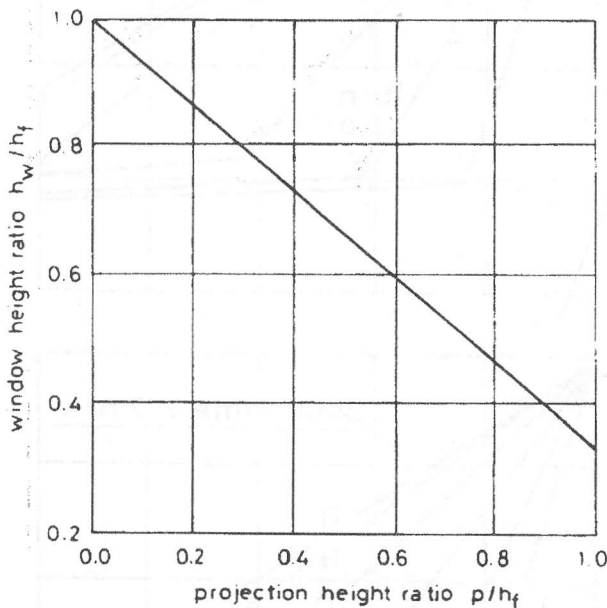


Figure 6. Dependence of window height on overhang projection for 31° north latitude.

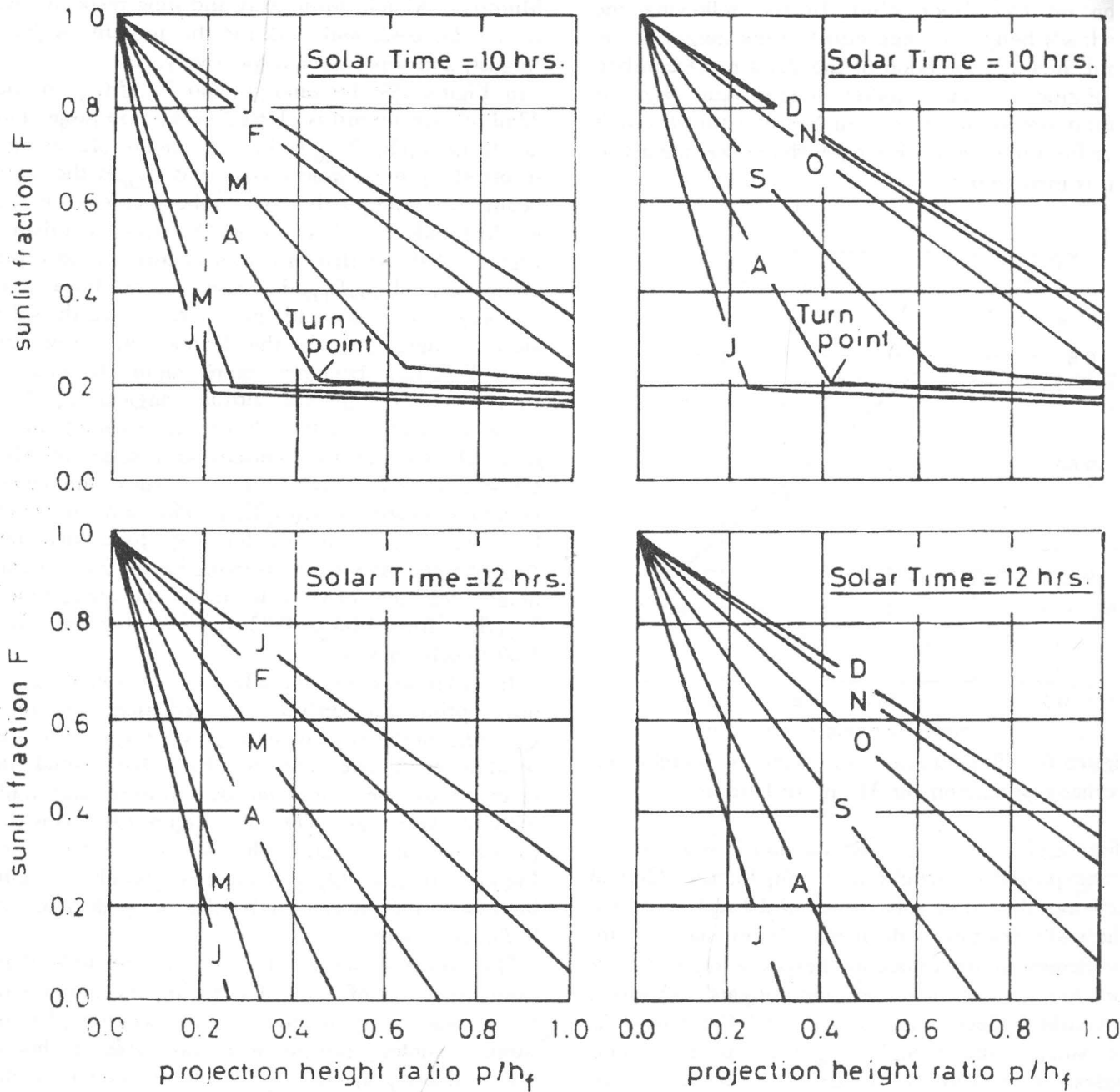
Figure (7) shows the sunlit fraction F as a function of the projection height ratio p/h_f for the 22nd of each month and at solar times of 10 and 12 hours. The sunlit fraction F decreases almost linearly with the increase of the projection height ratio p/h_f during the months of low altitude angles (October, November, December, January and February). As the sun becomes higher (greater solar altitude angles) F continues to decrease linearly with increasing ratio p/h_f till a certain turn value which depends on both the month and solar time. Further increase in p/h_f above the turn value causes only small change in F . At the turn value of the ratio p/h_f the sunlit area of the window is lighted only by the sun rays falling under the overhang's eastern edge. This causes a slight decrease in the sunlit area of the window which, in turn, results in the small decrease of the Sunlit fraction F . The turn value of p/h_f at

solar time 10 hrs is 0.63, 0.43, 0.25, and 0.21 for the Months of March, April, May and June respectively. It is 0.25, 0.42, and 0.63 for the months of July, August, and September respectively.

In Figure (8) the energy ratio $E_{w,b}/E_{f,b}$ on the 22nd of each month is plotted versus the projection height ratio p/h_f . $E_{w,b}$ is the daily beam solar energy received by the window glass and $E_{f,b}$ is the daily beam solar energy that would be incident on the whole facade (window and wall) without overhang. Figure (8) shows that in winter heating months, the energy ratio $E_{w,b}/E_{f,b}$ decreases almost linearly as the projection height ratio p/h_f increases. As the solar altitude angles increase the decrease in the energy ratio $E_{w,b}/E_{f,b}$ becomes more rapid. In cooling summer months (greater altitude angles) $E_{w,b}/E_{f,b}$ continues decreasing linearly with increasing p/h_f but relatively steeper to a certain turn value of p/h_f . Greater values of p/h_f than the turn one cause negligible change in $E_{w,b}/E_{f,b}$. The turn values are 0.32, 0.22, 0.33, and 0.47 for May, June, July, and August respectively. An overhang having a projection height ratio p/h_f of 0.30 decreases the energy ratio $E_{w,b}/E_{f,b}$ from 1.00 to 0.82 in December and from 1.00 to 0.14 in June.

In order to show the effect of the overhang on intercepting the diffuse sky radiation, the ratio $Q_{w,d}/Q_{f,d}$ of the sky radiation power received by the window with overhang to that, that would be received by the whole facade (window and wall) without overhang is plotted in Figure (9) versus the projection height ratio p/h_f . It can be seen from Figure (9) that $Q_{w,d}$ decreases steeply as p/h_f increases. This is caused by the steep decrease in h_w/h_f , see Figure (6).

The above discussion leads us to conclude that, from the point of view of reducing the amount of beam solar energy received by the window glass in summer cooling months, it is reasonable to choose the overhang projection so that the projection height ratio p/h_f is between 0.3 and 0.5 at latitudes near 31° . This does not significantly reduce beam solar energy gain during winter heating months. As for the diffuse sky radiation, it is necessary to keep the amount of this component received by the window in heating months as high as possible since it represents a great portion of the incident radiation in these months. This is achieved by choosing the ratio p/h_f possibly small. Therefore a value of p/h_f around 0.3 is recommended.



a from January to June

b. from July to December

Figure 7. Effect of overhang projection on sunlit fraction for 31° latitude.

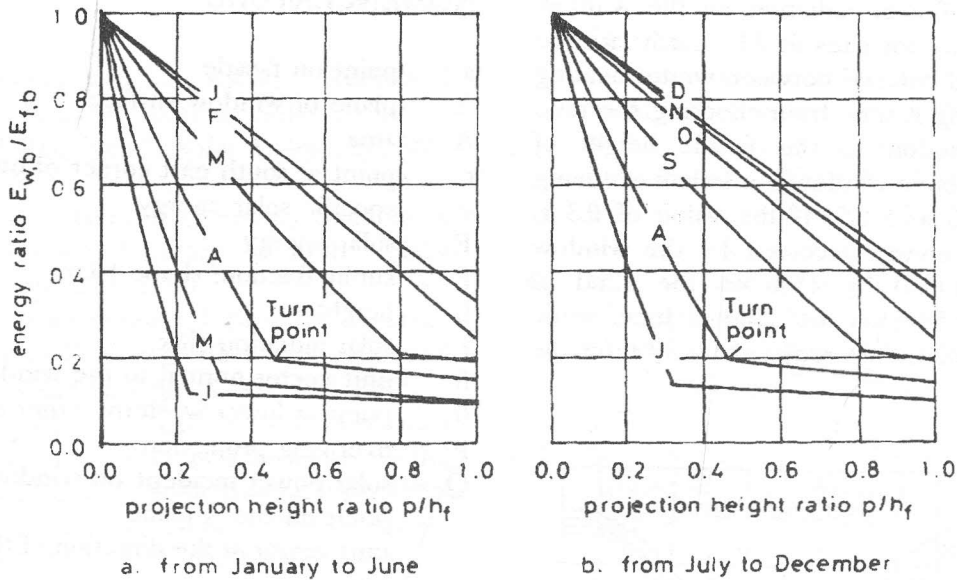


Figure 8. Effect of overhang projection on beam energy ratio for 31° north latitude.

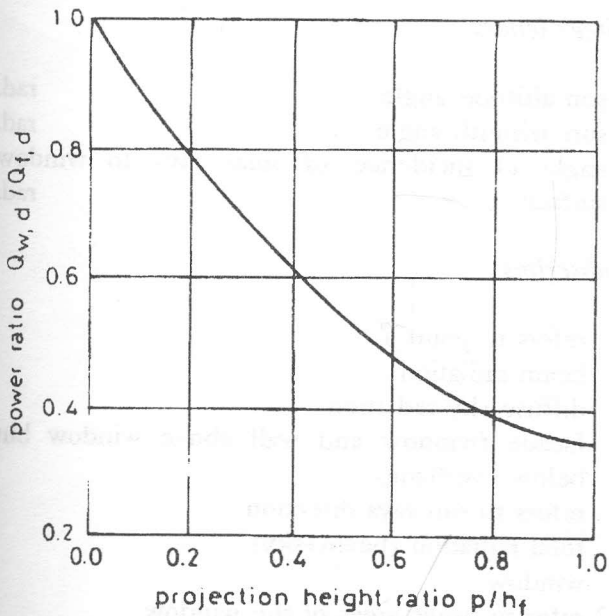


Figure 9. Effect of overhang projection on diffuse sky radiation power ratio for 31° north latitude.

The daily specific energies $e_{w,b}$, $e_{w,d}$, and $e_{w,t}$ of beam, sky and total radiation respectively have been calculated for a ratio $p/h_f = 0.3$ using the method developed herein and those reported in literature [1-3] and [9]. They are plotted in Figure (10) versus the day of the year. The upper diagram shows $e_{w,b}$,

$e_{w,d}$ and the lower one depicts the sum of $e_{w,b}$ and $e_{w,d}$; i.e $e_{w,t}$. In [9], a portion of the radiation coming from the sides of the overhang to the window was partially considered, but in [1,2,3] it was totally neglected. Therefore, as it is obvious from Figure (10), the method in [9] gives higher values of the specific energies than those obtained using the methods [1,2,3]. In the present method, all radiation passing by the overhang sides to the window is considered. This leads to higher values of the specific energies than those of the other methods.

Due to the simplicity of the analyses given in this work and the easiness of using them, they can be applied at any site of the world. However, the requirements of cooling and heating differ from one site to the other. Accordingly, it is not possible to find out a formula with which the ratio p/h_f can be determined for all sites of the world.

CONCLUSIONS

The arrangement of a south-facing vertical glazed window and an equal width overhang was studied in this work. A mathematical model was developed. with aid of this model the window area shaded by the overhang can be determined considering the overhang end effects due to its finite length. In addition, this model enables calculation of the

incident amount of solar radiation on the window surface. Calculations for sites at 31° north latitude showed that a good balance between winter heating and summer cooling results from choosing the ratio of overhang projection to the facade height of window and wall above window but below overhang to be between 0.3 and 0.5. If the value of 0.3 is chosen, the daily energy received by the window glass will be reduced by 15% on the 22nd of December and by 84% on the 22nd of June, while the diffuse sky radiation is reduced by 30% for the whole year.

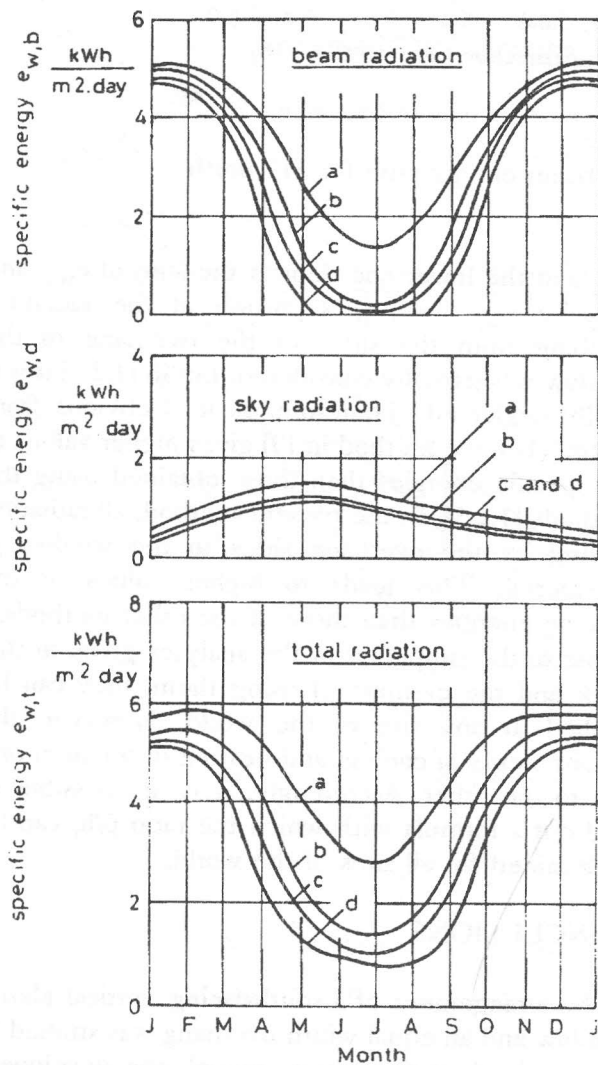


Figure 10. Predicted daily window specific energy for $p/h_f=0.3$ and 31° north latitude.
 a- without overhang b- present method
 c- Ref. [9] d- Refs. [1], [2] and [3].

NOMENCLATURE

a	point on facade	-
A	plane of window surface	-
A	area	m ²
c	point at south east corner of overhang	-
e	specific solar energy	kWh/m ²
E	solar energy	kWh
F	sunlit fraction; shape factor	-
h	height	m
I	solar radiation flux	kW/m ²
\bar{n}	unit vector normal to the window surface	-
O	point at lower western corner of window	-
P	overhang projection	m
Q	solar power incident on window	kW
r	point on the A-plane	-
\bar{s}	unit vector in the direction of the incident sun rays	-
t	parameter defined in Eqn.(2)	-
w	window width	m
x,y,z	coordinates	m

Greek letters

α	sun altitude angle	rad.
γ	sun azimuth angle	rad.
θ	angle of incidence of solar rays to window surface	rad.

Subscripts

a	refers to point a
b	beam radiation
d	diffuse sky radiation
f	facade (window and wall above window but below overhang)
s	refers to sun rays direction
t	total radiation (beam+sky)
w	window
1, 2	refer to sunlit areas of the window

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