GROUNDWATER ENVIRONMENT IN COASTAL LEAKY AQUIFERS WITH PUMPED WELLS

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ABSTRACT

For a semi-confined (leaky) coastal aquifer, the effect of a group of fully penetrating pumping wells on the interface between fresh and salt water is studied. It is assumed that these wells are located upstream of the toe of such an interface. The interface is assumed to be sharp and the aquifer is homogeneous and isotropic. A quasi-3D, steady state finite element (FE) model has been developed for simulating such a problem. The results of the developed model are compared to, and verified with, previous results of analytical and numerical models. The developed model is also used to study the effects of different wells' discharges, locations and arrangements; and aquifers conductivities; on the shape and location of the interface. Location of the hinge point has been also studied. The results show that wells' discharges and locations have more significant influence on the interfaces locations and shapes compared to wells' arrangements and aquifers conductivities.

Keywords: Groundwater environment-Coastal aquifers-Leaky- Salt-water intrusion-Sharp interface-Pumping Wells-Finite element-Areal model.

NOTATIONS

The following symbols are used in this paper, [Figure (1)]:

B	distance between two successive wells, in the y-direction;
b_a	thickness of the semi-confined(leaky) aquifer;
b_f,b_s	fresh and salt water thicknesses;
d	saturated thickness of the top ,semi-pervious layer(SPL);
d_{av}	average saturated thickness of the SPL;
d_o	saturated thickness of the SPL at sea;
f	a subscript denoting the fresh water domain;
H_o	sea water level above datum;
h_f,h_s	potential piezometric heads in fresh and salt water domains, respectively;
h_g	height of the ground water table in the SPL above datum;
KR	$=(k'_z.X_m)/(k_x.d_{av})$, flow conductance ratio;
k'_z	hydraulic conductivity of the SPL,in the Z-direction.
k_x,k_y	hydraulic conductivities in the x and y directions of the aquifer, respectively;
L	horizontal distance of the toe from the origin (intrusion length);
L_1	intrusion length for the case of no wells;
N_w	number of wells;
Q_1	total fresh water discharge into the aquifer at the land side,for the case of no wells;
Q_{wi}	discharge extracted from the i th well;
s	a subscript denoting the salt water domain;
s_b	slope of the bottom of the aquifer;
s_t	slope of the top of the aquifer;
TR	$=(N_w.Q_w)/Q_1$,trapping ratio;
X	horizontal distance from the origin,

	perpendicular to the coast;
X_{hp}	hinge-point distance from origin, in the X-direction
X_m	considered model length in X-direction;
X_w	well distance from origin, in the x-direction;
Y	distance from the origin, in the Y-direction, parallel to the coast;
Y_m	considered model length in Y-direction;
Z_B	height of the aquifer bottom above datum;
Z_I	height of the interface above datum;
Z_T	the height of the top of confined aquifer above datum;
$\delta(x)$	the Dirac delta function in X-coordinate;
$\delta(y)$	the Dirac delta function in Y-coordinate;
δ_1	$= \rho_f/(\rho_s-\rho_f)$;
ρ	density of water.
$*,o$	superscripts denoting values at landward side and seaward side boundaries, respectively.

INTRODUCTION

With ever increasing water requirements for irrigation and other uses, pumping from groundwater reservoirs, not far from the sea is becoming more feasible. Numerous field situations exist in which the main aquifer is overtopped by a rather thin semi-pervious layer(SPL), such as the case of most of the Nile Delta aquifer in Egypt. Boundary conditions for the salt water intrusion (SWI) problem in such semi-confined (leaky) aquifers are usually more complex when compared to confined or phreatic aquifers. They are further complicated by the introduction of a set of pumping wells.

Mualem and Bear (1974) derived an analytical solution for the linearized flow equation for a coastal aquifer with thin horizontal semi-pervious layer. They used Hele-Shaw model to verify the analytical results with good agreement. Amer, et al(1980) studied the steady-state, one dimensional flow in a coastal, leaky aquifer under different given inland conditions, aquifer geometry, and types of wedges (exterior wedge and interior wedge), but with no pumping wells. They developed an analytical model to represent the flow using a nonlinear set of differential equations. A linearization of the second type has been then utilized to solve these equations. They also developed a numerical solution to integrate them using fourth order Runge-Kotta method. These solutions were applied to a section in the Nile Delta aquifer. Analytical solution of the nonlinear differential equation describing the interface between flowing fresh ground water above stagnant saline water, in a semi-confined aquifer has been derived by Sikkema and Van Dam (1982). The resulting formulas were very complicated, even though no wells were included. Sherif et al (1988), developed a two dimensional (FE) model to simulate the SWI in confined and leaky coastal aquifers under steady state conditions. The governing equations are combined into two nonlinear coupled partial differential equations (PDE) in two variables , namely, the concentration and the equivalent fresh water piezometric head, in the vertical plane. No pumping from the aquifer was considered. The model is used to predict equi-concentration lines, and equi-potential lines in the aquifer. They assumed a mixed boundary at the seaward side consisting of zero concentration gradient at the upper part and a constant concentration, equal to that of the sea, at the lower one. Hassan(1988), developed a two-dimensional, unsteady-state areal (FE) model to solve the flow equation for both regional ground water flow and conservative solute transport in porous medium. The model was applied to a portion of the eastern Nile Delta leaky aquifer to simulate the problem of areal SWI. The model included effects of pumping from the aquifer on solute transport and dispersion. However, it cannot simulate the interface in the vertical direction since it uses vertically integrated values for the salt water concentration.

In the present work, a systematic analysis of the effects of pumped wells, as well as other parameters, on the SWI in a coastal, leaky aquifer is made. A quasi-3D, areal, FE model is developed for that purpose, in which the aquifer fresh water and the salt water piezometric heads are the two main unknowns.

The Governing Equations

For a three dimensional flow, the combination of Darcy's law equation with the continuity equation leads to a three dimensional system of equations, for the unknown piezometric heads ,h_f and h_s, in the fresh and salt water domains, respectively.

Vertical components of the velocity for flow within the leaky aquifer are usually very small and can be neglected (Depuit-Forchheimer assumption). The differential system of equations can then be integrated in the vertical direction resulting in two non-linear (2D or quasi-3D) partial differential equations in the X-Y plane (Bear and Verruijt, 1987). Permeability and storage coefficients are assumed to be constant along any vertical section. The top SPL is of a small thickness and relatively small permeability. Hence flow in the SPL can be assumed vertical. Considering a group of wells, fully penetrating the fresh water domain of the aquifer, at different locations (x_i,y_i), and of different abstractions, Q_{wi} (L^3T^{-1}), and for a sharp interface separating fresh and salt water domains, the following steady state equations are obtained:

(a) For fresh water domain:

$$\frac{\partial}{\partial x}C_1\frac{\partial h_f}{\partial x}+\frac{\partial}{\partial y}C_2\frac{\partial h_f}{\partial y}-\sum_{i=1}^{N_w}\delta(x-x_i)\,\delta(y-y_i)\,Q_{wi}+\frac{(k'_z)_f}{d}(h_g-h_f)=0 \qquad (1)$$

(b) For salt water domain:

$$\frac{\partial}{\partial x}C_3\frac{\partial h_s}{\partial x}+\frac{\partial}{\partial y}C_4\frac{\partial h_s}{\partial y}=0 \qquad (2)$$

in which

$$C_1=(k_x)_f\,b_f \qquad (3)$$
$$C_2=(k_y)_f\,b_f \qquad (4)$$
$$C_3=(k_x)_s\,b_s \qquad (5)$$
$$C_4=(k_y)_s\,b_s \qquad (6)$$
$$b_f=Z_T-Z_I \qquad (7)$$
$$b_s=Z_I-Z_B \qquad (8)$$
$$Z_I=(1+\delta_1)\,h_s-\delta_1\,h_f \qquad (9)$$

and

$$\delta_1=\rho_f/(\rho_s-\rho_f) \qquad (10)$$

Notations used in equations (1) through (10) are illustrated in Figure (1), k_x and k_y are values of hydraulic conductivity of the aquifer along the x and y directions (LT^{-1}); x and y are assumed to be parallel to the major axes of hydraulic conductivity; while k'_z is the vertical conductivity of the (SPL),(LT^{-1}), h_g is the ground water level in the SPL,(L); d is the saturated thickness of the SPL (L); h_f and h_s are the piezometric heads in the aquifer fresh water and salt water domains, respectively,(L); b_f and b_s are their corresponding saturated water thicknesses, respectively,(L); Z_I is the interface height above datum (L); $\delta(x-x_i)$ and $\delta(y-y_i)$ are the Dirac delta functions, (L^{-1}); Q_{wi} is the discharge of the "i" th well located at (x_i,y_i), positive for extracting wells and negative for recharging wells (L^3T^{-1}); N_w is the number of wells; ρ is the density of water,(ML^{-3}); and the subscripts (f) and (s) are notations for fresh water and salt water, respectively.

Boundary Conditions

Upstream (landward side) and downstream (seaward side) boundaries of the rectangular modelled area of dimensions X_m and Y_m, shown in Figure (1-b), represent constant-head surfaces.

(a) At the upstream boundary $(x=X_m,\ 0\le y\le Y_m)$:

$$h_f=h_f^{*} \qquad (11)$$

(b) At the downstream boundary $(x=0,\ 0\le y\le Y_m)$:

$$h_g=h_f=h_s=H_o \qquad (12)$$

(c) For symmetrical conditions, the other two boundaries will represent stream surfaces, hence:

$$\frac{\partial h_f}{\partial y}=\frac{\partial h_s}{\partial y}=0.0 \qquad (13)$$

for $(0\le x\le X_m)$, $y=0$ or $y=Y_m$

(d) With an impervious layer below the aquifer, its bottom will also represent a stream surface:

$$\frac{\partial h_f}{\partial n}=\frac{\partial h_s}{\partial n}=0.0 \qquad (14)$$

for $(0\le x\le X_m)$, $(0\le y\le Y_m)$, and $Z=Z_B^{\,o}+S_bx$

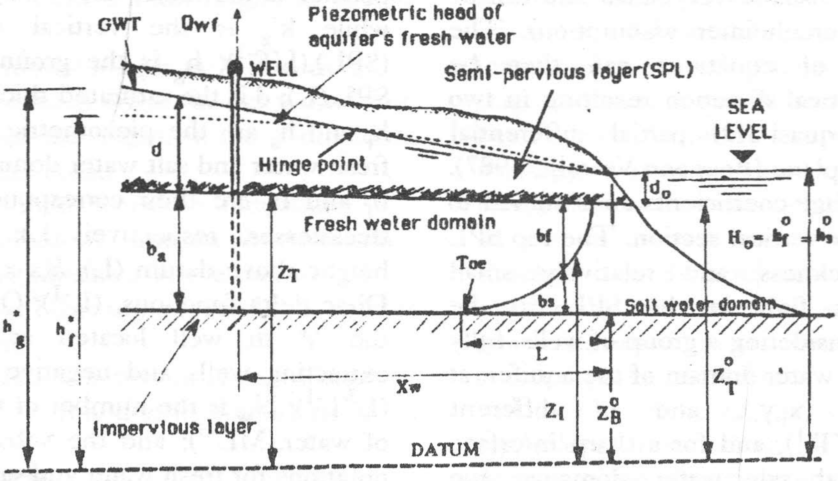


Figure 1-a. Longitudinal section A-A stream surface.

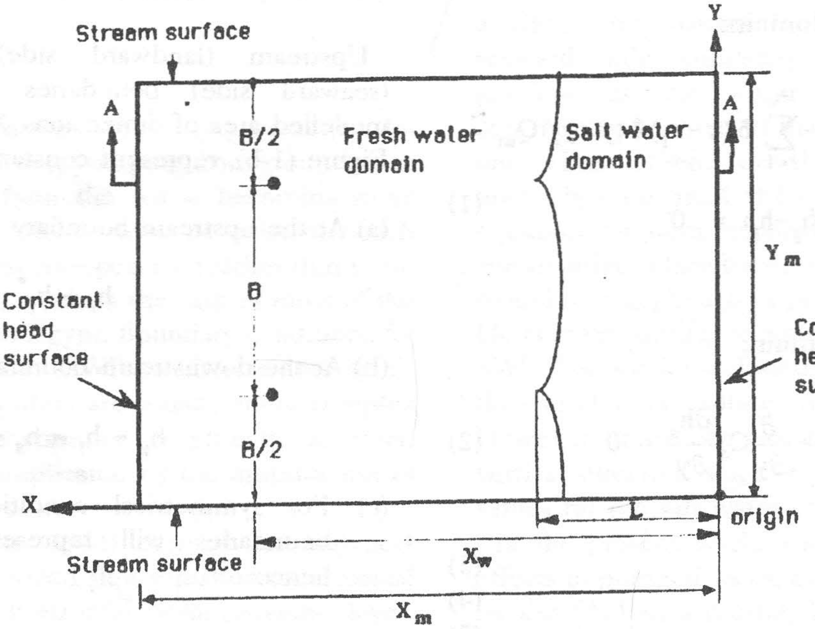


Fig. (1-b) **Plan**

Figure 1. Definition of different parameters for a leaky aquifer.

The aquifer top is a leaky surface, with a downstream flow (recharge), or an upward flow (abstraction), depending on the local values of h_f and h_g. This is represented by the last term in the L.H.S. of equation (1). The ground water table (GWT) levels within the SPL are known and can take any general shape.

Finite Element Equations

According to Galerkin's approximation, and by using Gauss divergence theorem and the integration of the divergence of the flux terms as developed by Warner (1987), the finite element algebraic equations equivalent to equations (1) and (2) are:

For fresh water domain:

$$[K_f]\{h_f\} + \{B_f\} = \{F_f\} \qquad (15)$$

For salt Water domain:

$$[K_s]\{h_s\} = \{F_s\} \qquad (16)$$

Where $[K_f]$ and $[K_s]$ are the element matrices given by Reddy(1993) and Warner(1987), $\{B_f\}$ is the source term vector in equation(15) and $\{F_f\}$ and $\{F_s\}$ are the boundary conditions vectors;

$$\begin{aligned}(K_{ij})_f = -\iint_A &(((C_1\frac{\partial N_i(x,y)}{\partial x})(\sum_{j=1}^{m} h_f(x,y)_j \frac{\partial h_f(x,y)_j}{\partial x})) + \\ &((C_2\frac{\partial N_i(x,y)}{\partial y})(\sum_{j=1}^{m} h_f(x,y)_j \frac{\partial h_f(x,y)_j}{\partial y}) \qquad (17) \\ &-\frac{k'_z}{b}(\sum_{j=1}^{m} h_f(x,y)_j N_i(x,y)))dxdy\end{aligned}$$

$$\begin{aligned}(K_{IJ})_s = -\iint_A &(((C_3\frac{\partial N_i(x,y)}{\partial x})(\sum_{j=1}^{m} h_s(x,y)_j \frac{\partial h_s(x,y)_j}{\partial x})) + \\ &((C_4\frac{\partial N_i(x,y)}{\partial y})(\sum_{j=1}^{m} h_s(x,y)_j \frac{\partial h_s(x,y)_j}{\partial y}))dxdy\end{aligned} \qquad (18)$$

$$\begin{aligned}(B_i)_f = -\iint_A &(N_i(x,y)\sum_{j=1}^{N_w} \delta(x-x_j)\ \delta(y-y_j) Q_{wfj}\,dxdy \\ &+\iint_A \frac{K'_z}{b} h_g\ N_i(x,y)\ dxdy\end{aligned} \qquad (19)$$

$$(F_i)_f = \int_B C_1 N_i(x,y)(\frac{\partial h_f}{\partial x})l_x + C_2 N_i(x,y)(\frac{\partial h_f}{\partial y})l_y \qquad (20)$$

$$(F_i)_s = \int_B C_3 N_i(x,y)(\frac{\partial h_s}{\partial x})l_x + C_4 N_i(x,y)(\frac{\partial h_s}{\partial y})l_y \qquad (21)$$

Where

m is the number of elements in the domain, and consequently the number of resulting equations; B is the domain surface boundary; A is the entire domain of calculations representing the flow; $N_i(x,y)$ is the shape function (linear interpolation function over the element); $(h_f(x,y))_j$ is true nodal value of the fresh water head; $h_s(x,y))_j$ is true nodal value of the salt water head and l_x,l_y are direction cosines at the boundary points in X and Y directions, respectively.

The Computer Program

The salt water intrusion(SWI) program, previously developed by the writers (El-Ganainy, et al,1995), for phreatic and for confined aquifers has been modified to include the current case of a leaky aquifer. Iteration schemes for the nonlinear coefficients are similar for both programs. To overcome the instability problems, the new value of the piezometric heads is taken as:

$$h^{new}\ (i) = 0.5[h^{cal}(i) + h^{old}(i)]$$

where; h^{new} is the new estimated value for the piezometric fresh or salt water heads in the iteration scheme, h^{old} is the old value, while h^{cal} is the calculated value due to substitution with h^{old} in the last iteration. The notation (i) denotes the node i in the modeled flow field. This method slows down conversion of the iteration scheme, but assures the stability of the solution. The elements at the interface region have smaller dimensions to improve the accuracy of solution in this region.

The input data for the new program are element topology, nodal coordinates, permeability coefficients, locations and discharges of wells, geometrical data of the aquifer, piezometric head of the aquifer fresh water at the upstream boundary, sea water level and GWT levels in the SPL for every node. The output of the program are values of piezometric heads of fresh and salt water at nodal points, interface height, intrusion length and hinge point location. The program has an option to give the flow balance between any two subsequent vertical sections through out the aquifer.

Verification of the model:

To verify the developed model, comparisons are made and presented in graphical forms for two test problems:

(1) A rather simple problem of a leaky aquifer, with a constant fresh water heads, h_f^* and H_o, at the

upstream and the downstream boundaries, respectively, Figure (2-a), i.e. no salt water calculations are involved. The SPL itself is subjected to a constant head, $h_g^* = h_f^*$, which is equivalent to a horizontal GWT. For the values of parameters shown in Figure (2-a), aquifer fresh water piezometric heads computed by the developed model are illustrated in Figure (2-b) along with results of Hassan's (1988) FEM model and Bennett's (1946) analytical solution. The results of the three solutions are identical.

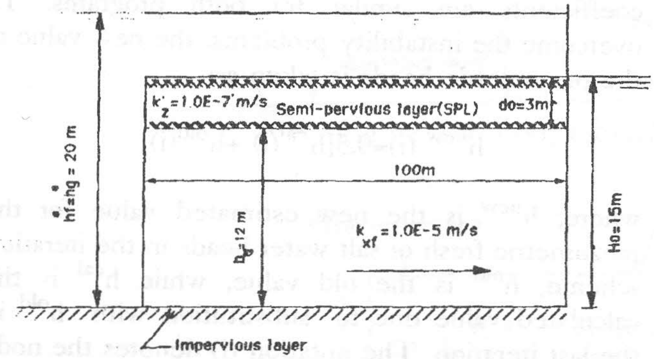


Figure 2-a. Parameters involved in the model.

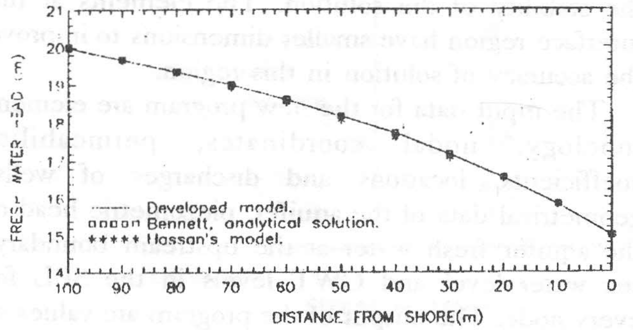


Figure 2-b. Comparison of computed piezometric heads with other solutions for the problem shown in Figure (2-a).

(2) Results of the developed model have been also compared with those of Amer's et al (1980) analytical solution of the linearized flow equations model for the problem shown in Figure (3-a). Values of different parameters used in the comparison are also shown in the same figure. The GWT in the SPL is assumed to vary linearly in both models whereas the piezometric heads of the aquifer are computed. Locations and shapes of the interface for two different values of vertical permeability coefficients, k'_z, of the SPL are presented in Figure (3-b). The resulting toe locations are very close for the two solutions but the interface shapes are slightly different which may be due to the linearization approach used in Amer's solution.

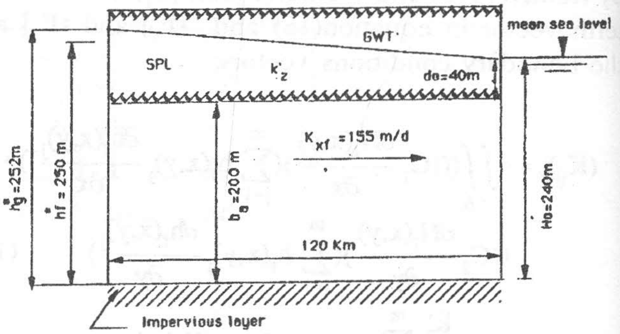


Figure 3-a. Parameters used in the comparison.

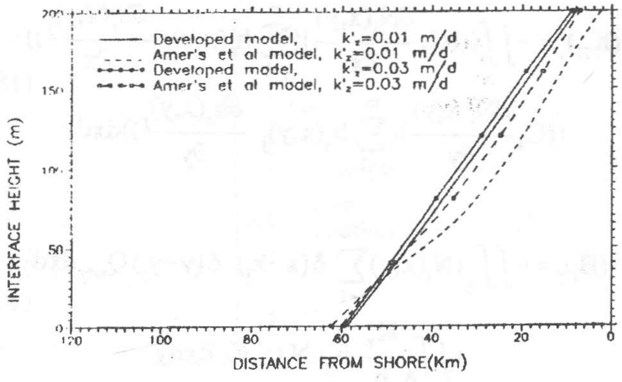


Figure 3-b. Comparison of computed interface location and shape with Amer's et al (1980) analytical solution for different values of k'_z.

RESULTS AND DISCUSSION

The physical model dimensions are shown in Figure (4), with $Y_m = X_m = 10.0$ Km. Two different wells' arrangements are studied in addition to the case of no wells, Figure (4-b). The shallow water surface in the SPL (the GWT) is assumed to vary linearly from h_g^* at the upstream (the landward side) to H_o at the downstream (the seaward side). The developed model has been used to study effects of the different parameters on the intrusion length, the interface shape and location of the hinge point (the vertical section at which the direction of leaking flow from the SPL to the aquifer is reversed). The considered parameters are:

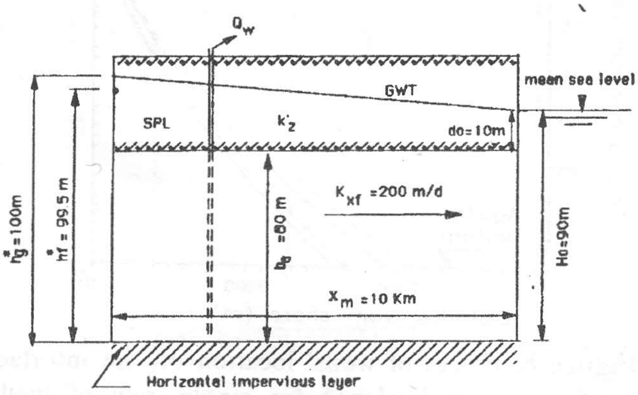


Fig.(4-a) Longitudinal section C-C

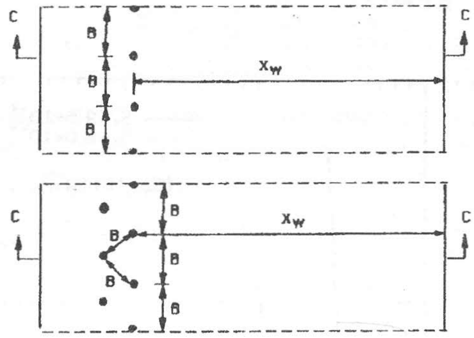


Figure 4-b. Plan of different studied wells' arrangements

(i) Single row, uniformly spaced, B/ba=2.5 and 6.25
(ii) Double row, stagered, B/ba=2.5, 6.25 and 12.5.

Figure 4. Physical model parameters used in the analysis.

(1) Trapping ratio, TR, which is defined as $TR = N_w Q_w / Q_1$, where $(N_w Q_w)$ represents the total discharge abstracted by N_w of similar wells.

(2) Wells' locations, X_w/L_1, where X_w is the distance of the nearest row of wells from the Y-axis, and L_1 is the intrusion length for the case of no wells.

(3) Flow conductance ratio, KR, which is defined as: $KR=(k'_z.X_m)/(k_x.d_{av})$, where d_{av} is the average saturated thickness of the SPL.

About sixty runs of the developed model are carried out. Results of these runs are presented in graphical forms, Figures (5 through 10). Analysis of such results are presented below.

Figure (5) shows the variation of the relative intrusion length, L/L_1, with the trapping ratio, TR. Intrusion length increases, with a decreasing rate, when more fresh water is trapped by the wells. Upconing starts at TR=0.75, approximately.

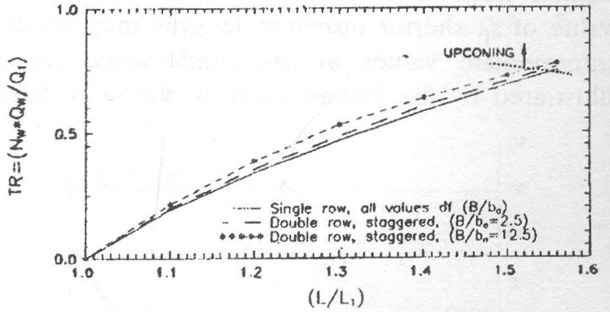


Figure 5. Effect of trapping ratio on the intrusion length (X_w/L_1 = 1.5, KR=0.1).

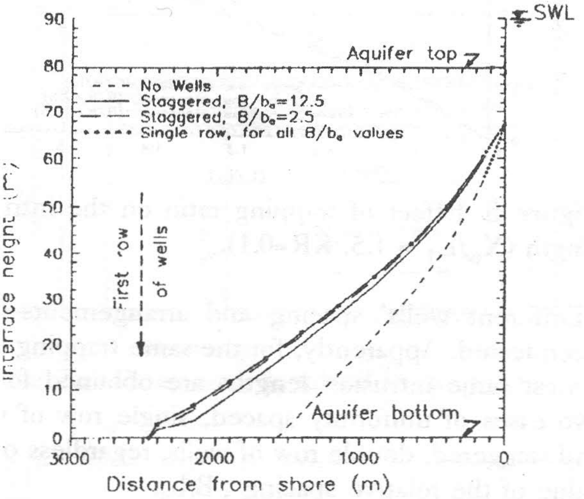
Different wells' spacing and arrangements have been tested. Apparently, for the same trapping ratio, almost same intrusion lengths are obtained for the two cases of uniformly spaced, single row of wells and staggered, double row of wells, regardless of the value of the relative spacing , B/b_a.

Figure (6) illustrates the interface surfaces corresponding to a trapping ratio TR=0.75, for different arrangements of wells, compared to that of no wells. It is to be noted that, within the range of the used data, the tip top points of these surfaces, determined by their general shapes, lie below the aquifer top i.e. exterior rather than interior downstream wedges will exist for the interface surfaces. Analyses of the aquifer fresh water balance for the modeled flow field supports this conclusion.

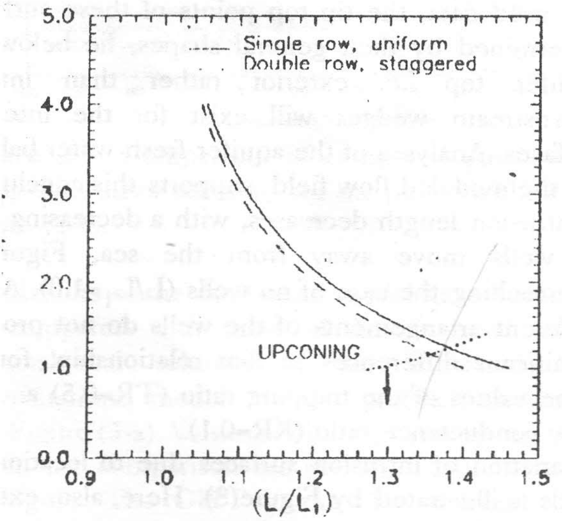
Intrusion length decreases, with a decreasing rate, as wells move away from the sea, Figure(7), approaching the case of no wells (L/L_1=1.0). Again, different arrangements of the wells do not produce significant differences in that relationship, for the same values of the trapping ratio (TR=0.5) and the flow conductance ratio (KR=0.1).

Variation of intrusion surfaces due to locations of wells is illustrated by Figure(8). Here, also, exterior wedge exists at the sea side.

Referring to figure (9), it is clear that the flow ductance ratio has no appreciable influence on intrusion length. This is particularly true when average slope of the aquifer fresh water zometric surface, $s_f=(h_f^*-H_o)/X_m$, is very close to average GWT slope in the SPL, $s_g=(h_g^*-H_o)/X_m$, hown by the solid curve, Figure (9). Effect of the ductance ratio is, therefore, directly related to the ntity of the leaking discharge, relative to the ifer flow. It is also noted that, for the smaller ıe of s_f, shorter intrusion lengths may result for rmediate values of the conductance ratio, as ıtrated by the dashed curve in the same figure.



gure 6. Interface surface (vertical section) for ferent wells' arrangements (TR=0.75, KR=0.1, $_w/L_1$=1.5)



gure 7. Effect of well's location on the rusion length (TR=0.5, KR=0.1, B/B_a=6.25).

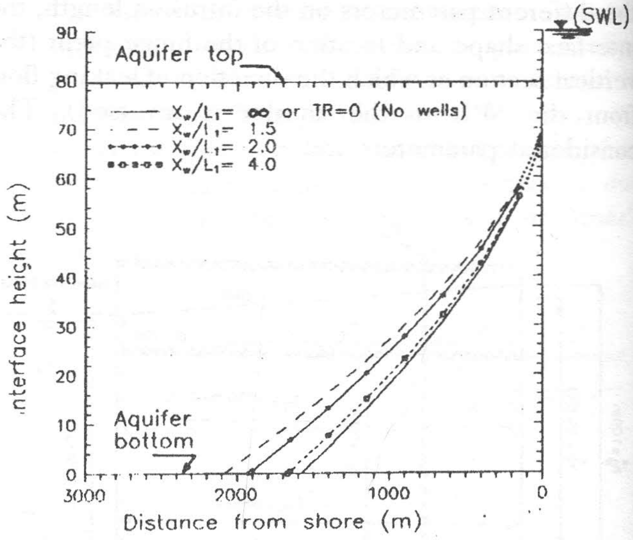


Figure 8. Effect of wells' location on the interface surafces (vertical plane) for single row of wells. (Tr=0.5, KP=0.1).

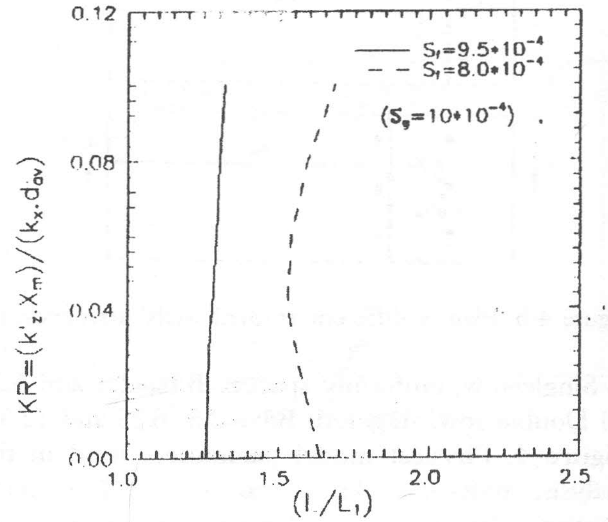


Figure 9. Effect of the flow conductance ratio on the intrusion length (TR=0.5, X_w/L_1=1.5, single row of wells).

Figure (10) demonstrates the sensitivity of the hinge point location due to variation of four different parameters: (X_w/L_1), TR, KR, and (s_g-s_f). The hinge point moves away from the coast, with a decreasing rate, as wells' locations become farther, approaching the no-wells case. On the other hand, increasing the trapping ratio will push the hinge point towards the coast, at a nearly constant rate. Conductance ratio, however, has a negligible effect on that location. The hinge point is shifted farther towards the landward side when the difference (s_g-s_f) increases.

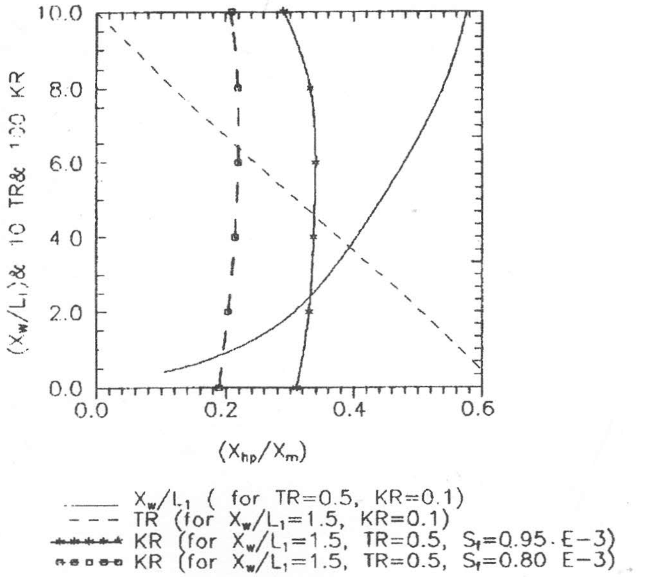


Figure 10. Variation of the hinge-point location with wells' locations, trapping ratio and conductance ratio. (for single row of wells, and $s_g=10*10^{-4}$).

CONCLUSIONS

The developed, quasi-3D, FEM model simulates the sea water intrusion conditions resulting from pumping through a group of wells, for a leaky aquifer. Its results have been compared to and verified with other analytical and numerical models. Analysis of the current results show that:

1- Arrangement of wells has a minor influence on the intrusion length and interface shape, as long as their locations from the coast and the total pumped discharge are kept constant.

2- As wells become closer to the coast, intrusion length increases, with an increasing rate. Upconing will, of course, occur for distances below certain limits, depending on the boundary conditions, aquifer physical parameters, and pumping rate.

3- Importance of relative values of hydraulic conductivities of the aquifer and the SPL, as well as their physical dimensions, represented here by the flow conductance ratio, is directly related to other boundary conditions that affect the leaking discharge.

4- For the parameters ranges used in this study, the resulting location of the tip-top point of the interface creates an exterior, not interior wedge i.e. an outlet face exists for the aquifer fresh water below its top level. This conclusion is in agreement with Kashef (1983).

REFERENCES

[1] A. Amer, M. Hashish and M. Rasmy, "One-dimensional steady state sea-water intrusion in coastal aquifers.", *MIT/Cairo University technological planning program*, in cooperation with the Ministry of Irrigation, Cairo, 1980.

[2] J. Bear and A. Verruijt, *Modeling Groundwater flow and pollution.*, Reiddel Publishing Company, Dordrecht, Holland, pp. 414, 1987.

[3] J.C. Bennett "The effect of blankets on seepage through pervious foundations" *ASCE*, No. 2270, 1946.

[4] M. El-Ganainy et al,"Effects of pumped wells on salt water intrusion in coastal aquifers.", *Alexandria Engineering J., Jan.* 1995.

[5] A. Hassan, "Management of wadis in arid regions with application to quality models.", *Ph.D. Thesis*, Faculty of Engineering, Ain Shams University, Cairo, Egypt, (1988).

[6] A.I. Kashef, "Salt-water intrusion in the Nile Delta", *Ground Water*, vol. 21, No.2, pp. 160-167, 1983.

[7] Y. Mualem and J. Bear, "The shape of the interface in steady flow in a stratified aquifer", *Water Resources Research*, 10(6): 1207-1215, 1974.

[8] J.N. Reddy, *An introduction to the finite element method*, McGraw-Hill Inc., pp. 684, 1993.

[9] M.M. Sherif V.P. Singh and A.M. Amer "A Two-dimensional finite element model for dispersion in coastal aquifers.", *J. of Hydrology*, vol. 103, pp. 11-36, 1988.

[10] P.C. Sikkema and J.C. Van Dam, "Analytical formulae for the shape of the interface in a semi-confined aquifer", *J. of Hydrology*, vol.56, pp. 201-220, 1982.

[11] J.W. Warner, *Mathematical development of the Colorado State University finite two dimensional ground water flow model*, CSU-GWFLOW, VERSION FEM2D3.1", Groundwater technical report No.2, Ground water program, Department of Civil Engineering, Fort Collins, pp. 92, 1987.