EFFECT OF LOAD POSITION ON THE ELASTIC LATERAL BUCKLING MOMENT OF STEEL COPED BEAMS

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ABSTRACT

The elastic lateral buckling moment capacity of simple beams, having copes at both ends and loaded at various positions on the same cross section, is studied. The critical case of a single concentrated load acting at mid-span of the simply supported beam is considered. The results show great influence of loading at top flange for the coped beams specially for short spans where the plastic moment capacity is critically affected. The general equation for coped beams with a load acting at top flange is presented and many solved examples are used to introduce the curves showing the behavior for different cross sections and spans.

Keywords: Lateral buc ,ıg, Coped beams, Steel-beams, Load position.

INTRODUCTION

Connections between steel elements are of crucial importance to the structural behavior of the whole system. This fact is due to the fancy and accurate methods used in both designing and constructing steel structures. While two elements are meeting at a point, the method used to connect them may lead to a roller, a hinged, or a fixed end for one of them relative to the other.

In some situations modifications have to be performed in order to construct the structure, one of these cases is cutting a part of the flange to allow connecting a beam with a larger, or same size, girder. This cutout which is widely used in simple connections is called a cope as shown in Figure (1). This cope is an important factor which controls the lateral-torsional buckling behavior of pin-pin simple beams and changes the problem from the case considered by Timoshenko (1) where the differential equation of a beam with two rotation prevented ends was investigated to that studied by du Plessis (2) where the effect of end-notches on the lateral-torsional buckling of beams had been examined.

In the case of uncoped beams the, most important, compression flange is assumed restrained against lateral movement at both ends, while for the case of coped ones this significant factor is not satisfied and the two ends are relatively free to move depending on the lateral torsional stiffness of the Tee section remaining beneath the cope. The reduction in beam elastic torsional buckling moment capacity may easily shrink and lose more than 90% in some cases specially for short beams with large cope length and width. This effect was studied by Gupta (3) and Cheng (4,5).

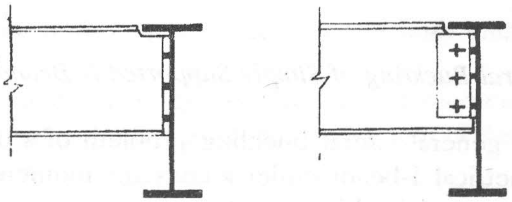

Figure 1. Examples for coped beam connections.

On the other hand, the problem of elastic lateral buckling of beams and the effect of the load position application was studied by other researchers as Nethercot (6). It was found that there is a considerable effect of the application point of the load on the overall lateral buckling of the beam. Figure (2) represents an easy to follow example for the reason of this phenomenon and how the load position is decreasing, or increasing, the stabilizing influence of an additional torque. This indirect torque leads to a substantial decrease, or increase, in the lateral torsional buckling capacity of beams, which may result in a more critical case than that of a constant moment acting along the entire span.

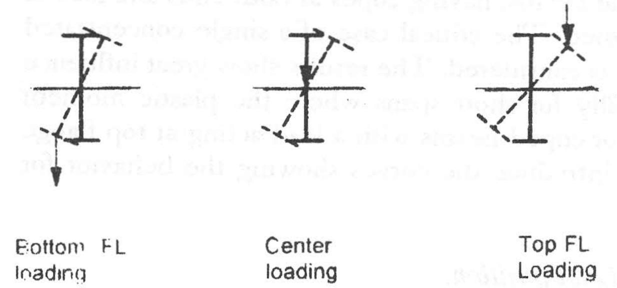

Figure 2. Effect of load position on the lateral stability of I-sections.

In the presented work, the combination of these two factors is introduced and many examples are solved to show the behavior of the beam under different load positions and end conditions. The reason of this study is to drag the attention of the designers to the importance of detailing their designs and how many factors may reduce , dramatically, the performance of their designs and shift them to real far away regions of the factor of safety they have in mind.

Lateral Buckling of Simply Supported I- Beams

The general lateral buckling problem of a doubly symmetrical I-beam under a constant moment may be illustrated by Figure (3).
The differential equations for this case will be:

$$EI_y \frac{d^2u}{dz^2} = M\varphi \qquad \text{(1-a)}$$

$$GJ\frac{d\varphi}{dz} - EC_w\frac{d^3\varphi}{dz^3} = \frac{du}{dz} \qquad \text{(1-b)}$$

Where E is the modulus of elasticity of the beam material, I_y is the moment of inertia of the beam section about its Y axis, x, y, and z are the considered coordinate axes shown in figure 3, u is the lateral deformation of the shear center, ϕ is the angle of twist, G is the shear modulus of elasticity, J is the torsion section constant, and C_w is the warping section modulus.
Considering the I-section shown in figure 3, the following approximate expressions for I_y, J, and C_w may be used:

$$I_y = \frac{t\,b^3}{6} \qquad \text{(2-a)}$$

$$J = \frac{2bt^3 + (d-t)t_w^{\ 3}}{3} \qquad \text{(2-b)}$$

$$C_w = \frac{I_y(d-t)^2}{4} \qquad \text{(2-c)}$$

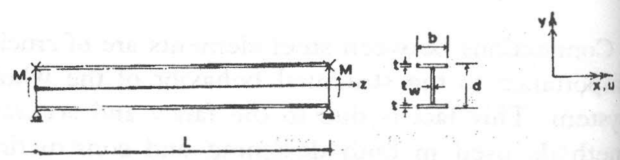

Figure 3. A simple span beam subjected to a constant moment.

Allowing the two ends of the beam to warp while preventing them from rotation, which is the case of two braced beam ends, and considering the two beam ends to be simply supported, the following boundary conditions are stated:

$$u_{(0)} = u_{(l)} = \varphi_{(0)} = \varphi_{(l)} = 0 \qquad \text{(3-a)}$$

$$\left(\frac{d^2u}{dz^2}\right)_{(0)} = \left(\frac{d^2u}{dz^2}\right)_{(l)} = \left(\frac{d^2\varphi}{dz^2}\right)_{(0)} = \left(\frac{d^2\varphi}{dz^2}\right)_{(l)} = 0 \qquad \text{(3-b)}$$

Using the boundary conditions of equation 3 with the differential equation 1, leads to the following expression for the moment required to cause elastic lateral buckling (M_{LTB}):

$$M_{LTB} = \frac{\pi}{L}\sqrt{EI_yGJ + \frac{\pi^2E^2I_yC_w}{L^2}} \qquad \text{(4)}$$

For the case of non constant moment, this equation of M_{LTB} is modified to:

$$M_{LTB} = C_b \frac{\pi}{L} \sqrt{EI_y GJ + \frac{\pi^2 E^2 I_y C_w}{L^2}} \quad (5)$$

Where:

C_b is the equivalent uniform moment factor which accounts for any case of acting moment. Values for this factor may be found for all cases of loading in texts and codes dealing with designing steel structures.

The transverse loads are generally considered acting at the level of beam centroid, regardless the load is free to move with the beam section while it is deforming, or not. If the load does not move with the beam as it buckles there will be no effect for the position of load application. On the other hand, if the load is moving with the beam section, there will be a great effect on the ·ability of the section as shown before in Figure). The equivalent uniform moment factor is modified by Nethercot (6) to be C_{b*} which may be represented as follows:

$C_{b*} = C_b / n$ for the case of load at top flange and

$C_{b*} = C_b * n$ for the case of load at bottom flange.

Where:

n is a factor depending on the type of loading. For the most critical case of a mid-span loaded simple beam, n is considered as follows:

$$n = 1.000 - 0.180K^2 + 0.649K \quad (6)$$

where

$$K = \sqrt{\frac{\pi^2 \, E \, C_w}{L^2 \, G \, J}} \quad (7)$$

To illustrate the effect of load position on the torsional buckling moment capacity, two I-beams are analyzed for the three load positions shown in Figure (3) and the results are shown in Figure (4). These two examples show how important the level of load application is, and how it may reduce the lateral buckling moment of beams.

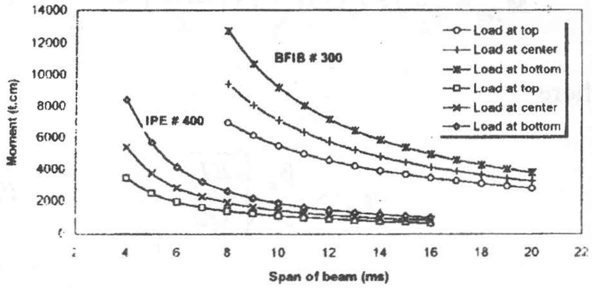

Figure 4. Effect of load position on the lateral buckling moment of a centrally loaded simple span.

Lateral Buckling of Simply Supported Coped Beams

For the coped beam shown in Figure (5), some practical limitations are considered. The cope depth, dc, is not exceeding 0.2 d and the cope length, c, is not exceeding 0.5 d where d is the beam depth. Cheng and Yura (4,5) proposed a method to deal with the coped beams, loaded at shear center, based on equating the lateral buckling moment (M_{LTB}) of the uncoped segment with that of the coped T-section region (M_{Tee}).

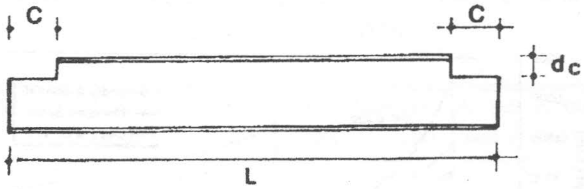

Figure 5. A typical coped beam at both ends.

The condition of getting the critical moment (M_{cr}) controlling the lateral buckling moment of the coped beam is:

$$\frac{1}{M_{LTB}} + \frac{1}{\frac{LM_{Tee}}{c}} = \frac{1}{M_{cr}} \quad (8)$$

where the lateral buckling moment of the Tee coped section, M_{Tee}, is represented as:

$$M_{Tee} = \frac{\pi}{2c}\sqrt{EI_yGJ}\left[\sqrt{1+\left(\frac{\pi\gamma_m}{2}\right)^2}+\frac{\pi\gamma_m}{2}\right] \quad (9\text{-a})$$

where

$$\gamma_m = \frac{\beta_x}{2c}\sqrt{\frac{EI_y}{GJ}} \quad (9\text{-b})$$

$$\beta_x = -\left[\frac{1}{I_x}\left[\frac{t_w}{4}[(h_o-y^*)^4 - (y^*-t)^4] - \left(y^*-\frac{t}{2}\right)\left[\frac{b^3t}{12}+bt\left(y^*-\frac{t}{2}\right)^2\right]\right]+2y^*-t\right] \quad (9\text{-c})$$

where:

y^* is the distance between the neutral axis and the extreme fiber of the Tee section flange. Figure (6) shows the effect of the cope length and width on the lateral buckling of I-beam. The reduction of moment capacity of coped beams is clear specially for short beams, where the coped T section is controlling the lateral buckling moment. While for long spans where the capacity of the uncoped region is controlling the behavior, there is a much less difference between the two moments.

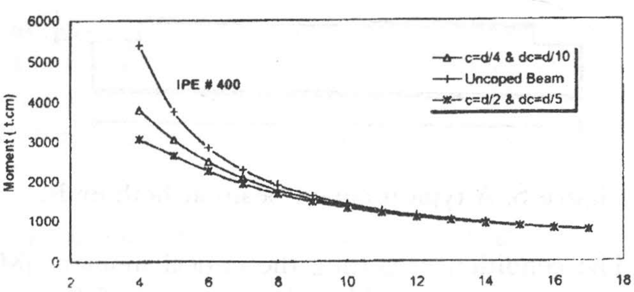

Figure 6. Effect of coping on the lateral torsional buckling moment of a centrally loaded simple span.

Effect of Load Level on Coped Beams

The combination between the coping and top flange loading is considered due to the fact that this is the most critical case affecting the behavior of the simple beam as shown in Figures (4) and (6). These two factors, load position and flange coping, are expected to change the behavior of any loaded beam. The most important points to be investigated are how far is the modified lateral buckling moment of the top flange loaded coped beam from that of the shear center loaded uncoped one and the factor of safety used in the design. For a simply supported beam, mid-span loaded, and coped at depth of 0.2 d and length of 0.5 d. Equation 8 is modified to be:

$$\frac{1}{M^*_{LTB}} + \frac{1}{\frac{LM_{Tee}}{c}} = \frac{1}{M_{cr}} \quad (10$$

where

$$M^*_{LTB} = C^*_b\frac{\pi}{L}\sqrt{EI_yGJ+\frac{\pi^2E^2I_yC_w}{L^2}} \quad (11$$

$$C^*_b = \frac{C_b}{n} = \frac{C_b}{1.0-0.18K^2+0.649K} \quad (12$$

and K is calculated using equation 7.

Numerous beams are investigated specially IPI sections, where web depth to thickness ratio i relatively high. For each beam the modified and th original lateral buckling moments (M_{LTB*} and M_{LTE} are calculated for different values of span, L. Th variation between the two values is of great interes because the beam, which is assumed to be stabl and capable of carrying loads till the plastic momen is achieved, practices a case of elastic lateral bucklin at an earlier stage of loading as shown in Figure (7 The beam which is thought to sustain plasti moment for a specific span till the behavior i controlled by elastic lateral buckling is changing it behavior at an earlier stage reducing that dependabl length significantly. In other wards, designing beam based on its plastic moment capacity may b questionable specially for short beams with copes a their ends and loaded at their top flanges.

The following example shows how important i considering the two factors. Two I beams ar considered, an IPE section and a BFIB section t represent the cases of different high and low we height to thickness ratios.

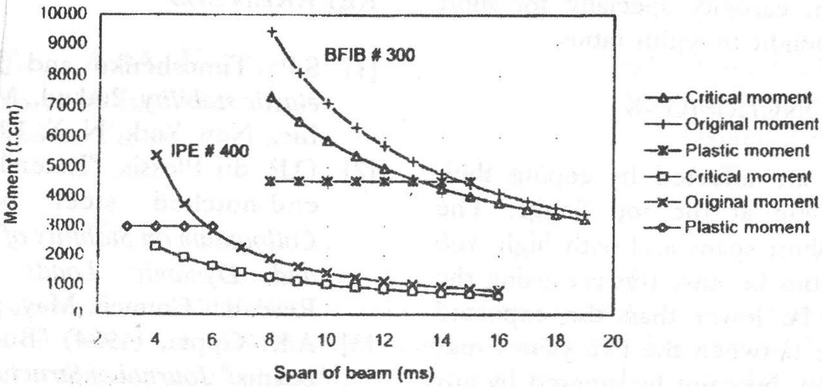

Figure 7. Effect of top flange loading and beam coping on the moment capacity.

Example:

A BFIB 300 and an IPE 400 are considered as simple beams loaded mid-span with a single concentrated load as shown in figure 8.

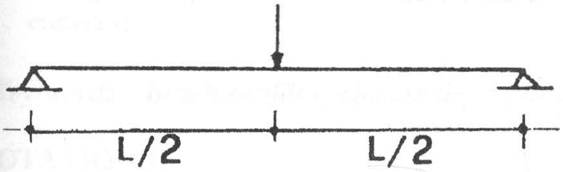

Figure 8. A simple span beam loaded at its mid-length.

The plastic moment is calculated for each case with the section moment capacity based on considering copes at ends, top flange loading, and the combination of the two cases. The results are given in tables 1 and 2. All calculations are based on the factors used in equation 2 to get the required section properties. The copes are assumed to be of 0.2 d in height and 0.5 d in length. Table 1 gives the plastic moment for each profile and the controlling span at which the ordinary equation of lateral buckling moment (equation 5) is used to get the moment capacity of the section. The elastic buckling moment capacity is calculated based on considering end copes, top flange loading, and the combination of these two modes for each of the sections at these controlling spans.

Table 1. Plastic Moments and Maximum Controlling Spans.

Section	Plastic moment Mp (t.cm)	Length to sustain Mp (cm)
BFIB # 300	4519	1613
IPE # 400	2972	582

Table 2. Effect of Coping, Top Loading, and Their Combination.

Section	Coped beam moment (t.cm)	Top loading moment (t.cm)	Copes & top load (t.cm)
BFIB 300	4452	3810	3762
IPE 400	2308	2028	1691

Table 2 indicates that while the two profiles are thought to fail by plastic moment at specific spans, the real capacity of the section at these spans are controlled by elastic lateral buckling behavior at lower moment values. IPE 400 which is thought to reach the plastic moment of 2972 t.cm at a span of 582 cm is failing by an elastic moment of 1691 t.cm at this span. The reduction, in this case of IPE, is about 75% which may not be ignored. However, the difference for the BFIB is only about 20% which is, also, a high value. The above example gives an indication that top flange loading combined with coping the beam at its ends is, seriously, reducing the I section moment capacity specially for short spans and high web height to width ratios.

SUMMARY AND CONCLUSIONS

The simple beams are affected by coping their ends and loading them at the top flange. The problem is clear for short spans and with high web height to thickness ratios because this is causing the moment capacity to be lower than the expected value. The difference between the two values may reach about 75% which may not be ignored by any means and may, easily, shift the designed section to an undesired domain.

In order to overcome this problem the coped end must be fully detailed so that the, relatively free to move, flange end must be restrained. Bracing elements or a vertical stiffener combined with a horizontal one at the borders of the cope may be used. If the situation does not allow for any modification to the cope, the cope effect must be considered. The position of the loading is to be carefully examined. If the load is moving with the section while it is deforming, the effect of load position must also be considered.

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