BEHAVIOUR OF A FLEXIBLE SURFACE FOOTING LOCATED ABOVE A CAVITY

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ABSTRACT

The paper presents the influence of creating a continuous cavity underneath a strip surface flexible footing, on the bearing capacity and settlement of that footing. The analysis is made by using the finite element method in which the soil was idealized as an elastic perfectly plastic material. The results indicated that for each footing-cavity configuration, there exists a critical depth below which the presence of the cavity has negligible influence on the footing performance. When the cavity is created within the critical depth, the bearing capacity and settlement of the footing above were found to be function in various factors such as the size and depth of the cavity in respect with footing size. The paper presents a data base which is useful for predicting the changes in bearing capacity and settlement of a such footing.

Keywords : Surface footing, Flexible, Cavity, Bearing capacity, and settlement.

NOTATION

P	applied load on the footing.
A	footing area.
B	footing width.
b	cavity width.
c_u	undrained soil shear strength.
N	load ratio ($N=P/Ac_u$).
P_u	ultimate bearing load of a footing.
N_c	bearing capacity factor ($N_c=P_u/Ac_u$).
γ	unit weight of soil.
Z	cavity embedment depth.
Z/B	cavity relative embedment depth.
$(Z/B)_{cr}$	critical cavity relative embedment depth.
p_{uc}	ultimate bearing load of a footing located above a cavity.
p_{uo}	ultimate bearing load of a footing without a cavity underneath.
p_r	stress level ratio (p_r = applied load on a footing / ultimate bearing load of that footing without cavity underneath).
S_c	settlement of a footing located above a cavity.
S_o	settlement of a footing without a cavity underneath.
C_r	bearing capacity reduction factor ($C_r=p_{uc}/p_{uo}$).
S_i	settlement ratio ($S_i=S_c/S_o$).
S_r	stiffness of the footing-soil system.

INTRODUCTION

In general, cavities created under existing structures resulting from mining, tunnelling or solution cavity in a soluble rock may lead to a serious structural damage specially when the cavity is created at a small depth below the foundation level, since the creation of a cavity may cause weakening of soil support and consequently settlement and differential settlement would occurr which results, of course, additional bending moments and shearing forces in the foundations as well as in the superstructures of the existing buildings. The geotechnical engineers aware of constructing on soil contains cavities or on soil which has the ability of cavities to be created in it. The available studies of the problem of the influence of a void underneath a footing are scare in the literature, for instance [1-7]. Atkinson and Cairncross [1] investigated the problem of the stability of a shallow tunnel in a weightless soil to which a uniform surface surcharge pressure was applied . Atkinson and potts [2] presented an investigation for predicting the radial applied pressures to be supported by compressed air, bentonite slurry, a shield or by other means in order to achieve the stability of a cavity. Also a number of model tests to examine the behaviour of shallow tunnels in sand have been carried out , Atkinson et al [3]. The purpose of these tests was to illustrate the way in which soil around a circular cavity deforms as the fluid pressure within the cavity is reduced since this approximates to the stress condition within a tunnel during construction. Baus and Wang [4] presented an experimental and theoretical study on the problem of a continuous strip footing located above a void. The experimental tests were carried out in a tank filled with a compacted silty clay. The experimental results were compared with those predicted using the finite element method. The results indicated that there is a critical depth above which the existence of the cavity causes a substantial decrease in its bearing capacity. Boscardin and Cording [5] proposed a procedure for predicting the tolerance of a brick-bearing wall and small frame structures to the ground displacement that may develop resulting from tunnelling and open cutting. Building orientation and building location in respect to excavation are also included. The authors presented a historical case to verify their study. Abdel-Salam [6] presents experimental and theoretical study to investigate the influence of loss of support on the behaviour of a strip footing. Photo-elasticity and finite element techniques were used. It was concluded that the contact pressure distribution was non uniform and proportional directly with the gap size and subgrade stiffness. The effect of soil type on the distribution of contact pressure was found to be less pronounced than the effect of footing rigidity and gap size. Hsieh [7] studied the problem of a strip footing centered above continuous circular void using the upper bound theorem of limit analysis. Equations were presented for predicting the collapse footing pressure as a function of soil type and different void conditions, however these equations are very complex so that computation of collapse footing pressure requires a numerical analysis through a nonlinear programming. However no generally accepted design method is presented to deal with the performance of the footing once a cavity is created underneath. Thus the main aim of this research is to investigate the changes in bearing capacity and settlement of a footing when a cavity is created underneath.

METHOD OF ANALYSIS

The problem was idealized using the finite element technique in which nonlinearity was introduced by the plasticity and accounted for using elastic-perfectly plastic model, Zienkiewicz [8]. The finite element formulation is presented elsewhere, Zienkiewicz [8] in which beyond the elastic range, the soil is assumed to behave as perfectly plastic in accordance with Tresca's criterion.

BOUNDARY CONDITIONS

The layout of the problem is shown in Figure (1), whereas the idealization of the problem is shown in Figure (2) in which half of the problem was considered, owing to symmetry. The boundary conditions are shown in Figure (2). The loads were applied incrementally through two stages. The first stage was done by application of equivalent own weigh forces (i.e body forces), whereas the second stage was done by application of a uniform surface stress, up to failure, along the contact surface between the footing and the soil mass (i.e flexible footing). A Fortran computer program was written to deal with this problem.

DISCUSSION OF RESULTS

The analysis was carried out on a surface flexible footing resting on a clayey soil with and without cavity underneath. Thus the influence of the cavity on the footing behaviour can be, easily, extracted. For the sake of comparison the bearing capacity reduction factor C_r and the settlement factor S_i were introduced and defined as,

$$C_r = P_{uc} / P_{uo} \qquad (1)$$

and

$$S_i = S_c / S_o \qquad (2)$$

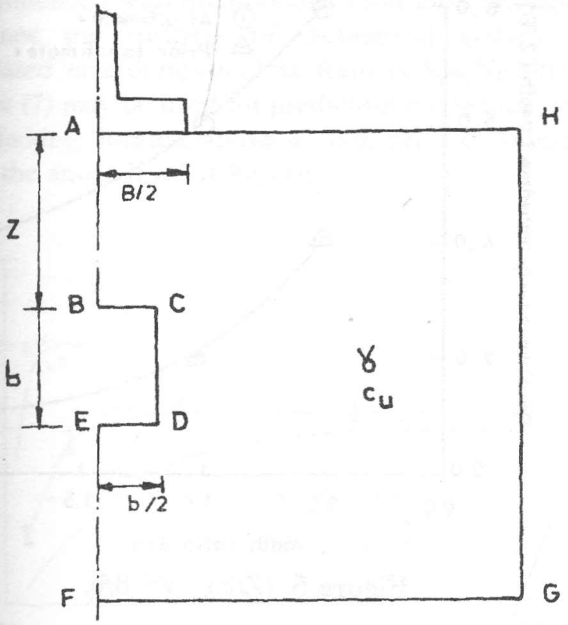

Figure 1. Physical problem.

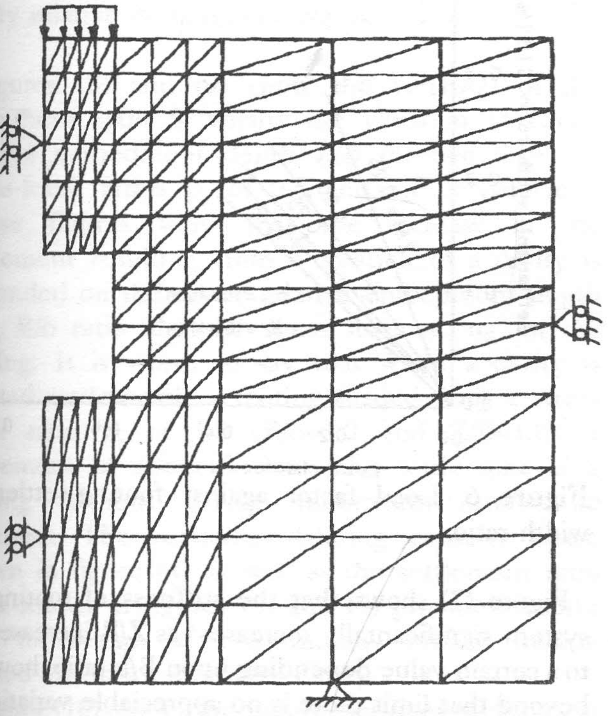

Figure 2. Idealization of the problem.

Figure (3) demonstrates a typical relation between the load ratio N against the footing settlement-width ratio S/B with different cavity relative embedment depths of 0.25, 0.5, 0.75, 1.0, 2.0, 4.0 and 6.0 respectively. The results of the footing without cavity underneath is denoted as Z/B=∞ and plotted on the figure as a dashed line. The figure shows that the load-displacement behaviour of the footing comprises three parts. The relation starts with a straight line in which the load is proportional to the footing settlement. This straight line is followed by a curve then ended by another straight line. The intersection point of these two straight lines was used for predicting both the bearing capacity factor N_c and the footing settlement ratio S/B at ultimate condition.

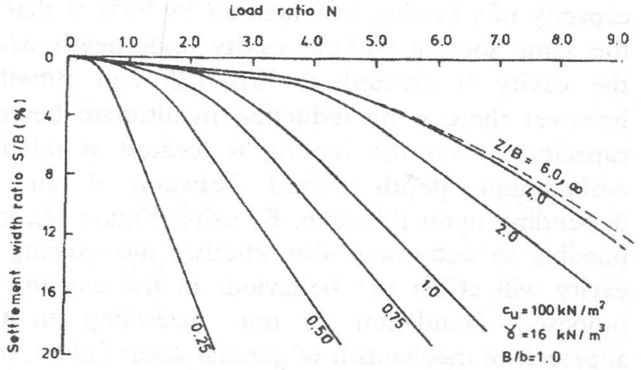

Figure 3. Load ratio against footing-settlement width ratio.

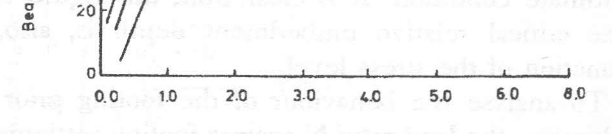

Figure 4. Bearing capacity reduction factor against cavity relative embedment depth.

Figure (4) shows the variation of the bearing capacity reduction factor C_r against the variation of cavity relative embedment depth Z/B for different values of B/b ratios of 0.5, 1.0 and 2.0 respectively. Figure (4) demonstrates the effect of existing of cavity on bearing capacity diminishes as the cavity relative embedment depth Z/B increases. In other words, the bearing capacity significantly decreases and becomes smaller when the cavity is located closer to the footing. This is because at smaller depth of the cavity, the soil mass underneath the footing becomes thin so that the mobilized soil shearing resistance is remarkably decreased. The effect of cavity vanishes when Z/B reaches a certain value which is defined as a cavity critical relative embedment depth $(Z/B)_{cr}$. It is worth to say that the reduction factor C_r is not a function in the undrained soil shear strength c_u. It is obvious that the bearing capacity of a footing is decreased by 95% of that of the same footing without cavity underneath when the cavity is created at Z/B=0.25 and B/b=0.5, however there is no reduction in ultimate bearing capacity when the footing is located at relative embedment depth varied between 4 and 8 depending upon B/b ratio. By using Figure (4), it is possible to determine also whether the existing of cavity will affect the behaviour of the existing or proposed foundation or not. According to the approach of mechanism of general shear failure, it is generally believed that there is a critical depth after which the influence of the cavity on the bearing capacity of footing is negligible. According to the general shear failure theories the critical depth is commonly taken at the bottom end of the radial shear zone and generally no more than two times the footing width. However the present results indicate that the critical relative cavity embedment depth is greater than this value and depends also on the ratio B/b. Figure (5) shows the variation of $(Z/B)_{cr}$ with the variation of B/b ratio at ultimate and also prior to ultimate condition. It is clear from this figure that the critical relative embedment depth is, also, a function of the stress level.

To analyse the behaviour of the footing prior to ultimate, the load ratio N against footing settlement ratio S/B of the initial straight lines of Figure (3) was magnified and plotted in Figure (6). The stiffness of footing-soil system S_r was calculated as the slope of the straight lines of the load ratio N against footing settlement-width ratio S/B. Thus Figure (7) was plotted to demonstrate variation of the stiffness of soil-footing system versus the variation of cavity relative embedment depth Z/B.

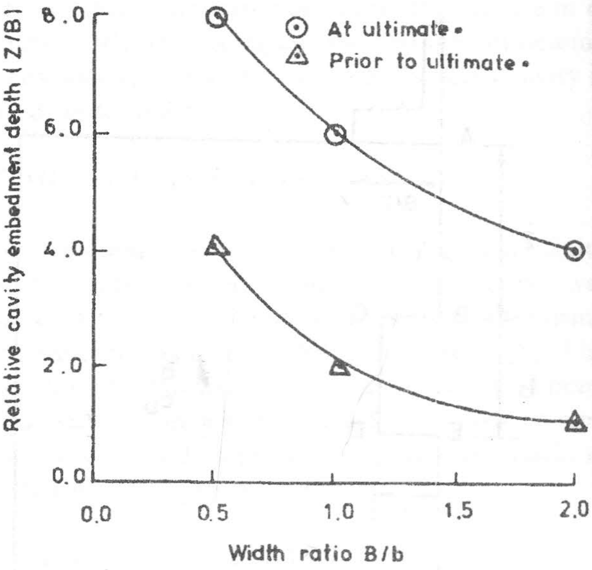

Figure 5. $(Z/B)_{cr}$ VS B/b.

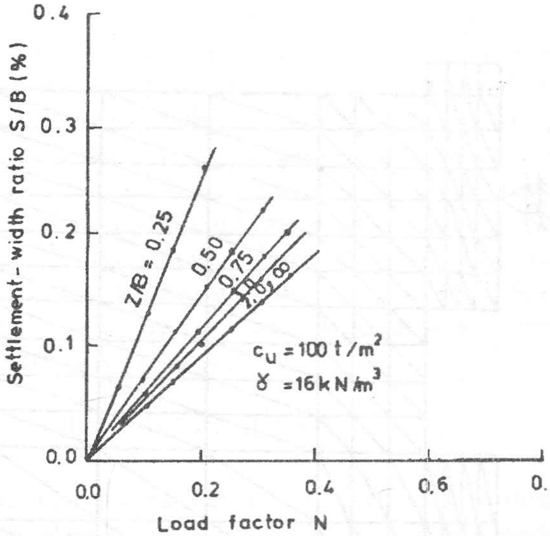

Figure 6. Load factor against footing-settlement width ratio.

Figure (7) shows that the stiffness of footing-soil system significantly increases as Z/B increases, up to a certain value depending upon B/b ratio, however beyond that limit there is no appreciable variation of the footing-soil stiffness with increasing of the cavity relative embedment depth Z/B. This imply that the critical cavity embedment depth depends on the load level acting on the footing. Figure (7) shows also that the stiffness of the footing-soil system is not influenced with the undrained soil shear strength c_u since the stiffness of footing-soil system is calculated in a dimensionless form as $S_r=(N/(S/B))$. Figure (7) may be used for predicting the settlement of a footing, located above a void, prior to failure, once the stress level is known.

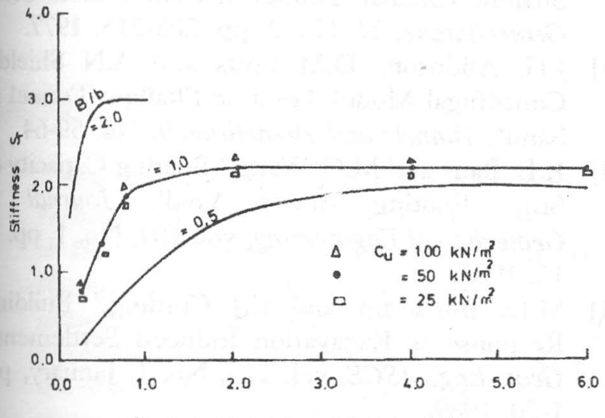

Figure 7. Stiffness of soil-footing system against cavity relative embedment depth.

Figures (8) and (9) show the variation of the settlement ratio S_i versus the variation of cavity relative embedment depth Z/B for two different stress-level ratios p_r of 0.5 and 1.0 respectively. These figures show that the increase in the settlement resulting from a creation of a cavity is depended on the cavity relative embedment depth Z/B, B/b ratio and the stress level acting on the footing. It is worth to say that when a cavity is created underneath a footing loaded with a stress level ratio p_r of 1.0 (B/b=1.0 and Z/B=1.0), a settlement of about 5 times the settlement of a footing without cavity underneath would be occurred. The reduction bearing capacity chart, shown in figure (4) as well as the settlement ratio chart shown in figures (8) and (9) provide a useful data for design of a continuous surface footing located above a continuous cavity.

Figure (10) shows the footing-settlement shape, at ultimate, with the variation of the cavity relative embedment depth Z/B. As it is anticipated the footing-settlement would be in a dished pattern since the footing is flexible (i.e the contact pressure is distributed uniformly) and the occurrence of high stresses below the center of footing in addition to the existing of the cavity underneath, give high settlement at the center and less settlement at the footing edges. It is obvious from figure (10) that as Z/B increases the distribution of settlement tends to be uniform. It is worth to say in spite of the footing settlement of figure (10) was calculated at ultimate condition, the footing settlement shape is in a shape of settlement of a flexible footing resting on a semi elastic mass. This is, mainly, the results of existing of a cavity underneath. Figure (11) shows the settlement shape of the cavity top surface. The figure shows that the settlement shape of the cavity top surface is also in a dished shape.

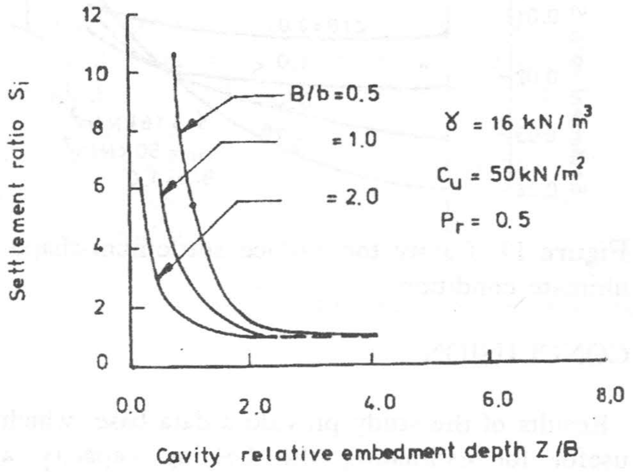

Figure 8. S_i vs Z/B.

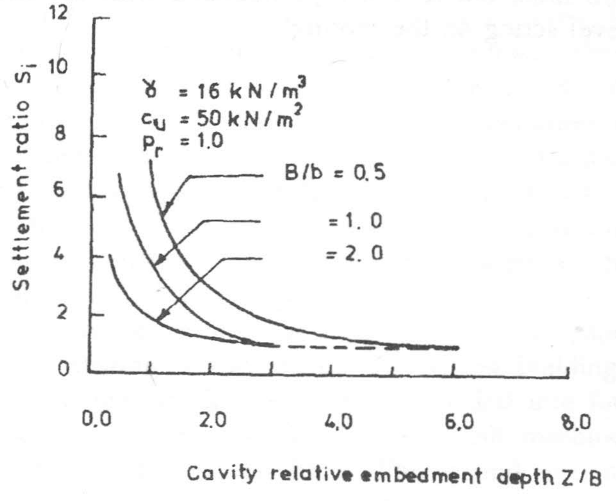

Figure 9. S_i vs Z/B.

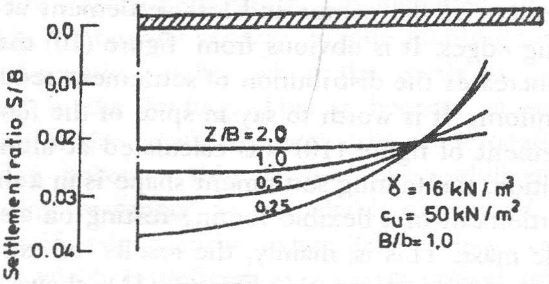

Figure 10. Footing-settlement shape at ultimate condition.

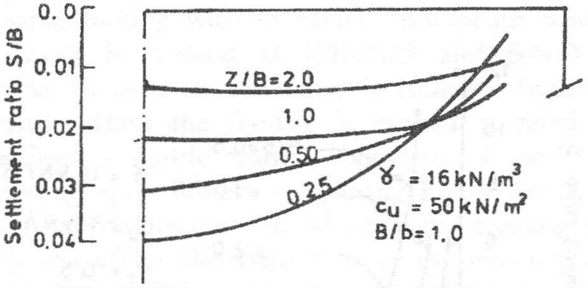

Figure 11. Cavity top surface settlement-shape at ultimate condition.

CONCLUSION

Results of the study provide a data base, which is useful for estimating the bearing capacity and settlement of a continuous flexible surface footing centered with a cavity created in cohesive soil. The critical embedment depth is not only in function of B/b ratio, but it is also pronounced with the stress level acting on the footing.

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