A SIMPLE METHOD FOR OPTIMIZING FUNCTION GENERATING MECHANISMS DESIGNED BY PRECISION POINT APPROACH

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ABSTRACT

A simple method is proposed for minimizing the maximum absolute error resulting from the function generating mechanisms designed by the precision point approach. The prescription of the desired function and the input-output relation of the mechanism are used in optimizing the mechanism parameters. The method may be applied to the optimization of any planar or spatial function generating mechanism for which the input-output relation can be expressed as a continuous function. Two numerical examples are included to illustrate the application of the proposed method.

Keywords:

INTRODUCTION

Different techniques have been developed for designing function generating mechanisms such as precision point [1], least square [2, 3], nonlinear programming [4] and graphical [5, 6] methods. In the precision point approach, a number of discrete points, equal to the number of the unknown independent parameters of the mechanism, are chosen within the function generation interval, and the mechanism is designed to yield the value of the desired function at these points. The main disadvantage of this method is that away from the precision points large deviation between the generated and desired output functions may result. Freudenstein [7] showed that the maximum absolute value of this error is minimum when the local peaks of the absolute error within the function generation interval are equal. To synthesize this mechanism, he developed an iterative optimization technique in which the precision points are respaced and the mechanism is redesigned. In the method presented here, the mechanism parameters, which minimize the maximum absolute error are determined directly by using the desired function and the input-output relation of the mechanism without redesigning the mechanism, which usually includes solution of nonlinear trigonometric equations.

THEORY

In the function generation problem the desired output, y is prescribed as a function of the input, x for a given interval $[x_s , x_f]$. The prescription may be in the form of an explicit or implicit function, or a plot. The generated output, Y is a function of x, as well as the n independent mechanism parameters, p_j, j = 1 to n. Usually, it is difficult to express this function explicitly and the input-output relation is described by an equation in the form,

$$G(x, Y, p_1, p_2, .., p_n) = 0. \quad (1)$$

Both desired and generated functions are assumed to be continuous throughout the function generation interval. The error, E is defined by,

$$E = Y - y. \quad (2)$$

The general behavior of the error curve resulting from a function generating mechanism designed by the precision point technique is presented in Figure

(1) for n = 5. Special cases, such as those which have extra precision points, are not considered in the present work. The error is zero at the precision points, and has n + 1 peaks with alternating signs at points (x_i , E_i), i = 0 to n, where $x_0 = x_s$ and $x_n = x_f$. At the peaks between the precision points, we have,

$$\frac{(\partial G/\partial x)_i}{(\partial G/\partial Y)_i} = -\left(\frac{dy}{dx}\right)_i, (i=1 \text{ to } n-1), \qquad (3)$$

where subscript i indicates that the derivative is evaluated at the i th peak point. For manipulation convenience, the absolute values of the error at the peak points, thereafter referred to as the peak values, are expressed as,

$$A_i = SU_i E_i\,, \quad (i=0 \text{ to } n)\,, \qquad (4)$$

where,

$$S = E_0 / |E_0|\,,$$

$$U_i = (-1)^i\,.$$

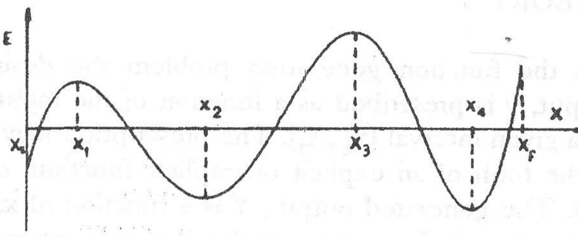

Figure 1. General behavior of the error curve for n = 5.

Now, we assume that the mechanism parameters are given small finite increments, δp_j , j = 1 to n, from their initial values, and we find the effect of this perturbation on the peak values. In this respect the peak points are divided into the following two groups:

a) *First Group* ($x = x_0 , x_n$)

At these points, x_i and y_i are not affected by the perturbation. Ignoring the partial derivatives of G with orders higher than the first, the change in the i th peak value, δA_i can be derived from equations (1), (2) and (4) as,

$$\delta A_i = -\frac{SU_i}{(\partial G/\partial Y)_i}\sum_{j=1}^{n}\left(\frac{\partial G}{\partial p_j}\right)_i \delta p_j, (i=0,n). \qquad (5)$$

b) *Second group* ($x = x_i$, $i = 1$ *to* n-1)

In general, the perturbation causes x_i, y_i and Y_i to be given increments δx_i, δy_i and δY_i. Ignoring the higher order derivatives of y and G, the equations relating these increments can be written as,

$$\delta y_i = \left(\frac{dy}{dx}\right)_i \delta x_i, (i=1 \text{ to } n-1)\,, \qquad (6)$$

$$\left(\frac{\partial G}{\partial x}\right)_i \delta x_i + \left(\frac{\partial G}{\partial Y}\right)_i \delta Y_i + \sum_{j=1}^{n}\left(\frac{\partial G}{\partial p_j}\right)_i \delta p_j = 0, (i=1 \text{ to } n-1). \qquad (7)$$

Eliminating δx_i from equations (6) and (7), and substituting by equation (3), we get,

$$\left(\frac{\partial G}{\partial Y}\right)_i (\delta Y_i - \delta y_i) + \sum_{j=1}^{n}\left(\frac{\partial G}{\partial p_j}\right)_i \delta p_j = 0, (i=1 \text{ to } n-1). \qquad (8)$$

Using equations (2) and (4), equation (8) gives,

$$\delta A_i = -\frac{SU_i}{(\partial G/\partial Y)_i}\sum_{j=1}^{n}\left(\frac{\partial G}{\partial p_j}\right)_i \delta p_i, (i=1 \text{ to } n-1). \qquad (9)$$

Equations (5) and (9) show that the changes in all peak values are related to the increments of the mechanism parameters by the same relationship. From equations (4), (5) and (9), the new peak values are expressed as,

$$A_i' = SU_i\left[E_i - \frac{1}{(\partial G/\partial Y)_i}\sum_{j=1}^{n}\left(\frac{\partial G}{\partial p_j}\right)_i \delta p_j\right], (i=0 \text{ to } n). \qquad (10)$$

The accuracy of this equation depends on the magnitude of the increments of the mechanism parameters, which were assumed to be small, as well as the higher order partial derivatives of the desired and generated functions, which were disregarded in the foregoing derivation.

OPTIMIZATION

It is shown in Ref. [7] that, the maximum absolute error of a function generating mechanism throughout a certain function interval is minimized if the absolute values of the error at its local peaks within the interval are made equal. Based on this rule, the goal of the optimization algorithm is to determine the mechanism parameters which yield an error curve with equal peak values.

In general, the peak values may differ widely from each other. In such cases the increments in the mechanism parameters necessary for optimization are not small enough to apply equation (10) with acceptable accuracy. Therefore, the optimization is carried out in N successive steps whereby the deviation between the peak values is reduced gradually until it reaches a final value, which is almost zero. The number of the optimization steps is chosen in view of the deviations of the peak values from their mean value. In each step only small reduction in these deviations is considered so that the required increments of the mechanism parameters are sufficiently small for the application of equation (10). Thus, the deviation of the i th peak after the m th optimization step may be expressed as,

$$A_i' - M' = R_m (A_i - M), \quad (i=0 \text{ to } n) \,, \qquad (11)$$

where,

- A_i and A_i' = values of the i th peak before and after the m th optimization step,
- M and M' = mean values of the peak values before and after the m th optimization step
- R_m = the desired reduction ratio of the deviations due to the m th optimization step, ($R_m<1$).

Substitution by equations (4) and (10) into equations (11) gives,

$$M' - R_m M = S U_i [E_i (1 - R_m) - \frac{1}{(\partial G/\partial Y)_i} \sum_{j=1}^{n} (\frac{\partial G}{\partial p_j})_i \delta p_j], (i=0 \text{ to } n). \qquad (12)$$

M,M' and S can be eliminated from the n+1 equations represented by equation (12) to yield n equations represented by,

$$U_i [E_i (1 - R_m) - \frac{1}{(\partial G/\partial Y)_i} \sum_{j=1}^{n} (\frac{\partial G}{\partial p_j})_i \delta p_j] - E_0 (1 - R_m) + \frac{1}{(\partial G/\partial Y)_0} \sum_{j=1}^{n} (\frac{\partial G}{\partial p_j})_0 \delta p_j = 0,$$

$$(i=1 \text{ to } n) \qquad (13)$$

This equation can be expressed in a matrix form as,

$$[B]\,\{V\} = \{C\} \,, \qquad (14)$$

where,

$$B_{ij} = \frac{(\partial G/\partial p_j)_0}{(\partial G/\partial Y)_0} - \frac{U_i (\partial G/\partial p_j)_i}{(\partial G/\partial Y)_i} \,, \qquad (15)$$

$$\{V\}^T = [\delta p_1, \delta p_2, ..., \delta p_n] \,, \qquad (16)$$

$$C_i = (1 - R_m)(E_0 - U_i E_i) \,. \qquad (17)$$

In order that the deviations of the peak values from their mean value decrease gradually and almost vanish after the last step of optimization, ratio R_m is reduced uniformly in the N steps to its final value, which is zero. This gives,

$$R_m = (N - m)/N \,. \qquad (18)$$

The procedure of the optimization algorithm may be described as follows:

1. Derive the expressions which give $\partial G/\partial Y$ and $\partial G/\partial p_j$, j = 1 to n, by differentiating the equation describing the input-output relation of the mechanism.
2. Do the following for the N steps of optimization (i.e. for m = 1 to N):
   a) Determine x , Y and E at the points of maximum absolute error by using equation (1) and the description of the desired function.
   b) Using the values of x and Y obtained above, calculate [$(\partial G/\partial Y)_i$ and $(\partial G/\partial p_j)_i$, j = 1 to n], i =

0 to n.

c) Calculate R_m from equation (18).

d) Calculate the elements of [B] and [C] from equations (15) and (17).

e) Solve equation (14) to determine {V} which defines δp_j, j = 1 to n.

f) Calculate the new values of the mechanism parameters ($p_j+\delta p_j$, j = 1 to n).

3. Determine the peak values resulting from the optimized mechanism.

EXAMPLES

In order to check the accuracy of the proposed method, the optimization algorithm was applied with different values of N for optimizing various function generating mechanisms. In all cases, five optimization steps were found sufficient for producing a mechanism with almost equal peak values. Two examples are presented below to illustrate the application of the proposed method. In these examples, the optimization algorithm was applied in five steps. The input and output were represented by input and output angles θ and ϕ (Figure (2)), which are defined by,

$$\theta = \theta_s + \frac{\Delta\theta}{x_f - x_s}(x - x_s) \ ,$$

$$\phi = \phi_s + \frac{\Delta\phi}{y_f - y_s}(Y - y_s),$$

where,

y_s, y_f = starting and final values of the desired output,

θ_s, ϕ_s = starting values of the input and desired output angles,

$\Delta\theta$, $\Delta\phi$ = ranges of the input and desired output angles.

Example 1

A four-bar mechanism (Figure (2)) was designed in Ref. [7] to generate the function y=sin(x), for the interval $0° \leq x \leq 90°$, with $\Delta\theta=\Delta\phi=90°$, using 5-point synthesis, which yielded,

a_1/a_4 = 2.075, a_2/a_4 = 2.411, a_3/a_4 = 0.757, θ_s = -63.75°, ϕ_s = 254.05°.

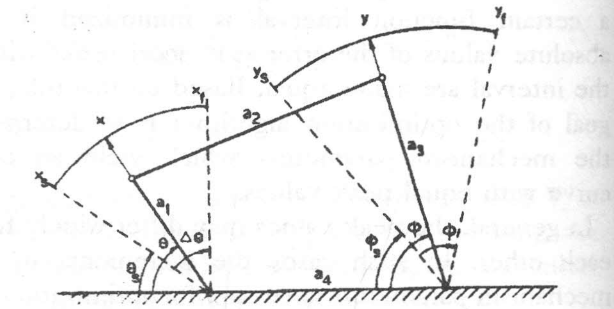

Figure 2. A four-bar planar mechanism.

The error curve resulting from this mechanism is represented in Figure (3) by a dashed line.

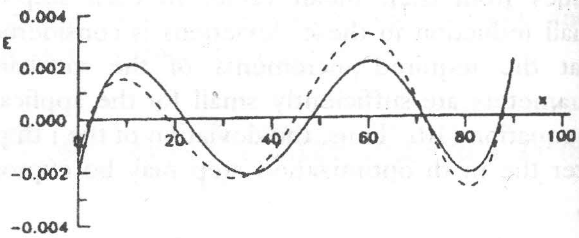

Figure 3. The error curves of Example 1.

In order to apply the optimization algorithm, the loop closure equation of the mechanism was written as [1],

$$p_1 \cos\theta - p_2 \cos\phi + p_3 - \cos(\theta - \phi) = 0 \ ,$$

where,

$$p_1 = a_4/a_3 \ ,$$

$$p_2 = a_4/a_1 \ ,$$

$$p_3 = \frac{a_1^2 - a_2^2 + a_3^2 + a_4^2}{2a_1a_3}$$

The fourth and fifth mechanism parameters were $p_4=\theta_s$ and $p_5=\phi_s$. The partial derivatives of G were derived from the loop closure equations as,

$$\frac{\partial G}{\partial Y} = \frac{\Delta\phi[p_2 \sin\phi - \sin(\theta - \phi)]}{y_f - y_s},$$

$$\frac{\partial G}{\partial p_1} = \cos\theta ,$$

$$\frac{\partial G}{\partial p_2} = -\cos\phi ,$$

$$\frac{\partial G}{\partial p_3} = 1 ,$$

$$\frac{\partial G}{\partial p_4} = -p_1 \sin\theta + \sin(\theta - \phi) ,$$

$$\frac{\partial G}{\partial p_5} = p_2 \sin\phi - \sin(\theta - \phi) .$$

The specifications of the optimized mechanism were,

$a_1/a_4 = 1.836$, $a_2/a_4 = 2.240$, $a_3/a_4 = 0.694$,
$\theta_s = -65.02°$ $\phi_s = 251.28°$.

The error curve produced by this mechanism is represented in Figure (3) by a continuous line. The figure shows that the maximum absolute error within the function generation interval has been reduced to 66% of its original value. The same results were obtained in Ref. [7] by respacing the precision points and redesigning the mechanism four times. Another four-bar mechanism was designed in Ref. [3] to generate the same function, using an improved least square method, with nineteen design points spaced at 5° intervals. The maximum absolute error resulting from this mechanism was 20% greater than that produced by the optimized mechanism. This indicates that the least square methods do not minimize the maximum absolute error.

Example 2

The RSSR spatial mechanism represented in Figure (4) was designed to generate the function $y=x^2$, for the interval $1 \leq x \leq 3$, using 3-point synthesis, with $a_4=a_5$, $\theta_s=\theta_s = -45°, \Delta\theta=\Delta\phi=90°$. The mechanism unknowns were a_1/a_5, a_2/a_5 and a_3/a_5. The precision points were chosen at x=1.2, 2, 2.8. According to Ref.[8], the loop closure equation of the mechanism can be written as,

$$p_1 \sin\theta - p_2 \sin\phi + p_3 - \cos\theta \cos\phi = 0,$$

where,

$$p_1 = a_4/a_3 ,$$

$$p_2 = a_5/a_1 ,$$

$$p_3 = \frac{a_1^2 - a_2^2 + a_3^2 + a_4^2 + a_5^2}{2a_1a_3} .$$

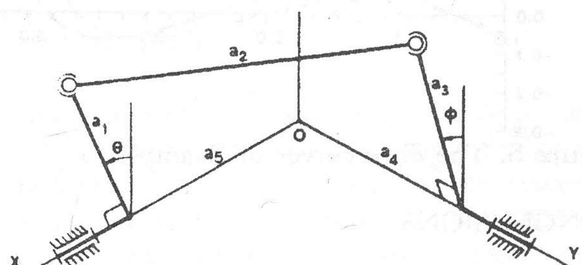

Figure 4. The RSSR spatial mechanism of Example 2.

Substituting by the values of x and y at the precision points, the loop closure equation yielded three simultaneous equations in the mechanism parameters, p_1, p_2 and p_3. Solving these equations, the link proportions were specified by,

$a_1/a_5 = 0.421$, $a_2/a_5 = 1.472$, $a_3/a_5 = 0.412$.

In order to apply the optimization algorithm, the partial derivatives of G were derived from the loop closure equation of the mechanism. The results were,

$$\frac{\partial G}{\partial Y} = \frac{\Delta\phi(-p_2\cos\phi + \cos\theta\sin\phi)}{y_f - y_s},$$

$$\frac{\partial G}{\partial p_1} = \sin\theta \, ,$$

$$\frac{\partial G}{\partial p_2} = -\sin\phi \, , \frac{\partial G}{\partial p_3} = 1 \, .$$

Application of the optimization algorithm gave,
$a_1/a_5 = 0.382$, $a_2/a_5 = 1.465$, $a_3/a_5 = 0.377$.
The error curves before and after the optimization are represented in Figure (5) by dashed and continuous lines respectively. The figure shows that the maximum absolute error has decreased to 28% of its original value.

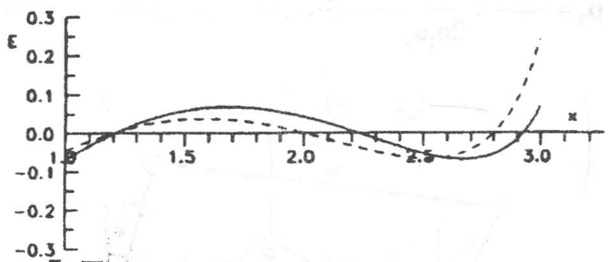

Figure 5. The error curves of Example 2.

CONCLUSIONS

The following conclusions may be drawn regarding the proposed method of optimization.

1. Five optimization steps are sufficient for optimizing mechanisms with parameters far from their optimal values.
2. In general, the method gives results better than those of the least square methods, although in these methods many design points are considered.
3. The method is simple, since it does not require redesigning the mechanism, which usually includes solution of nonlinear trigonometric equations.
4. The method does not require expressing the desired function explicitly, and therefore, it can be applied when this function is prescribed by an implicit function or a plot.
5. The method is general, and can be applied to any mechanism for which the input-output relation can be expressed in the form of equation (1).

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