# ASYMPTOTICALLY STABLE ACTIVE PARAMETRIC CONTROL OF A SLEWING THIN FLEXIBLE BEAM

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## ABSTRACT

This paper investigates the inplane motion of a slewing thin flexible beam clamped at one end to a rotating rigid hub. Both flexural and axial effects are included in the derived model. The paper presents an active parametric control which uses the beam tensile stress as a distributed time-dependent parameter to suppress the transverse vibration. The control is carried out by applying an axial force at the beam free-end. This force is determined according to a synthesized asymptotically stable modified bang-bang control algorithm. The optimum values of the axial control force are evaluated in terms of beam dimensions and material properties. Numerical simulation of the closed-loop control system showed that, in the absence of passive damping and uncontrolled slewing time, the amplitude of vibration is reduced to 18% of its uncontrolled value. The proposed control is proven to be efficient in computation, concise in formulation, and effective in hardware realizable application.

*Keywords: Parametric control, Flexible beam.*

## INTRODUCTION

Development in large space structures has been focusing on nonlinear rigid-body motion coupled with flexible body elastic vibration. Recent research in large angle maneuvering of flexible spacecraft, addressed the structure model of a flexible beam clamped into a rigid hub mounted on a motor shaft at one end. The motion of this system can serve as a model of robotic arms, helicopter blades, turbine blades, spacecraft antenna, and solar panel.

The control design for either rigid-body rotational maneuver or vibration suppression of a flexible manipulator has been considered by many investigators [1-10]. Although most of the investigations were aimed at some typical maneuver missions, both the complexity of the control law formulation and the associated computational burden represented the major hurdle to successful hardware implementation. Murotsu et al. [11] suggested that the controller design for vibration suppression of highly flexible structures should be based on a two-level control architecture: high-authority control (HAC) and low-authority control (LAC). HAC is designed so that the controller perform the given mission and the subsequent vibration be suppressed by LAC. Motivated by this approach, Liu and Yang [12] proposed the constrained motion method as a high-authority controller. It was proven to be effective in achieving the rotational maneuver.

Two approaches to the active control of vibration in flexible beam, LAC, are currently considered: Modal control, and Distributed parameter feedback. Modal control uses a reduced order finite dimensional model. This model is obtained by considering and retaining only the first few arbitrary number of vibrational modes. This choice is based on the belief that the energy content in the higher modes is insignificant compared to the lower modes. Once such a reduced order model was derived, conventional control strategies, including regulator theory [13,14] and pole-placement method [15], were applied. However, such modal truncations may lead to instabilities due to the phenomenon of spillover

[14,16]. Spillover is a manifestation of either the neglected modes becoming unstable, the controller modes becoming unstable, or both. In distributed parameter feedback control, the system model retains an infinite number of modes. Plump et al. [17] applied distributed piezoelectric-polymer for the active vibration control of a non-rotating cantilever beam. Balles et al. [18] introduced active parametric control and studied a finite number of modes for finite observation. Habib and Radcliffe [19] employed active vibration control for a simply supported Bernoulli-Euler beam using one of the distributed, time-dependent parameters of the system. Liu and Yang [20] presented a coupled active damping control and optimal control for vibration suppression of rotating beam during and after the slewing motion. Although the method was shown to be effective in reducing both the transient vibration and the settling time, many computations for the optimal feedback gain were required. From the previous review, it is clear that there is a need for a control law that minimizes the computational burden, yet satisfies the performance required. The present study is directed toward this goal.

In this paper, the vibration control of a rotating thin flexible beam is studied. A coupled model for the hub-beam system that includes axial and flexural effects is derived. An active vibration control, using the axial force as a distributed time-dependent parameter of the system, is employed. Asymptotically stable , modified bang-bang control is applied as a control algorithm.

## HUB-BEAM SYSTEM MODEL

The plane mechanism shown in Figure (1) consists of a rigid hub of radius $r_o$, mass moment of inertia $I_h$, and a flexible beam of length L, cross sectional area A, and constant material properties. The hub angular rotation is $\theta(t)$ measured counter-clockwise with respect to the x-axis of the fixed reference frame. The transverse and longitudinal displacements v, and u, respectively, are measured with respect to the current body frame. It is assumed that the plane sections remain plane during deformation. The effects of shear deformation, rotary inertia, and passive damping are assumed negligible. An energy method is employed to generate the governing equations of motion.

Considering Figure (1), the position vector r(x,t) can be written as

$$\mathbf{r}(x,t) = (r_o + x + u)\, e_1 + (v)\, e_2 \tag{1}$$

where $e_1$, $e_2$, and $e_3$ are the unit vectors defining the body frame.

Noting that $\dot{e}_1 = \dot{\theta} e_2$ and $\dot{e}_2 = -\dot{\theta} e_1$, the velocity vector is given by:

$$\dot{\mathbf{r}}(x,t) = [\, u_t - v\dot{\theta}\,]\, e_1 + [(r_o + x + u)\dot{\theta} + v_t]\, e_2 \tag{2}$$

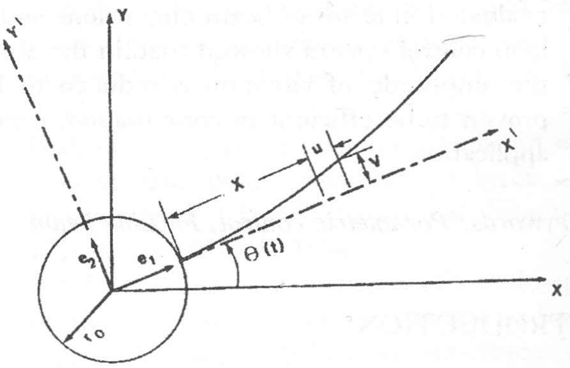


**Figure 1.** Hub-beam system.

where $u_t$, and $v_t$ are, respectively, the partial derivatives of u and v with respect to time.

The total kinetic energy of the system, T, is:

$$T = T_{beam} + T_{hub} \tag{3}$$

where

$$T_{hub} = 0.5\; I_h \dot{\theta}^2 \tag{4}$$

$$T_{beam} = 0.5\, \rho \int_A \int_0^L \dot{\mathbf{r}} \cdot \dot{\mathbf{r}}\; dx\, dA \tag{5}$$

Substituting Eq.(2) into Eq.(5) ,then Eq.(3) yields,

$$T = 0.5 I_h \dot{\theta}^2 + 0.5\rho A \int_0^L [u_t^2 + v_t^2 + (v\dot{\theta})^2 + (r_o + x + u)^2\, \dot{\theta}^2 - 2v\dot{\theta}u_t + 2\dot{\theta}(r_o + x + u)v_t]\, dx \tag{6}$$

Since the beam is assumed to be linearly elastic, the total potential energy is given by:

$$V = 0.5 \int_0^L (p^2/EA)\, dx + 0.5\, EI \int_0^L v_{xx}^2\, dx \quad (7)$$

The first term in Eq.(7) represents the total strain energy. p(x,t) is a parametric time varying force resulted from a time dependent compression force P(t) acting on the beam end. The axial-strain-displacement relation is given by [18] as:

$$p/EA = u_x + 0.5\ v_x^2 \quad (8)$$

The second term in Eq.(7) corresponds to the total bending energy. Considering Hamilton's principle,

$$\int_{t_1}^{t_2} (\delta T - \delta V + \delta W)\, dt = 0.0 \quad (9)$$

where $\delta W$ is the work done by the torque $\tau$ at the hub, and it is given by:

$$\delta W = \tau\, \delta\theta \quad (10)$$

Computing the variation $\delta T$ and $\delta V$ from Eqs.(6) and (7), and substituting into Eq.(9) results in the following Euler-Bernoulli's beam equations

$$EIv_{xxxx} - (pv_x)_x + \rho A[v_{tt} + 2\dot{\theta}u_t + (r_o+x+u)\ddot{\theta} - v\dot{\theta}^2] = 0 \quad (11)$$

$$p_x + \rho A[(r_o+x+u)\dot{\theta}^2 + v\ddot{\theta} + 2\dot{\theta}v_t - u_{tt}] = 0 \quad (12)$$

$$\begin{aligned}\tau(t) = {} & [I_h + \rho A\int_0^L [v^2 + (r_o+x+u)^2]dx]\ddot{\theta} \\ & + 2\rho A\dot{\theta} \int_0^L [v_t v + u_t(r_o+x+u)]\, dx \quad (13) \\ & + \rho A \int_0^L [(r_o+x+u)v_{tt} - vu_{tt}]\, dx\end{aligned}$$

with the boundary conditions

$$\begin{aligned} u(0,t) = v(0,t) = v_x(0,t) = 0.0 \\ v_{xx}(L,t) = v_{xxx}(L,t) = 0.0 \quad (14) \\ EA\,[u_x(L,t) + 0.5(v_x(L,t))^2] = P(t) \end{aligned}$$

and initial conditions

$$v(x,0) = u(x,0) = v_t(x,0) = u_t(x,0) = 0.0 \quad (15)$$

## CONTROL STRATEGY

Active Parametric Control theory is based on using one of the time-dependent distributed parameters to control transverse displacement. Equation(11) contains the parametric axial tension p(x,t) as a coefficient. If the time-dependent force P(t) is applied at the free end of the beam with an appropriate control algorithm, the induced parametric force p(x,t) can be used to control the beam transverse stiffness and to form an asymptotically stable closed-loop system. Liapunov function will be used to derive a control law for p(x,t) which guarantees asymptotic stability.

The main difficulty with distributed parameter systems is the identification of an appropriate Liepunov function, *LY*. Liepholz [21] showed the close connection between Liapunov's stability criterion and the classical energy criterion expressed via the Hamiltonian,H, for autonomous, dynamic continuous system. He proved that for a nonconservative system, if *LY* is chosen as the Hamiltonian, then

$$\frac{d\,LY}{dt} = \frac{dH}{dt} \leq \int_{V_o} Q\, \frac{dq}{dt}\, dV_o \quad (16)$$

where Q is the vector of the generalized force, q is the generalized vector, and $V_o$ is the volume of the system. Choosing *LY* as the total energy of the rotating beam system,

$$LY = T - V \quad (17)$$

Form Eqs.(6) and (7), one can notice that *LY* is positive definite which admits an infinitely small upper bound in the neighborhood of the equilibrium state of the beam. The time derivative of *LY* is:

$$\begin{aligned}\frac{\partial LY}{\partial t} = {} & I_h\dot{\theta}\ddot{\theta} + \int_0^L [\rho A[u_t u_{tt} + v_t v_{tt} + vv_t\dot{\theta}^2 + v^2\dot{\theta}\ddot{\theta} + (r_o+x+u)u_t\dot{\theta} \\ & + (r_o+x+u)^2\dot{\theta}\ddot{\theta} - vu_{tt}\dot{\theta} - vu_t\ddot{\theta} - v_t u_t\dot{\theta} + (r_o+x+u)v_{tt}\dot{\theta} \quad (18) \\ & + (r_o+x+u)v_t\ddot{\theta} + u_t v_t\dot{\theta}] + EIv_{xx}v_{xxt} + pp_t/EA]\, dx\end{aligned}$$

Using the constitutive relationship (8), integrating by parts, and applying the boundary conditions (14) yield:

$$\frac{\partial LY}{\partial t} = \int_0^L [[\rho A[u_{tt} - (r_o + x + u)\dot{\theta}^2 - v\ddot{\theta} - 2v_t\dot{\theta}] - p_x]u_t$$
$$+ [\rho A[v_{tt} - v\dot{\theta}^2 + (r_o + x + u)\ddot{\theta} + 2\dot{\theta}u_t] + EIv_{xxxx} - (pv_x)_x]v_t \quad (19)$$
$$+ [\rho A[v^2\ddot{\theta} + (r_o + x + u)^2\ddot{\theta} + 2(r_o + x + u)\dot{\theta}u_t + (r_o + x + u)v_{tt}$$
$$+ 2v\dot{\theta}v_t - vu_{tt}]]\dot{\theta}]dx + I_h\dot{\theta}\ddot{\theta} + (pu_t)_0^L + (pv_x v_t)_0^L)$$

Applying the asymptotic stability condition (16), then

$$\frac{\partial LY}{\partial t} = \tau\,\dot{\theta} \qquad (20)$$

Substituting equations of motion (11-13) in Eqs.(19) and (20) yields,

$$(p\,u_t)_0^L + (p\,v_x\,v_t)_0^L \le 0.0$$

noting that

$$u_t(0,t) = 0\ ,\ v_x(0,t) = 0$$

$$\therefore\ P(t)[u_t(L,t) + v_x(L,t)\ v_t(L,t)\ ] \le 0.0 \qquad (21)$$

Only at equilibrium state, the term CP,[ CP= $u_t(L,t)+v_x(L,t)v_t(L,t)$], is equal to zero. Therefore, considering P(t) as any negative function of CP will yield to asymptotic stability. The modified bang-bang control force algorithm shown in Figure (2) satisfies the condition in Equation(21). In this case, both the axial and transverse velocities at the free end of the beam are observed and then the control force is evaluated according to Figure (2). The control structure is presented in Figure (3).

## DISCUSSION OF RESULTS

A computer simulation was performed to examine the control algorithm on rest-to-rest maneuver. The dimensions and material properties of the hub-beam system, used in the calculations, are listed in Table (1). Equations (11) and (12), which represent the slewing motion of the flexible beam, were solved using an explicit finite difference scheme. A mesh of 10 nodes was considered along the length of the beam, and convergence parameter, $\Delta t\ /(\Delta x)^2$ = 0.00086 s/m$^2$, resulted in a stabilized solution ( the time increment was $5*10^{-6}$ s). The hub-rotational speed was approximated with an angular velocity profile similar to that in Figure (4). The results were obtained at maneuvering speeds of 0.15 rad/s, 0.3, and 0.45 rad/s. A modified bang-bang control force (Fig.2) was implemented with control force saturation magnitude, $\alpha$, varied from 0 to 150 N and $\epsilon = 3*10^{-8}$ m/s.

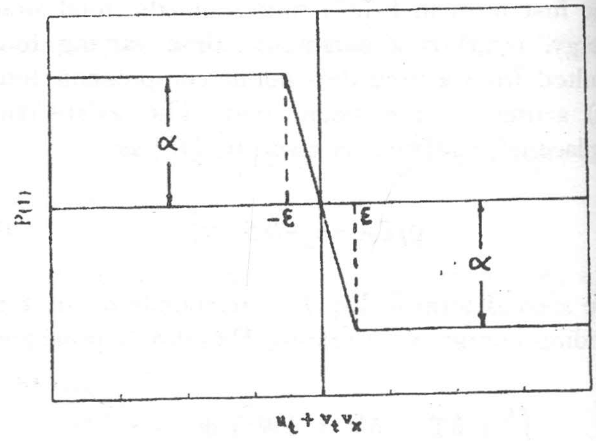


Figure 2. Asymptotically stable, modified bang bang, control force.

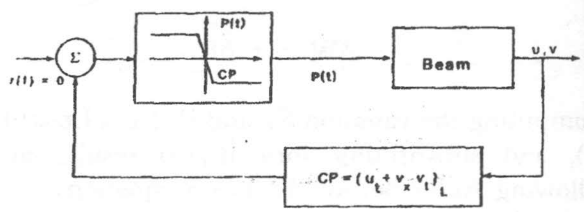


Figure 3. Active parametric vibration control structure.

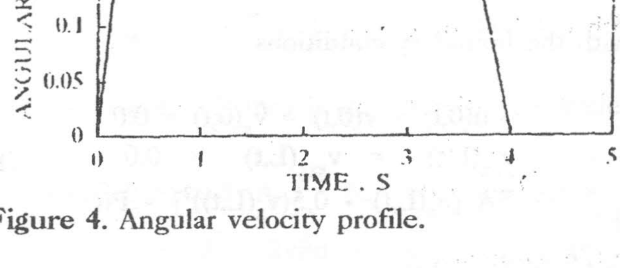


Figure 4. Angular velocity profile.

Figure (5) shows the tip deflection of the uncontrolled beam, $\alpha$=0, at different hub speeds. It appears from the figure that the amplitude of the vibration increases as rotational speed increases. Meanwhile, the increase in rotational speed results in an increase in the centrifugal force which reduces beam effective stiffness and lowers the frequency of the vibration. The frequencies

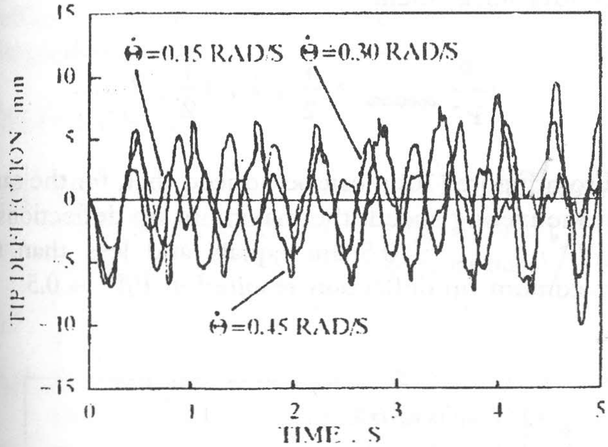


Figure 5. Tip deflection for $\alpha$=0.0.

Table 1. The physical parameters of the Hub-Beam System.

| | |
|---|---|
| Material | Aluminum |
| Density, $\rho$ | 2710.0 kg/m$^3$ |
| Young's modulus, E | $71.0*10^9$ N/m$^2$ |
| Beam thickness | $8.467*10^{-4}$ m |
| Width | $1.905*10^{-2}$ m |
| Length, L | 0.762 m |
| Cross sectional area, A | $1.613*10^{-5}$ m$^2$ |
| Moment of inertia, I | $9.3631*10^{-13}$ m$^4$ |
| Hub mass moment of inertia, $I_h$ | $1.7628*10^{-3}$ kg.m$^2$ |
| Hub radius, $r_o$ | 0.09525 m |

of vibration are 2.80 Hz, 2.15, and 1.70 Hz at $\theta$ = 0.15 rad/s, 0.30, and 0.45 rad/s, respectively.

Upon implementing the control strategy, the tip responses with $\alpha$ =0, 25, and 100 N are illustrated in Figures (6), (7), and (8) for $\theta$ = 0.15, 0.30, and 0.45 rad/s, respectively. The figures indicate that the amplitudes of vibration decreases as the magnitude of the axial control force increases. At $\theta$ =0.15 rad/s, the maximum tip deflections were 2.40 mm, 1.00, and 0.44 mm at $\alpha$ =0, 25, and 100 N , respectively. At $\theta$ = 0.3 rad/s, the amplitude of oscillation decays from 4.8 mm at $\alpha$=0 to 0.8 mm at $\alpha$=100 N. Also at $\theta$ = 0.45 rad/s, the maximum tip deflection at $\alpha$=100 N was reduced to 18% of its uncontrolled value. The saturating nature of the bang-bang control dominated the character of the control forces as shown in Figure (9) because of the high frequency components in the controlled response. It can be noticed from Figures 6 to 9 that the vibration was not completely suppressed after slewing motion. That is because the material damping was not incorporated in the developed model. Also the switch time and final time of the maneuver mission shown in Fig. 4 were not adjusted for optimal slewing motion. Since the feasibility of the developed control strategy is the main objective of this paper, neither the effect of the passive damping nor slewing time is included in the study.

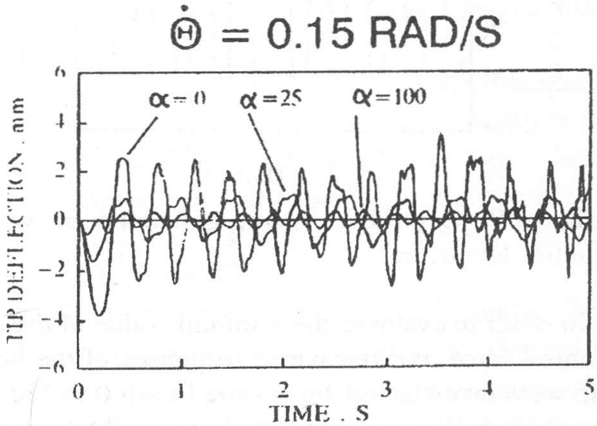


Figure 6. Controlled tip deflection at $\theta$=0.15 rad/s.

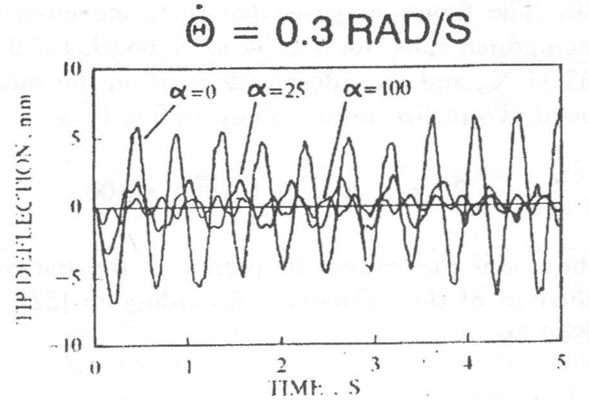


Figure 7. Controlled tip deflection at $\theta$=0.30 rad/s.

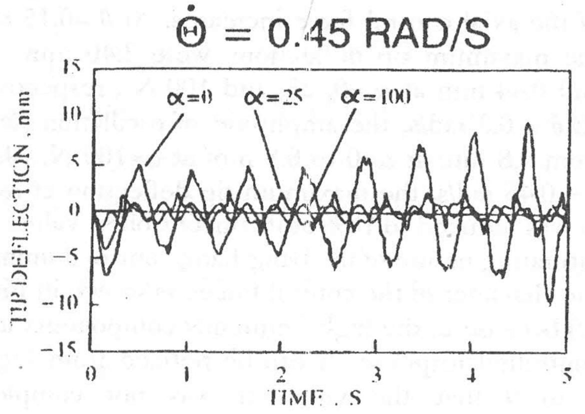


**Figure 8.** Controlled tip deflection at $\theta$=0.45 rad/s.

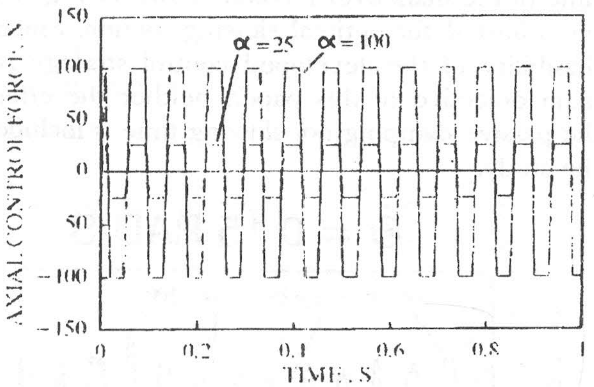


**Figure 9.** Controlled force, generated at various control levels, $\alpha$.

In order to evaluate the optimum value of the axial control force, the transverse responses of the beam-tip were investigated for $\alpha$ varied from 0 to 150 N at $\theta$= 0.15 rad/s, 0.30, and 0.45 rad/s. The resulted maximum tip deflections were plotted in Figure (10). The figure suggests that there are more than one optimal value for $\alpha$ at 34.33 N, 68.67, 103.0, and 137.33 N., and they do not depend on the rotating speed. To realize these values, define $P^*$ as

$$P^* = \rho \, A \, L \; ( L \, \omega^2 ) * 100$$

where $\omega$ is the natural frequency of the transverse vibration of the cantilever. According to [22], $\omega$ is given by:

$$\omega = \frac{\pi^2}{4 \, L^2} \sqrt{\frac{E \, I}{\rho \, A}}$$

$$P^* = \frac{\pi^4}{16 \, L^2} \, E \, I * 100$$

Using the values in Table (1), $P^*$ is equal to 68.( N. Considering $P/P^*$ as a nondimensional axi control force, then,

$$\left(\frac{P}{P^*}\right)_{optimum} = \frac{1}{2} \, , \, 1 \, , \, 1\frac{1}{2} \, , \, 2 \, ,.....$$

From Figure (10) it can be noticed that, for the sar maneuvering speed, the maximum tip deflections $(P/P^*)_{optimum}$ > 0.5 are equals and less than t maximum tip deflection resulted at $P/P^*$ = 0.5.

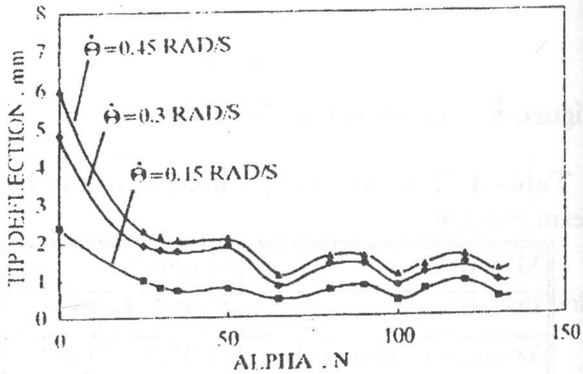


**Figure 10.** Maximum tip deflection at various $\alpha$, different maneuvering speeds.

## CONCLUSIONS

The structure model of a thin flexible bea connected to a rigid driven by a torque motor w investigated. A set of governing different equations was derived for the inplane motion of t system using energy method. The beam w assumed elastic and both flexural and axial effec were included in the model. Active paramet control that utilized beam tensile stress as distributed time-dependent parameter, was applie to control beam transverse vibration. Asymptotical stable, modified ban-bang control algorithm w synthesized. The parabolic partial differenti equations of the closed-loop control system were solved numerically to demonstrate the effectiveness of the control at different maneuvering speeds. The results showed that the proposed control reduces the amplitude of vibration to 18% of its uncontrolled value in the absence of passive damping. The optimum values of the axial control force were evaluated in terms of the dimensions of the beam and its material properties. From an engineering point of view, the suggested system is simple, efficient, and can be realized in hardware implementation.

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