ESTIMATION OF THE STRUCTURAL EXPRESSIONS FOR THE INSPECTION DETECTABILITY AND THE FALSE INDICATION PROBABILITY

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ABSTRACT

In this paper, the structural expressions for the distributions of the probability of crack detection (POD) and the probability of false crack indication (POF) in the visual inspection method are presented. The scatters in these distributions are caused by many factors related to the applied inspection method and the defect conditions in the structure. Multiple visual inspection method combined with the successive precise inspections which are carried out for limited locations is developed to estimate the POD and the POF properties. The POD and POF properties can be estimated from the results of field inspections carried out for a structure by an inspection team under a specified inspection condition. The crack classification method is also introduced to improve the accuracy of the inspection capability evaluation. The applicability of the proposed method is numerically examined for a structural model which has crack-form defects. The main advantage of the method is that the mean values of POD and POF as well as their distribution properties can be easily estimated without using any information other than the results of field inspections (Records).

Key Words: Inspection probability, False crack, Visual inspection, Success rate, Non-distrcutive inspection

INTRODUCTION

It is well known that Non-Destructive Inspections *(NDI)* are usually carried out for the structures when initially constructed and during service to minimize risk of failure. At the same time, no inspection procedure would provide 100% assurance that all cracks greater than detectable length will be found. Therefore, It is recognized that *NDI* is not always perfect and in general the missing of defects, the indication of false cracks and the erroneous measurement of true crack sizes possibly occur. However, it is important to grasp the capability of the inspection method, because the successive maintenance actions and the reliability analysis are carried out by comparing the inspection results with the capability of applied inspection method[13]. This paper treats the problems related to the capability evaluation of the visual inspection method which is carried out by inspector eyes and optical tools applied to the structures which have the crack-form defects.

The probability of crack detection curve *(POD)* is usually used to assess the inspection capability and many *POD* curves have been obtained using various inspection methods[4]. The detection probability itself in the *POD* curve is only characterized in terms of crack length[5][6] which must be considered the most important physical parameter governing the structure safety. For an inspection method, if the detection probabilities of individual crack are strongly correlated with crack lengths, the *POD* curve for the method can be widely utilized for the analyses of inspection reliability of structures. However, for visual inspections all the cracks of the same length do not have the same detection probability. The location and the orientation of crack, the gap of crack width, the surface condition of crack and environment in which the inspection takes place all influence the chance of crack detection. Therefore, the detection probability itself has a scatter owing to the influences of the above mentioned factors. Although this has been pointed out by many researchers[7], most of the discussions were limited to the mean value of *POD* and very few papers dealt with the shape of the distribution.

In this study the structural expressions for the distributions of *POD* and *POF* in the visual inspection of structures are presented, where the scatter in these distributions is caused by all factors other than crack length. If the capability of an applied inspection method is influenced by both of the inspection condition and the crack condition of the structure, then the problem arising is how to estimate the inspection capability. The inspection capability is usually evaluated through an experiment in which representative structures with known artificial cracks are inspected[4][6]. When the estimated *POD* curve is applied to the analysis of the inspection reliability of other structures, if either the inspection condition or the crack condition in the structure is different between experiment and the field inspection, the accuracy of the analysis will be lost. Ideally, the probabilities of crack detection and false crack indication of visual inspections should be estimated directly from the result of field inspection of the objective structures.

However, the estimations by this method are quite difficult, because true crack conditions in the structure are never being informed from the results of field inspections. Therefore, a method using only detected crack data was presented to estimate the *POD* curve[8]. Furthermore, the method is effective only when the number of true cracks and the crack length distributions can be roughly assumed according to the previous inspection experiences. This assumption may not always hold good in practice.

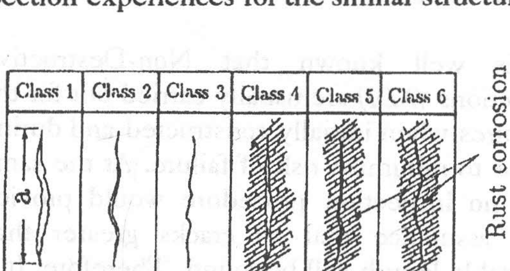
In this study, an inspection procedure employing multiple visual inspection method combined with successive precise inspection is developed to make it possible to estimate the *POD* and *POF* properties only from the results of field inspection of the structures.

THE PROPERTIES OF POD AND POF OF VISUAL INSPECTION METHOD

The POD property

As mentioned before, the cracks of the same length existing in the structure have not always the same detection probabilities by visual inspections. However, these cracks exert the same bad influence on the structural safety. The detection probability depends on the location and orientation of crack and feature of crack such as the shape of crack line, the gap of crack width, rust or corrosion which covers the crack surface, the surface roughness condition, and the environment in which the inspection takes place and so on.

Figure (1) shows a schematic sketch of the surface cracks of length *a*, in which cracks are classified into six groups from the viewpoint of detectability level. In the figure only the differences of the gap of the crack width and surface conditions are compared. This classification is carried out based on the subjective judgment of inspectors referring their inspection experiences for the similar structures.



q_i : Probability of existence of 'Class i' cracks

$p_i(a)$: Probability of detection of 'Class i' cracks

Figure 1. Schematic sketch of surafce cracks of length a.

For the crack of length *a* existing in the structure to be inspected, if the probability of existence of cracks belonging to class *i* condition is q_i. If the detection probability of class *i* cracks by visual inspections is $p_i(a)$. Then the detection probability for all the classes of cracks of length *a* existing in the structure is expressed by the following equation.

$$POD(a)=q_1\, p_1(a)+q_2\, p_2(a)+.....+q_i\, p_i(a) \quad (1)$$

where,

$$q_1+q_2+.....+q_i = 1.0 \quad (2)$$

$$1.0 \geq p_1(a) \geq p_2(a) \geq ..... \geq p_i(a) \geq 0.0 \quad (3)$$

Figure (2-a) shows an appearance of the detection probability defined by Eq.(1). It is obvious that *POD(a)* is the mean value of the distribution of the detection probability $p_i(a)$. If the classification of crack is carried out in more and more detail, a continuous distribution of $p_i(a)$ will be obtained as shown in Figure (2-b). The shape of the distribution will be influenced by both $p_i(a)$ and q_i. *POD(a)* can still be defined as the expectation, even for such a continuous distribution. Here in after we refer to the *POD* defined by Eq.(1) as the mean *POD* and the $p_i(a)$ as the *POD* for class *i* crack.

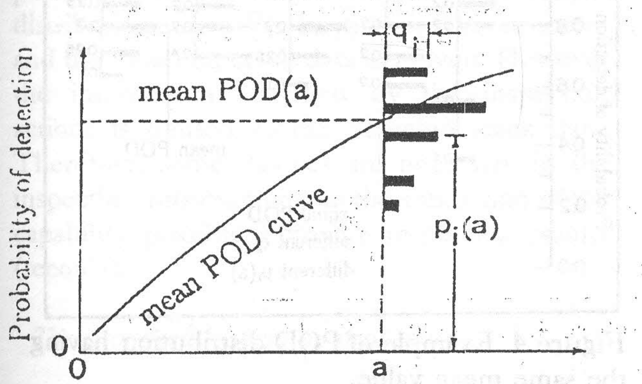


Figure 2-a. Discrete expression of probability of crack detection.

The q_i and p_i *(a)* are interpreted in the following way:

(i) The values of q_i's depend on the structural geometry, the feature and the cause of cracks (fatigue cracks, welding defects, etc.), the corrosion environment, the surface condition, the loading condition, etc. That is, q_i 's are settled when the structure (including environmental effect) and the number of inspections are specified. Therefore, all the factors related to the structure side other than crack length are to govern q_i's. However, the values of q_i's are though to be different depending on the objective structure.

(ii) The values of $p_i(a)$'s, which are the functions of crack length, depend on the inspector's ability (his experience, attitude toward the inspection), the allowed time for inspection, the performance of the inspection tool, etc. Therefore, all the factors related to the inspection side are to govern p_i *(a)* 's. That is, p_i *(a)* 's are determined by who inspects the structure using which method, following which way, under which condition.

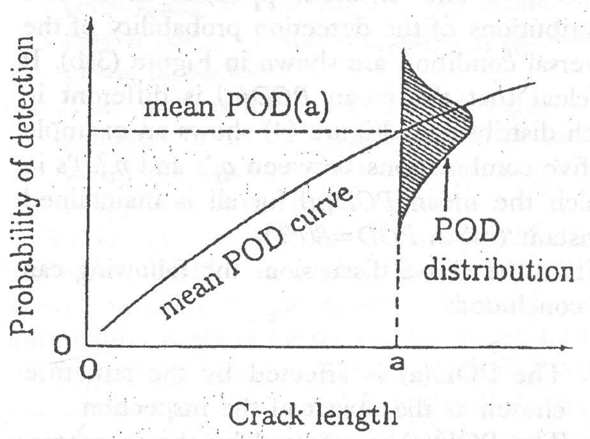


Figure 2-b. Continuous expression of probability of crack detection.

At this point, it must be noted that, when we apply the mean *POD* curve obtained by inspection of a structure with known crack conditions to the inspection reliability analysis of another structure. All the conditions with respect to the inspection method and the latent cracks must be nearly the same between the two structures. It is ideal that *POD* curve can be estimated through field inspections of objective structures.

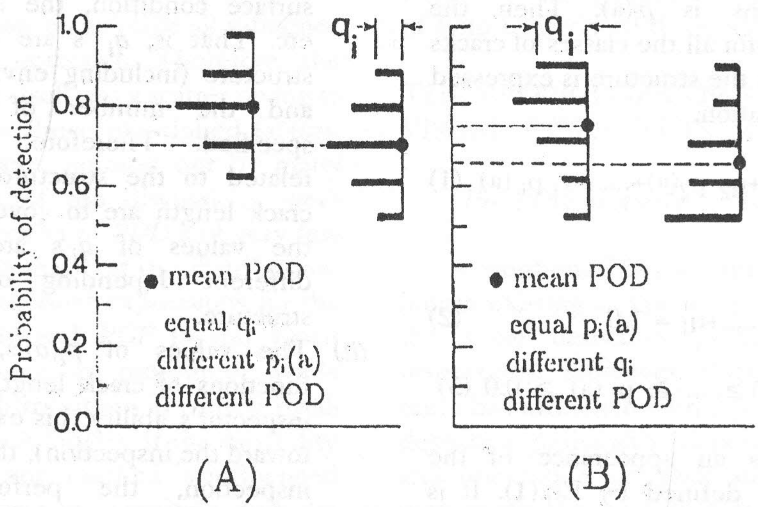


Figure 3. Example of POD distributions with different q_i and $p_i(a)$.

Figure (3-a) shows two distributions of the detection probability with the condition of the same q_i 's and different p_i *(a)*'s. Other two distributions of the detection probability of the reversal condition are shown in Figure (3-b). It is clear that the mean *POD(a)* is different in each distribution. Figure (4) shows an example of five combinations between q_i's and $p_i(a)$'s in which the mean *POD(a)* for all is maintained constant (mean *POD*=80%).

From the above discussion, the following can be concluded:

(i) The POD(a) is affected by the structure chosen as the object of the inspection.
(ii) The POD(a) is affected by the inspector, the applied inspection method, and the inspection condition including environmental effect.

The POF property

Many causes of false crack indications in the visual inspection of structures could be obtained such as mistaking the dirt or scratch for crack, the close resemblance between weld toe configuration and crack and so forth. When all the causes of the false crack indications are classified into certain groups from the viewpoint of how the cause is prone to false indication, the *POF* can be expressed by the following equation.

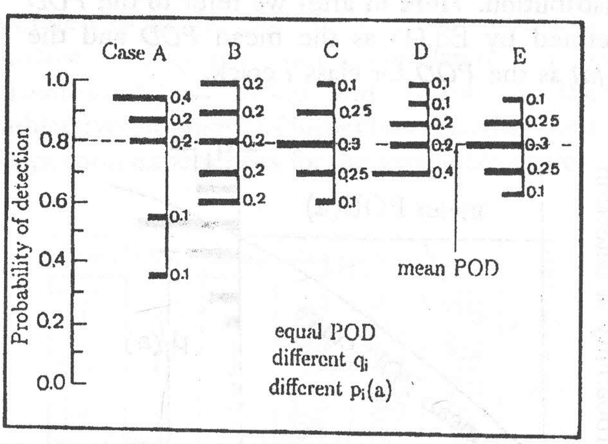


Figure 4. Example of POD distribution having the same mean value.

$$POF= r_1 s_1+r_2 s_2+.....+r_j s_j \tag{4}$$

where,

$$r_1+r_2+.....+r_j = 1.0 \tag{5}$$

$$1.0 \geq s_1 \geq s_2 \geq ..... \geq s_j \geq 0.0 \tag{6}$$

In the above, false indications are classified into *j* groups. In which r_j is the probability of existence of the causes belonging to class *j* condition in all the causes liable to false indications. The s_j is the probability of the false crack indication due to the causes belonging to class j condition. The structure of Eq.(4) is completely the same as that of Eq.(1). It is also noticed that the probabilities of existence of the causes belonging to class j are though to be different depending on the structure characteristics while the probability of the false crack indication depending on the accuracy of the applied inspection method. Therefore, the following becomes clear about the *POF* property of inspection. The value of *POF* will be affected by both the structures to be inspected and the applied inspection method to the structure.

ESTIMATION OF VISUAL INSPECTION CAPABILITY BASED ON THE RECORDS OF FIELD INSPECTION OF STRUCTURES

In the previous section, it is pointed out that the capability of visual inspection should be evaluated after an objective structure and an inspection method is specified. In general, the estimation of the inspection capability becomes possible only when both the true crack length distributions actually existing in the structure and the detected crack data are given. However the information provided by the inspection actions is limited to the detected crack data. Therefore, some devices are necessary in the inspection process to make the estimation of the capability possible from the inspection results (records).

The proposed inspection procedure

The proposed inspection procedure consists of two steps; multiple visual inspections and successive precise inspection using instrument. The inspection procedure and the relevant assumptions are summarized as follows:

(1) *n* inspectors carry out visual inspections independently for the whole structure using the same inspection method and report the inspection results respectively.

(2) The inspection abilities of all inspectors are assumed to be identical and the same ability is maintained throughout the inspection.

(3) There are possibilities of missing cracks and false indications in the visual inspections. Furthermore, the measured crack sizes are usually not accurate and sometimes uncertain.

(4) Precise inspections are carried out for the locations where at least one or more inspectors got positive indications by visual inspections. It is assumed that the missing of cracks and false crack indications never occur in the precise inspections. Furthermore, the measured crack sizes are perfectly accurate.

(5) The detected cracks and the false cracks are classified into several groups from the viewpoint that to detect a crack is easy or difficult and to obtain a false indication often occurs or seldom occurs, respectively. These classifications are carried out considering all the factors other than crack length based on the subjective judgments of the inspectors.

(6) The cracks belonging to the same class are assumed to be the samples from the same statistical population. The same assumption is applied for the false cracks[8].

Table (1) shows an example of the inspection results (records) obtained by the above inspection procedure. Although no information is given for the cracks missed by all the inspectors in the visual inspections *(k=0)*, one can deduce the ratios of *(k/n)* and the true lengths of the cracks detected by one or more inspectors' *(k>0)*. Also the false cracks can be distinguished from the true cracks by these results. In the table the true cracks and the false cracks are classified into three groups and two groups, respectively.

Estimation of p_i *(a)* curves

As the function of the detection probabilities $p_i(a)$ the following form was assumed[4].

$$p_i(a) = \frac{\exp(\alpha_i + \beta_i \times \ln(a))}{1 + \exp(\alpha_i + \beta_i \times \ln(a))} \qquad (7)$$

Table 1. Example of inspection results.

Structure member	Location	Visual inspection [k/n]	Visual inspection Measured crack length (mm)	Precise inspection Ture crack length (mm)	Precise inspection Crack condition
A	12	2/3	about 50	65	Class 3
	25	1/3	not clear	F.I.	" " 2F
	27	3/3	60 ~ 70	75	" " 1
B	6	3/3	45 ~ 55	60	Class 2
	9	2/3	not clear	37	" " 2
	17	1/3	20 ~ 35	23	" " 1
	20	2/3	about 60	55	" " 1
	28	3/3	80 ~ 90	80	" " 3
C	1	2/3	30 ~ 45	50	Class 2
	13	1/3	25 ~ 30	F.I.	" " IF
	10	3/3	about 90	112	" " 3
⋮	⋮	⋮	⋮	⋮	⋮
Z	4	1/3	about 60	64	Class 2
	11	2/3	80 ~ 80	103	" " 2
	16	3/3	50 ~ 65	40	" " 1
	25	1/3	not clear	F.I	" " 2F
	27	2/3	25 ~ 35	22	" "3

F.I. False indication.

In which i means the cracks of class i condition, a is the crack length and are the unknown parameters.

When n inspectors with the same inspection ability inspect a crack. If the event of the inspection results is that k inspectors succeed to detect the crack and $(n-k)$ fail. The probability of occurrence of the event (likelihood function) can be expressed by the following equation under the condition that at least one inspector found the crack $(k>0)$.

$$P_m\,(k/n)= \frac{C_k^n \times p_i^k(a)(1.0-p_i(a))^{(n-k)}}{(1-(1.0-p_i(a))^n)} \qquad (8)$$

Figure (5) shows the inspection procedures, events, and results in case that three inspectors are carrying out the inspection. It is noticed that Eq.(8) is applicable only for the straight line events in Figure (5). The relationship between the $P_m\,(k/n)$ and $p_i(a)$ given in Eq.(8) is shown in Figure (6).

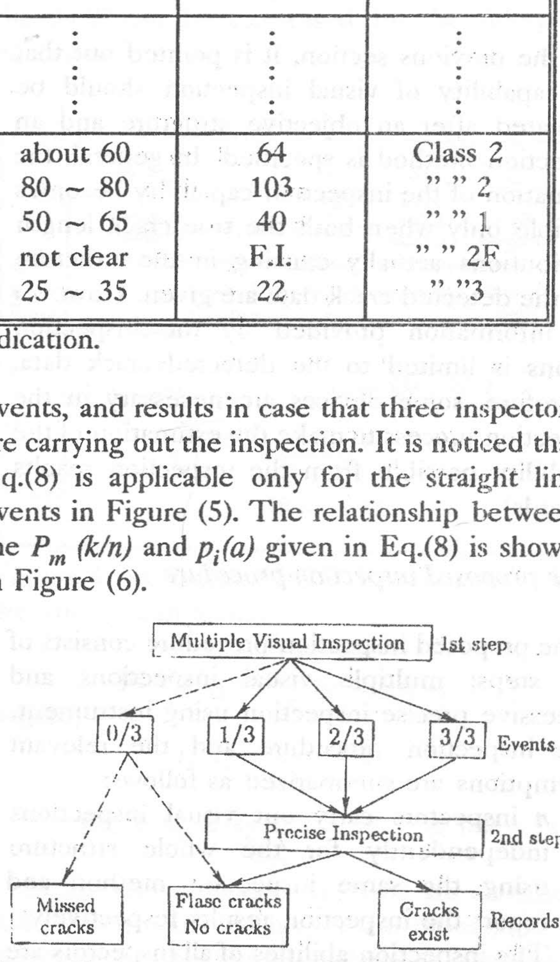


Figure 5. Inspection procedures and events.

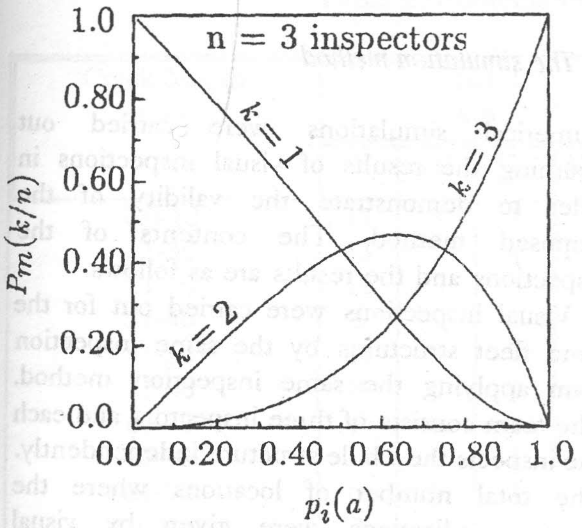


Figure 6. The relationship between $P_m(k/n)$ and $p_i(a)$.

The likelihood function of for all detected cracks belonging to class i condition is given by.

$$L(\alpha_i, \beta_i) = \Pi_{m=1}^{m_D} P_m(k/n) \tag{9}$$

Where m_D is the number of detected class i cracks. Of course the value of k and a are different for each crack in the calculation of $p_i(a)$.

The most probable values of are to be estimated by the Bayes's theorem[9].

$$g_{(\alpha_i,\beta_i)}(x,y) = \frac{L(\alpha_i,\beta_i)\times F_{(\alpha_i,\beta_i)}(x,y)}{\sum_x\sum_y L(\alpha_i,\beta_i)\times F_{(\alpha_i,\beta_i)}(x,y)} \tag{10}$$

The prior joint density function of denoted by $F_{(\alpha_i,\beta_i)}(x,y)$, in the above equation, was determined such that true values of (α_i,β_i) included in the range of the assumed distribution. The prior density was assumed to be two dimensional uniform distribution. The posterior joint density function of (α_i, β_i), denoted by, $g_{(\alpha_i,\beta_i)}(x,y)$ was estimated by substituting Eq.(9) into Eq.(10). Then the expected values of (α_i,β_i) were calculated by performing the marginal integration for the posterior density function. Substituting the expectation into Eq.(7), the $p_i(a)$ curve for the class i cracks can be obtained.

Estimation of probabilities q_i

The probabilities q_i's reveal the ratios of class i cracks to all the cracks existing in the structure. This probability is determined irrespective of crack length. However, the value of q_i may not be constant throughout the whole range of the crack length. For example, such event may happen that most of the long cracks belong to the high detectability class and most of the short cracks belong to the low detectability class.

Let m_i be the number of detected class i cracks whose lengths are between a_1 and a_2 as shown in Figure (7). The number of class i cracks actually existing in the structure, denoted by M_i can be estimated approximately by the following equation.

$$M_i = \frac{m_i}{1.0-(1-p_i(\frac{a_1+a_2}{2.0})^n)} \tag{11}$$

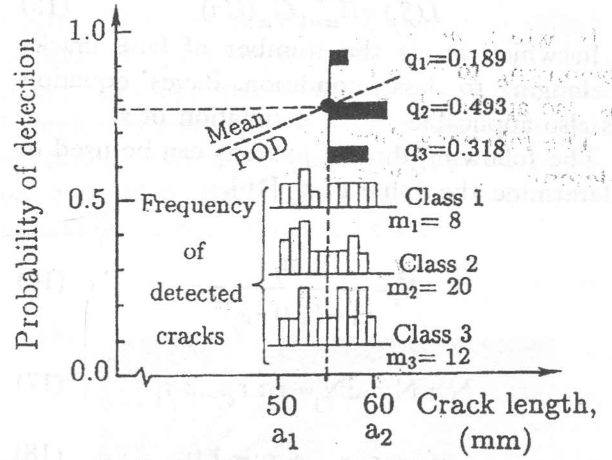


Figure 7. Estimation of probabilities q_i.

According to the definition, q_i must satisfy the following two equations.

$$M_1 : M_2 : \ldots\ldots : M_i = q_1 : q_2 \ldots\ldots : q_i \tag{12}$$

$$q_1 + q_2 + \ldots + q_i = 1.0 \tag{13}$$

Solving the above two equations, the

probabilities $q_1, \ldots, q_i$ can be determined[10] for the cracks with lengths between a_1 and a_2. The probabilities q_i's for other crack lengths can be calculated similarly. The number of divisions of the crack lengths will be determined by the subjective judgment based on the quantity of the crack data.

Estimation of s_j and r_j

For the false crack indications, the similar method is applicable to the estimation of s_j and r_j. Assume a false crack for which k inspectors out of n reported incorrect positive indication by visual inspection. The likelihood function of this event can be written in the following form under the condition that $(k>0)$.

$$Q_m(k/n) = \frac{C_k^n \times s_j^k (1.0 - s_j)^{(n-k)}}{1 - (1.0 - s_j)^n} \quad (14)$$

The likelihood function for all of the false cracks of class j condition is given by.

$$L(S_j) = \Pi_{m=1}^{m_F} Q_m(k/n) \quad (15)$$

In which m_F is the number of false cracks belonging to class j condition. Bayes' equation is also applicable to the estimation of s_j.

The following three equations can be used to determine the value of r_j [10].

$$N_i = \frac{m_j}{1 - (1.0 - s_j)^n} \quad (16)$$

$$N_1 : N_2 : \ldots : N_j = r_1 : r_2 : \ldots : r_j \quad (17)$$

$$r_1 + r_2 + \ldots + r_j = 1.0 \quad (18)$$

Where N_j is the number of causes belongs to the false indication existing in the structure. m_j is the number of false cracks obtained by the inspection and suffix j expresses class j condition.

NUMERICAL SIMULATION

The simulation method

Numerical simulations were carried out assuming the results of visual inspections in order to demonstrate the validity of the proposed method. The contents of the inspections and the results are as follows.

Visual inspections were carried out for the same fleet structures by the same inspection team applying the same inspection method. The team consists of three inspectors and each one inspects the whole structure independently. The total number of locations where the positive indications were given by visual inspections is 400. Among them the correct positive crack indications (defects) are 320 and the incorrect positive indications (false cracks) are 80. In the precise inspections, the cracks were classified into three conditions, namely Class 1 (detection is easy), Class 2 (detection is moderate), and Class 3 (detection is difficult), considering all the factors except crack length. For the false cracks no classification was performed.

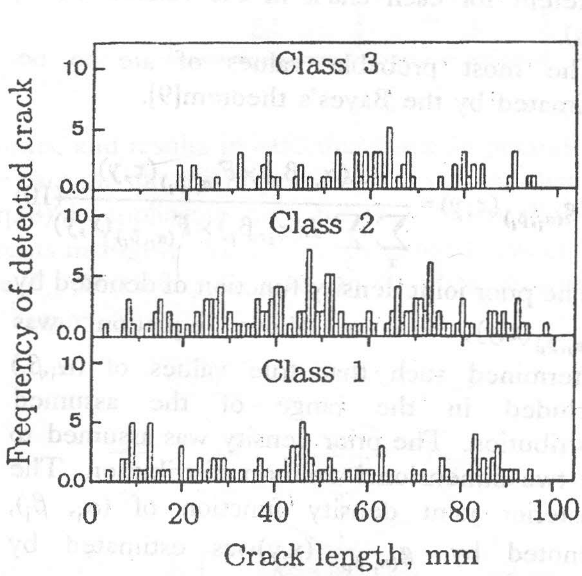


Figure 8. Histogram of the detected crack length.

Table 2. Contents of the detected cracks by VI's.

Crack length mm	Class 1 k/n				Class 2 k/n				Class 3 k/n			
	Σ	1/3	2/3	3/3	Σ	1/3	2/3	3/3	Σ	1/3	2/3	3/3
0.0-10	11	9	2	0	6	6	0	0	0	0	0	0
11-20	13	9	4	0	11	8	3	0	2	2	0	0
21-30	12	3	5	4	19	13	4	2	4	4	0	0
31-40	6	1	1	4	18	1	7	10	11	5	6	0
41-50	22	3	7	12	24	6	9	9	5	2	2	1
51-60	8	0	1	7	20	4	12	4	12	5	6	1
60-70	6	0	1	5	16	4	5	7	19	7	9	3
71-80	4	0	1	3	20	3	11	6	6	1	2	3
81-90	15	0	2	13	10	1	5	4	6	0	3	3
91-100	3	1	0	2	6	1	5	0	5	0	3	2
Σ	100	26	24	50	150	55	57	38	70	26	31	13

The contents of the 320 detected cracks and the 80 false indications are listed in Table (2) and Table (3), respectively. For detected cracks of each class, Figure (8) shows the histograms of the distributions of crack lengths. The number of cracks in Class 1, Class 2 and Class 3 conditions are 100, 150, and 70 respectively. Those inspection results were simulated by Monte Carlo method, where the number of cracked locations and the distributions of crack lengths, the detection probability curves and the probability of false crack indications had been assumed for the three crack conditions.

Table 3. Contents of the false indications.

k/n	Number of flase crack indication
1/3	69
2/3	11
3/3	0

Results of the Simulation

Figure (9) shows many $p_i(a)$ curves given by Eq.(7) with different values of α_i and β_i (α_i=-1 ~-12 step -1, β_i= 1 ~4 step 0.15). Three *POD* curves were picked up from Figure (9) to obtain the assumed probabilities of crack detection for Class 1, Class 2, and Class 3 respectively. Figure (10) and Figure (11) show the estimated and the assumed p_i *(a)* curves for the three classes. The estimated and the assumed values of and are given in the upper table of each figure. Through the comparison of these two figures, it is seen that the estimated $p_i(a)$ curves are closed to the assumed one. Therefore, it can be concluded that p_i *(a)* can be estimated accurately by the present method.

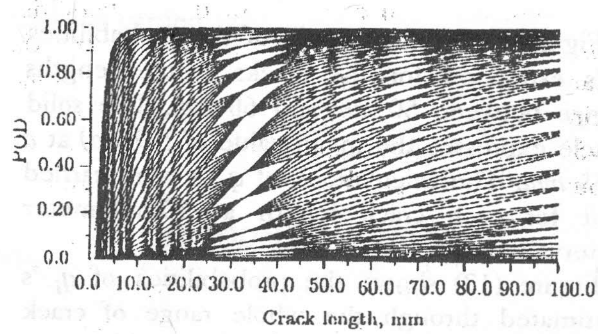


Figure 9. $P_i(a)$ curves.

	Class 1	Class 2	Class 3
α	−6.215	−6.536	−9.041
β	2.107	1.794	2.280

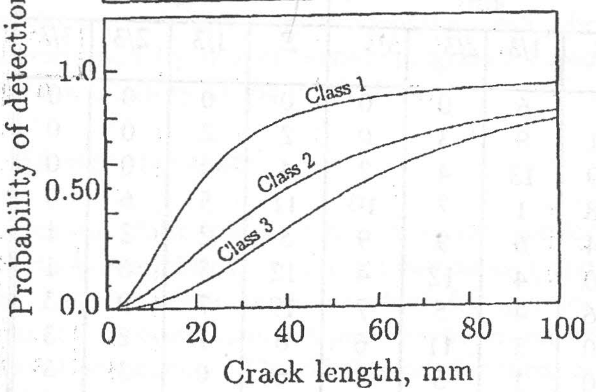


Figure 10. Estimated $p_i(a)$ curves.

	Class 1	Class 2	Class 3
α	− 8.5	− 7.0	−9.0
β	2.5	1.9	2.2

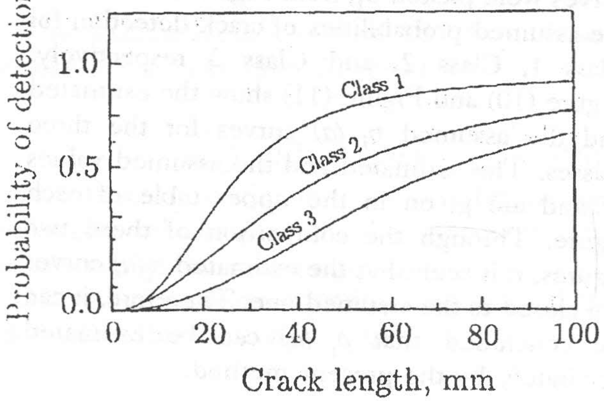


Figure 11. Assumed $p_i(a)$ curves.

Figure (7) is an example of the probabilities q_i's estimated for the cracks whose lengths range between 50 *mm* and 60 *mm*. The solid circle expresses the mean value of *POD(a)* at a =55 *mm*. The calculations of q_i 's were carried out for each crack length at 10-millimeter intervals.

Figure (12) shows the probabilities of q_i 's estimated through the whole range of crack length. The mean *POD(a)* curve which was calculated by substituting the p_i (a) curves and probabilities q_i 's into Eq.(1) is also shown in the figure. One can comprehend precisely the characteristics of the applied inspection method and the crack conditions in the structure. That is, the three p_i *(a)* curves express respectively the capabilities of the inspection method for class i cracks, and the height of the histograms represent the percentages of class i cracks existing in the structures.

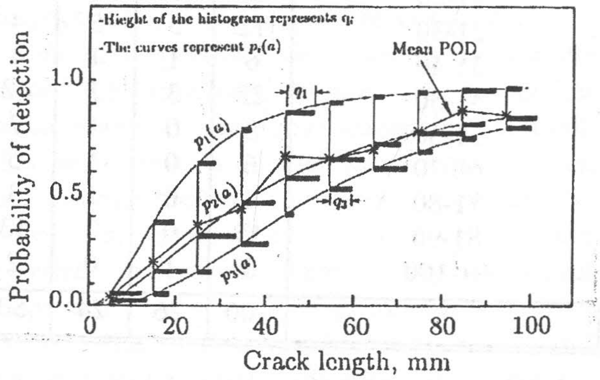


Figure 12. Capability of applied inspection method and crack condition in the structure.

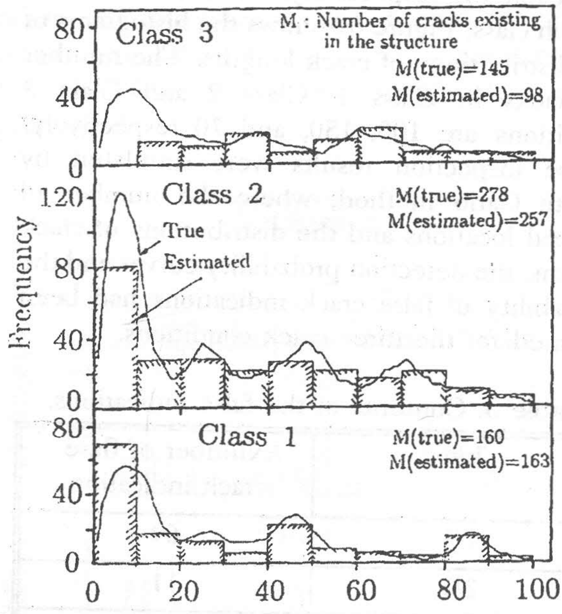
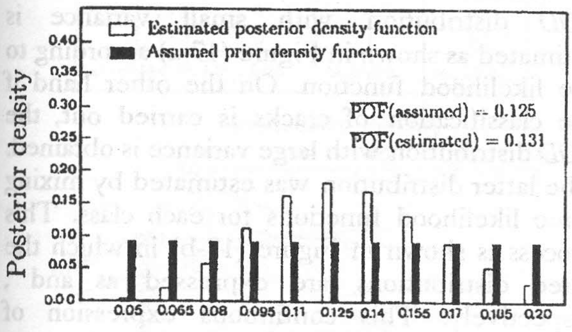


Figure 13. Estimated anmd assumed distributions of crack length.

Figure (13) shows the distributions of the number of cracks existing in the structure estimated by Eq.(11). The distributions of the true cracks assumed in the simulation are also shown in the figure. Fairly good agreement can be seen between the estimated and true distributions of the number of cracks. By using this figure, the number and length distributions of cracks which remain after inspection can be easily estimated.



Probability of false crack indication (POF)

Figure 14. Probability of fals indication.

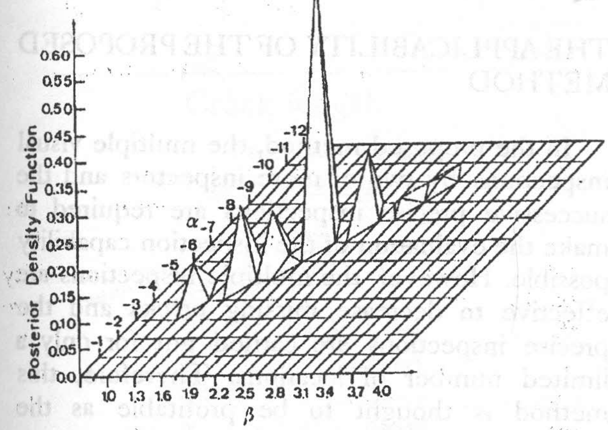


Figure 15. Estimation of α and β.

Figure (14) shows the posterior probability density function of the false crack indications calculated by the Bayes' equation as well as the prior density function of that. The true value assumed as the probability of false crack indication is 0.125. The expectation of the posterior distribution is 0.131 which is close to the true value. For the true crack indications, two dimensional uniform distributions were used as prior density for the parameters α_i and β_i in Eq.(7). Figure (15) shows the peak value at which the posterior probability density function of the true crack indications for Class1 is calculated by Bayes' equation. From the figure, the expected values of α and β of that class can be easily estimated.

The Benefit of the Crack Classification Method

The value of the crack classification method can be understood clearly from the following simple examples.

Example 1: Figure (16) is a result of visual inspection carried out by three inspectors A, B and C independently for a structural model in which *M* artificial cracks of the same length are prepared. The number of cracks detected by each inspector is equally six, and totally ten cracks are detected.

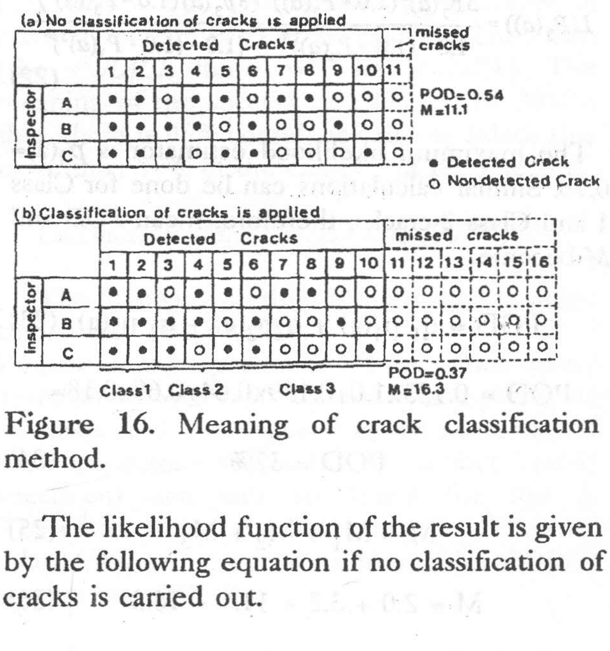


Figure 16. Meaning of crack classification method.

The likelihood function of the result is given by the following equation if no classification of cracks is carried out.

$$L(POD) = \left(\frac{POD^3}{1.0-(1-POD)^3}\right)^2 X \left(\frac{3XPOD^2(1.0-POD)}{1.0-(1-POD)^3}\right)^4 X \left(\frac{3XPOD(1.0-POD)^2}{1.0-(1-POD)^3}\right)^4 \quad (19)$$

The maximum likelihood estimator of *POD* and *M* are estimated as follows.

$$\text{POD} = 54\% \tag{20}$$

$$M = \frac{10}{1.0-(1.0-0.54)^3} = 11.1 \tag{21}$$

This means that about one crack is missed by all the three inspectors (see Figure (16-a)).

On the other hand, if the cracks are classified into three groups as shown in Figure (16-b) from the viewpoint of detectability level, three likelihood functions are obtained. For example the likelihood function for Class 3 cracks is given by.

$$L(P_3(a)) = \frac{3p_3(a)^2(1.0-P_3(a))}{1.0-(1.0-P_3(a))^3}\,\frac{(3p_3(a)(1.0-P_3(a))^2)^4}{(1.0-(1.0-P_3(a)^3)^4} \tag{22}$$

The maximum likelihood estimator is p_3(a)= 0.18. Similar calculations can be done for Class 1 and Class 2 cracks, therefore, mean *POD* and *M* become.

$$\text{POD} = q_1\, p_1(a) + q_2 p_2(a) + q_3\, p_3(a) \tag{23}$$

$$\text{POD} = 0.123\text{x}1.0+0.139\text{x}0.64+0.68\text{x}0.18=$$

$$\text{POD} = 37\% \tag{24}$$

$$M = M_1 + M_2 + M_3 \tag{25}$$

$$M = 2.0 + 3.2 + 11.1 = 16.3 \tag{26}$$

This suggests that, if the crack condition is good (Class 1), the detection probability is almost 100%; however if it is bad (Class 3), the detection probability is only 18% and about six cracks are missed by all the three inspectors.

The difference between the above two results is due to the subjective classification of cracks. Therefore, the accuracy of the estimated inspection capability can be increased if the difference of crack conditions is well reflected in the classification of cracks.

Example 2 : The likelihood function of *POD* or p_i(a) can be used as the posterior probability density function of detection probability by adjusting the area of the likelihood distribution at unity. Let the table in the Figure (17) be the inspection result for a specified crack length. If no classification of cracks is carried out, the *POD* distribution with small variance is estimated as shown in Figure (17-a) according to the likelihood function. On the other hand if the classification of cracks is carried out, the *POD* distribution with large variance is obtained. The latter distribution was estimated by mixing three likelihood functions for each class. This process is shown in Figure (17-b) in which the three distributions are expressed as and , respectively. This continuous expression of *POD* distributions must be useful when we discuss the confidence level of *POD* such as 90/95 level. And the more accurate estimation of confidence level can be expected by the application of crack classification method.

THE APPLICABILITY OF THE PROPOSED METHOD

In the proposed method, the multiple visual inspections by two or more inspectors and the successive precise inspections are required to make the evaluation of the inspection capability possible. However, the multiple inspections are effective to decrease missing cracks and the precise inspections are carried out for only a limited number of locations. Therefore, this method is thought to be profitable as the inspection method of actual structures.

The following are thought to be the merits of the proposed method.

(1) The mean of *POD* curve and the distribution property of the *POD* can be estimated from the records of the field inspections carried out for a structure by an inspection team under a specified inspection condition.

(2) The number of cracks remaining after the inspections and the length distribution of residual cracks can be easily estimated without using any information other than inspection results.

(3) Once the detection probabilities are evaluated in the manner shown in Fig.12,

the capability of applied inspection method can be understood more precisely as compared with the ordinary mean *POD* curve.

		Class 1	Class 2	Class 3
k/n	3/3	5	4	2
	2/3	4	4	4
	1/3	0	1	2
Estimated M_i		M_1=9.05	M_2=9.15	M_3=8.49

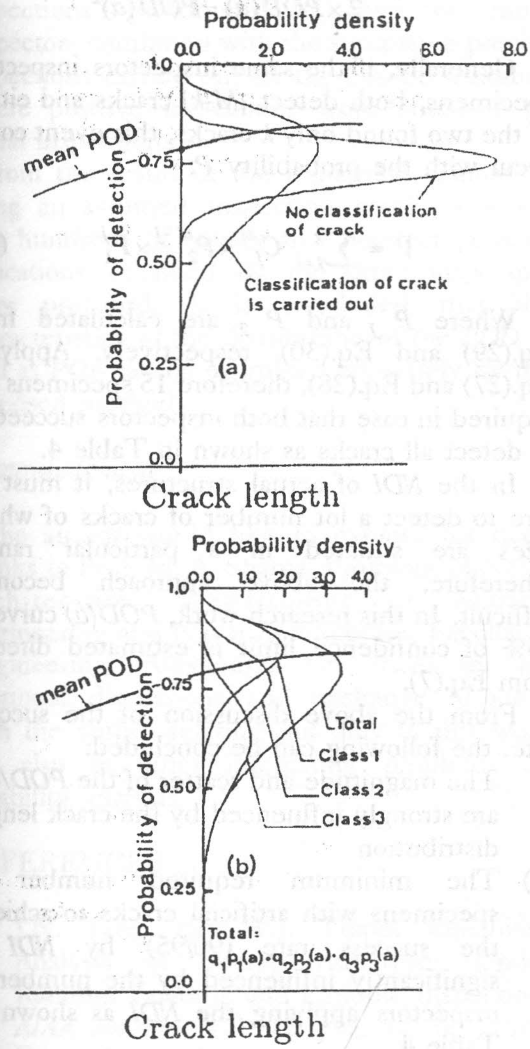


Figure 17. Application of crack classification method to the continuous distribution of POD.

When the proposed method is actually applied, further considerations must be given to the following issues:

(1) The inspectors' abilities to detect cracks have to be the same and independent for a crack. Whether these conditions are satisfied or not in the current inspections, can be judged to a certain extent by careful observation of the inspection results.

(2) The difference of the detectability must be well reflected in the classification of cracks, because the estimated result is influenced considerably by the classification.

SUCCESS RATE (90/95) BY *NDI*

As mentioned before, no inspection procedure could provide 100% assurance that all cracks greater than some useful length will be detected. The current capabilities and uncertainty resulting from the *NDI* dictate that the minimum detectable crack length must be specified in term of a high confidence level (*CL*) that a high percentage of probability of crack detection *POD* of all cracks greater than [the *POD/CL* limit] will be found[4]. The meaning of the success rate (90/95) by *NDI* is that, there is 95% confidence (lower sided) that more than 90% of the cracks will be detected.

Calculation of the success rate (90/95)

The following method is often used to judge the success rate (90/95):

Prepare many specimens M in which many artificial defects are prepared. The specimens are inspected successively by an inspector. If the inspector succeeds to detect *(M-k)* specimens and fails to detect the rest *k*. Therefore, *M* and *k* must satisfy the following relationship in order to achieve the *POD(a)* success rate (90/95):

$$\sum_{j=1}^{k} C_j^M POD(a)^{M-j}(1.0-POD(a))^j \geq \sum_{j=1}^{k} C_j^k \times 0.9^{M-j} \times 0.1^j = p^* \qquad (27)$$

If *POD(a)* is more than 90%, the probability that the inspector fails to detect *k* or less specimens is given by the above expression. The right hand side of the above equation expresses the probability that such event occurs when the *POD(a)* is just 90%. Even if *POD(a)* is less than 90%, actually the obtained event could occur within a probability p^*. When it is said that $POD(a)$ is equal to 90% or more, this expression has an accuracy of $(1-p^*)$.

The minimum required number of specimens M, which satisfies the following expression for a specified k, is the necessary condition in order to achieve the success rate (90/95).

$$\sum_{j=1}^{k} C_j^M \times 0.9^{M-j} \times 1.0^j < (1.0-0.95) \quad (28)$$

When an inspector inspects M specimens and detects all the cracks $(k=0)$, the required numbers of specimens M is 29 to satisfy Eq.(28). In this case it can be said that the success rate (90/95) is satisfied. However, in case that one crack is missed $(k=1)$, then M becomes 46 to maintain the same success rate. Table (4) shows the minimum required number of specimens to be inspected in different conditions.

Table 4. Minimum required number of specimens to achieve the success rate (90/95) by the NDI.

Number of inspectors	The obtained event by the inspection	Minimum M
One	Scucces to find M cracks	29
	Scucces to find M-1 cracks & fails to find the rest crack	46
Two	Both Scuccess to find M-1 cracks &	15
	Both success to find m-1 cracks & either fails to find the rets crack	25
Three	All sucess to find M cracks	10
	All success to find M-1 cracks & either of three fails to find the rest crack	16

This method is useful to estimate the success rate (90/95) in the *NDI* using artificially prepared specimens, for which the crack data in each specimen is known before the inspection. However, this method is not applicable to the *NDI* of the actual structures, because it is difficult to distinguish between the missed crack and no crack indications. But when the proposed inspection method is used, the judgment of the success rate (90/95) becomes possible.

For Example, if two inspectors inspect one specimen independently, the events that could be obtained are either of both found crack or either of the two found crack. The probabilities of these two events are given respectively by P_1 and P_2 in the following two equations.

$$P_1 = \frac{2 \times POD(a) \times (1.0 - POD(a))}{2 \times POD(a) - POD(a)^2} \quad (29)$$

$$P_2 = \frac{POD(a)^2}{2 \times POD(a) - POD(a)^2} \quad (30)$$

Generally, if the same inspectors inspect M specimens, both detect $(M-k)$ cracks and either of the two found only k cracks, this event could occur with the probability P:

$$P = \sum_{j=1}^{k} C_j^M \times P_2^{M-j} \times P_1^j \quad (31)$$

Where P_1 and P_2 are calculated from Eq.(29) and Eq.(30), respectively. Applying Eq.(27) and Eq.(28), therefore 15 specimens are required in case that both inspectors succeeded to detect all cracks as shown in Table 4.

In the *NDI* of actual structures, it must be rare to detect a lot number of cracks of which sizes are situated in a particular range. Therefore, the above approach becomes difficult. In this research work, $POD(a)$ curve of 95% of confidence limit is estimated directly from Eq.(7).

From the above discussion of the success rate, the following can be concluded:

(i) The magnitude and scatter of the *POD/CL* are strongly influenced by the crack length distribution.

(ii) The minimum required number of specimens with artificial cracks to achieve the success rate (90/95) by *NDI* is significantly influenced by the number of inspectors applying the *NDI* as shown in Table 4.

(iii) Increasing the sample size in the real structures, would increase the precision of the *POD/CL* estimates and also would decrease the scatter in *POD/CL* estimates.

CONCLUSION

In this study, the structural expression for the distributions of the *POD* and *POF* in the visual inspection method has been presented. The scatters in the distributions are caused by many factors related to the applied inspection method and the defect condition in the structure.

For estimating the *POD* and the *POF* properties from the results of field inspections of the structures, an inspection procedure and its analysis method have been presented. The inspection consists of the multiple visual inspections performed by two or more inspectors combined with the successive precise inspections carried out for the limited locations where positive indications were obtained by visual inspections.

From the result of the numerical simulation using an assumed inspection result in which four hundreds of correct and incorrect positive indications obtained by the three inspectors were prepared. It is concluded that the characteristics of the distribution of the *POD* as well as *POF* can be estimated accurately by the proposed method.

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