# EFFECTS OF PUMPED WELLS ON SALT WATER INTRUSION IN COASTAL AQUIFERS


**M.A. El-Ganainy, F.A. El-Fitiany**
Hydraulics & Irrigation Dept., Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

**M.M. Elafify**
Unclear Eng., Dept., Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

**and**

**Z.M. Shuluma**
Hydraulics & Irrigation Dept., Faculty of Engineering,
Alexandria University, Alexandria, Egypt.



## ABSTRACT

**A quasi-3 D, steady state, areal, finite element model has been developed for simulating the effects of a group of fully penetrating wells, of general coordinates, on the fresh-salt water interface in coastal regions, for confined and unconfined aquifers. In the present work, the Reddy's 2-D finite element code, (FEM2D), [12] (1993) which is usually used to solve problems of single phase flow through homogenous isotropic soil domains, is modified using the vertical integration approach to study the problem of fresh salt water intrusion in the presence of pumped wells. The results of the developed model are compared with previous results of analytical and numerical models. The model can also be used to study the effects of uniform surface recharge as well as recharge to ; and/or extraction from; different patterns of wells on the shape and location of the interface.**



*Keywords*: Salt water intrusion, Pumped wells, Coastal Aquifers, Unconfined and Confined aquifers, Finite element.


## NOTATIONS

The following symbols are used in this paper:

- $a$ an indicating factor, $a = 1.0$ for unconfined aquifers and $a = 0.0$ for confined aquifers;
- $B$ distance between two successive wells, in the y-direction;
- $b$ distance between two successive rows of wells, in the x-direction;
- $b_f, b_s$ fresh and salt water thicknesses;
- $f$ a subscript denoting the fresh water domain;
- $H_o$ sea water level above datum;
- $h_f, h_s$ potential heads in fresh and salt water domains;
- $h_f^o$ potential head at the top of the outlet face;
- $k_x, k_y$ hydraulic conductivities in the x and y directions;
- $k$ hydraulic conductivity for homogeneous, isotropic aquifer, for fresh water;
- $L$ horizontal distance of the toe from the origin;
- $N_w$ number of wells;
- $q_i$ upstream fresh water discharge into the flow domain,per unit length
- $q_o$ downstream fresh water discharge into the sea, per unit length;
- $Q_{wi}$ discharge extracted from the ith well;
- $R$ uniform recharge per unit surface area;
- $s$ a subscript denoting the salt water domain;
- $SF$ a factor equals to 1.0 if an outlet face exists and equal to 0.0 if not;
- $X$ horizontal distance from origin, perpendicular to the cost;
- $X_w$ well distance from origin, in the x-direction;

| | |
|---|---|
| y | horizontal distance from origin, parallel to the coast; |
| $Z_B$ | height of the aquifer bottom above datum; |
| $Z_I$ | height of the interface above datum; |
| $Z_I^o$ | height of the interface at the coast above datum; |
| $Z_T$ | the height of the top of confined aquifer above datum; |
| $\beta$ | a parameter which depends upon the slope of the sea bottom; |
| $\delta$ (x) | the Dirac delta function; |
| $\delta_1$ | $= \rho_f / (\rho_s - \rho_f)$; |
| $\rho$ | density of water. |

## INTRODUCTION

In coastal regions, heavy summer pumping for irrigation makes sea water intrusion a concern. The change in the location of salt water interface is an important task when planning for fresh water abstractions from coastal aquifers. Muskat [9] (1937) derived an analytical solution for the case of steady flow to an array of wells parallel to the coast, operating above a sharp interface. The solution was based on the Ghyben-Herzberg formula and the Dupuit's assumption of no head gradients in the vertical direction which implies an essentially horizontal flow. These assumptions are only justified for aquifers of small thicknesses relative to the well and the toe distances from the coast, and for relatively small interface rises. Bear and Dagan [1] (1963) used Muskat's relationships to determine the efficiency of fresh water interception by a coastal collector. They also modified these relationship for an unconfined aquifer with accretion. Strack [15] (1976) studied the case of a single well, fully penetrating a shallow aquifer, confined or unconfined. He employed the Dupuit's assumption and Herzberg's formula to derive a two-dimensional potential function in the horizontal plane, for the flow in the fresh water zone. The salt water was assumed stationary with a sharp interface and the well is far enough from toe such that no upcoming occurs. Taigbenu et al [16] (1984) developed a numerical model for the same problem, based on the boundary integral method. Their results showed good agreement with Strack's analytical model. Kashef and Smith [6] (1975) studied the areal growth of the salt water zone due to various rates and durations of pumping from a single well, using Thais equations (transient state) for wells in confined aquifers, combined with the natural groundwater flow equations (steady state). Based on the same approach, Kashef [7] (1975) studied also the use of a battery of recharge wells, arranged parallel to the shore line to control salt water intrusion. Pinder and Page [10] (1977) simulated salt water intrusion on the South Fork of Long Island with a group of wells located upstream of the sharp interface zone. Their finite element model utilizes the Dupuit assumption as well as Hubbert's equation [5] (1940) to calculate the position of the sharp interface. Contractor and Srivastava [3] (1990) modified a two-dimensional (areal) finite element model of salt water intrusion, previously developed by Contractor [4] (1983), so it can be run on a microcomputer. They utilized the depth-averaged differential equations for fresh water and salt water, derived by Sa da Costa and Wilson [13] (1979), based on the Dupuit's and sharp interface assumptions as well as Hubbert's equation at the interface. Contractor and Srivastava used the model to study transient salt water intrusion due to combined seasonal variations in rainfall, ocean level and pumping rates, for the Northern Guam Lens.

The main objective of this research is to simulate the steady pumping of fresh water from different patterns of wells as well as the natural fresh water flow to the sea.

A quasi 3-D, steady state, finite element model has been developed to simulate the effects of multiple well system of general coordinates pumping from a coastal aquifer (unconfined or confined) and the effects of uniform rainfall and/ or evaporation from the ground water.

### *The Governing Equations*

Generally, the mathematical formulation of the problem of sea water intrusion with multiple well system is obtained by combining Darcy's equation with the continuity equation, for both fresh and salt water zones, which leads to a three dimensional system of equations. The following assumptions are considered here:

1- Darcy's law is valid,
2- Fresh and salt water are immiscible and separated by a sharp interface and they are of constant densities and viscosites,
3- The Dupuit's approximation holds for unconfined aquifers and essentially horizontal flow occurs in confined aquifers,
4- The wells fully penetrate the aquifer. They are located far enough from the toe of the interface such that no upcoming occurs, and far enough from the shoreline such the discharge, $q_o$, to the sea can be considered uniform,
5- Permeability and storage coefficients are constant along any vertical section,
6- Saturated flow occurs in the fresh water and salt water zones.

The third assumption implies that no head gradients exist in the vertical direction which permits vertical integration of the governing, 3D system of equations, resulting in the following two non-linear, 2D (or quasi, 3D) equations, Bear and Verruijt [2] (1987):

*(a) For fresh water domain:*

$$\frac{\partial}{\partial x}C_1\frac{\partial h_f}{\partial x}+\frac{\partial}{\partial y}C_2\frac{\partial h_f}{\partial y}-\sum_{i=1}^{N_w}\delta(x-x_i)\delta(y-y_i)Q_{wi}+aR=0 \qquad (1)$$

*(b) For salt water domain:*

$$\frac{\partial}{\partial x}C_3\frac{\partial h_s}{\partial x}+\frac{\partial}{\partial y}C_4\frac{\partial h_s}{\partial y}=0 \qquad (2)$$

in which

$$C_1 = (k_x)_f\, b_f \qquad (3)$$

$$C_2 = (k_y)_f\, b_f \qquad (4)$$

$$C_3 = (k_x)_s\, b_s \qquad (5)$$

$$C_4 = (k_y)_s\, b_s \qquad (6)$$

$$b_f = a\, h_f + (1\text{-}a)\, Z_T - Z_I \qquad (7)$$

$$b_s = Z_I - Z_B \qquad (8)$$

$$Z_I = (1 + \delta_1)\, h_s - \delta_1\, h_f \qquad (9)$$

and

$$\delta_1 = \rho_f / (\rho_s - \rho_f) \qquad (10)$$

Equation (9) represents Hubbert's equation [5] (1940) for the boundary condition at the interface. Notations used in equations (1) through (10) are illustrated by Figures (1-a, b,c), $k_x$ and $k_y$ are values of hydraulic conductivity in the x and y directions ($LT^{-1}$); h is the potential head (L); $Q_{wi}$ is the discharge of the "i" th well located at $(x_i,y_i)$ positive for extracting wells and negative for recharging wells ($L^3T^{-1}$); R is the uniform recharge per unit surface area ($LT^{-1}$); $b_f$ is the fresh water thickness (L); $b_s$ is the salt water thickness (L); $Z_I$ is the interface height (L); $Z_T$ is the total height above the datum for confined aquifer (L); a is an indicating factor equals to 1.0 for unconfined aquifers and to 0.0 for confined aquifers; $\delta(x\text{-}x_i)$ and $\delta(y\text{-}y_i)$ are Dirac delta functions centered at the point $(x_i,y_i)$; $N_w$ is the number of wells; $\rho$ is the density of water ($ML^{-3}$) and the subscripts (f) and (s) are notations for fresh water and salt water; respectively. The non-linearity of the differential equation arises from the fact that both thicknesses $b_f$ and $b_s$ are functions of the unknown hydraulic heads $h_f$ and $h_s$.

## *Boundary Conditions*

Referring to Figure (1), there are three types of boundary conditions for the studied area, 1-2,3-4, in the xy plane, which can be classified as follows:

(1) At the upstream boundary (2-3), the value of the fresh water discharge per unit length, $q_i$, is given. This represents Neumann's type of boundary conditions, for which:

$$(k_x)_f\, b_f\, \frac{\partial h_f}{\partial x}+q_i=0 \qquad (11)$$

(2) At the downstream boundary (1-4), the potential head of the salt water along the coast, $H_o$, is assumed to be constant whereas the potential head at the top of the fresh water seepage face, $h_f^o$, is given by [11]:

a- For an unconfined aquifer:

$$h_f^\circ = SF\left(\frac{\delta-1}{\delta+1}\right)^{1/2}\left(\frac{q_o\beta}{k_f}\right) + H_o \qquad (12)$$

b- For a confined aquifer:

$$h_f^\circ = SF\left(\frac{q_o\beta}{k_f}\right) + \left(\frac{\delta_1+1}{\delta_1}\right)H_o - \frac{Z_T}{\delta_1} \qquad (13)$$

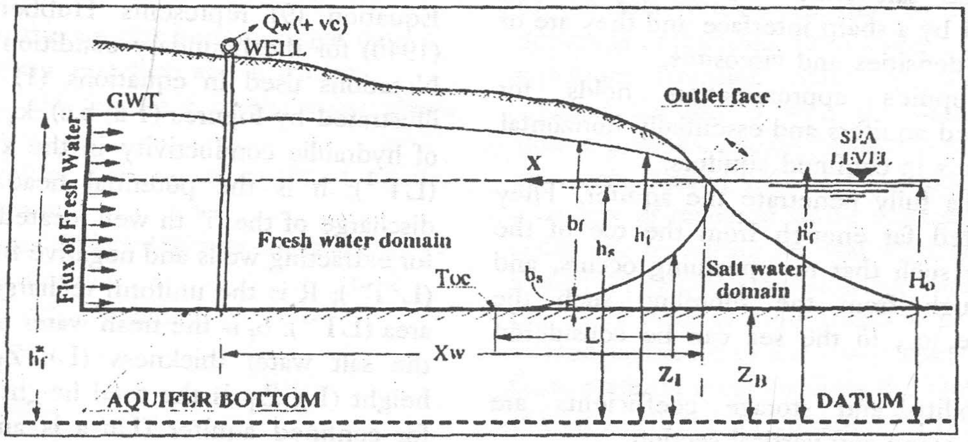


Figure 1-a. Longitudinal section A-A, for an unconfined aquifer.

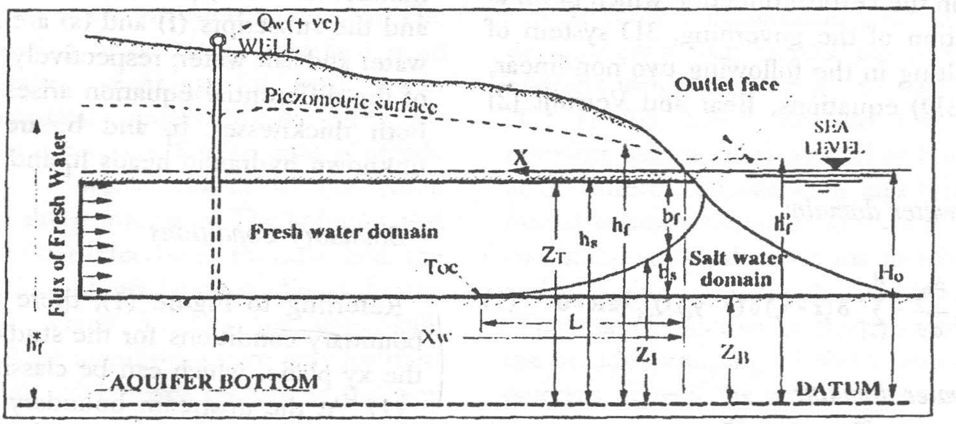


Figure 1-b. Longitudinal section A-A, for a confined aquifer.

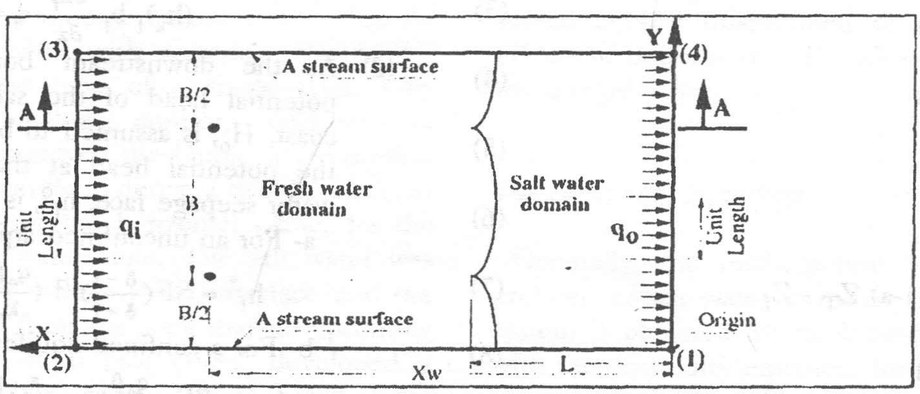


Figure 1-c. Plan, for both unconfined and confined aquifers.

This represents Dirichlet's type of boundary conditions. The interface height at the coast, $Z_i^\circ$, can be directly obtained from Hubbert's equation. Referring to equations (12) and (13), $q_o$ is the rate of fresh water discharge into the sea, per unit length of the coast, SF is a factor equals to 1.0 if an outlet face exists and equals to 0.0 of there is no outlet face, $k_f$ is the hydraulic conductivity for a homogenous isotropic aquifer, for fresh water and $\beta$ is a parameter which depends upon the slope of the sea bottom, taken here as unity. The value of $q_o$ is directly related to the inlet discharge, $q_i$, and the pumping wells discharge, $q_w$:

$$q_o = q_i - q_w \tag{14}$$

where $q_w$ is the total discharge of the wells per unit length of the aquifer, parallel to the coast.

(3) The boundaries (1-2) and (3-4) are assumed to be stream surfaces with no cross flow, hence head gradients in the normal direction are zero, representing Neumann's type of boundary conditions:

$$\frac{\partial h_f}{\partial y} = 0 \tag{15}$$

$$\frac{\partial h_s}{\partial y} = 0 \tag{16}$$

*Finite Element Equations*

According to Galerkin's approximation, and by using Gauss divergence theorem and the integration of the divergence of the flux terms as developed by Warner [17] (1987), the finite element algebraic equations equivalent to equations (1) and (2) are:

Fresh water domain:

$$[K_f]\{h_f\} + \{B_f\} = \{F_f\} \tag{17}$$

Salt Water domain:

$$[K_s]\{h_s\} = \{F_s\} \tag{18}$$

Where $[K_f]$ and $[K_s]$ are the element matrices given by Reddy and Warner [12,17], and;

$$(B_i)_f = -\int\int_A (N_i(x,y) \sum_{j=1}^{N_w} \delta(x-x_j)\, \delta(y-y_j)\, Q_{wfj})\, dxdy$$

$$+ \int\int_A (aR)\, N_i(x,y)\, dxdy \tag{19}$$

$$(F_i)_f = \int_B \left\{ C_1 N_i(x,y) \left(\frac{\partial h_f}{\partial x}\right) l_x + C_2 N_i(x,y) \left(\frac{\partial h_f}{\partial y}\right) l_y \right\} \tag{20}$$

$$(F_i)_s = \int_B (C_3 N_i(x,y) \left(\frac{\partial hs}{\partial x}\right) l_x + C_4 N_i(x,y) \left(\frac{\partial h_s}{\partial y}\right) l_y) \tag{21}$$

where

$N_i(x,y)$ is the shape function (linear interpolation function over the element).

$(h_f(x,y))_j$ is true nodal value of the fresh water head.

$(h_s(x,y))_j$ is true nodal value of the salt water head.

$l_x, l_y$ are direction cosines at the boundary points.

*The Computer Program*

The existing FEM2D code developed by Reddy [12] (1993) for the linear differential equations associated with ordinary seepage problems is modified to solve the non-linear system of partial differential equations (equations 1 and 2), using an iteration scheme for the determination of the coefficients $C_1$ to $C_4$. A guess of unity is found to be reasonable to start the iteration procedure. Subsequently, by calling FEM2D program for fresh water then for salt water, new fresh and salt head values, $(h_f)_{new}$ and $(h_s)_{new}$, at each node can be computed using the initial guess. The new calculated head values are used to determine the subsequent set of values for the C-coefficients at each node, and the average values for element. This set is used instead of the initial guess to compute the values of head of salt water and fresh water at each node. Once the desired accuracy for computed head values is achieved, the interface height can be determined. Flow chart of the main program is shown in Figure (2). Initial guesses for the interface and the phreatic (or the piezometric) surface are not needed as a part of the data since they are generated by the program. Input data can be summarized as follows:

1- Element topology.
2- Nodes co-ordinates.
3- Permeability coefficients.
4- Type of aquifer (confined or unconfined).
5- Boundary conditions (specified head boundary, no flow boundary, specified flow boundary and location and discharge of wells)

The output of the program includes the following:-

1- Potential fresh and salt water heads at the nodes.
2- The interface height.
3- Length of intruded zone.

*Verification of the Developed Model*

Comparisons are made and presented in graphical forms, Figures (3 to 5), between the calculated results of the developed, quasi-3D, finite element model and those using other published solutions, for both confined and unconfined aquifers. The comparisons included the simple case of fresh water to the sea as well as the case of pumping from a well.

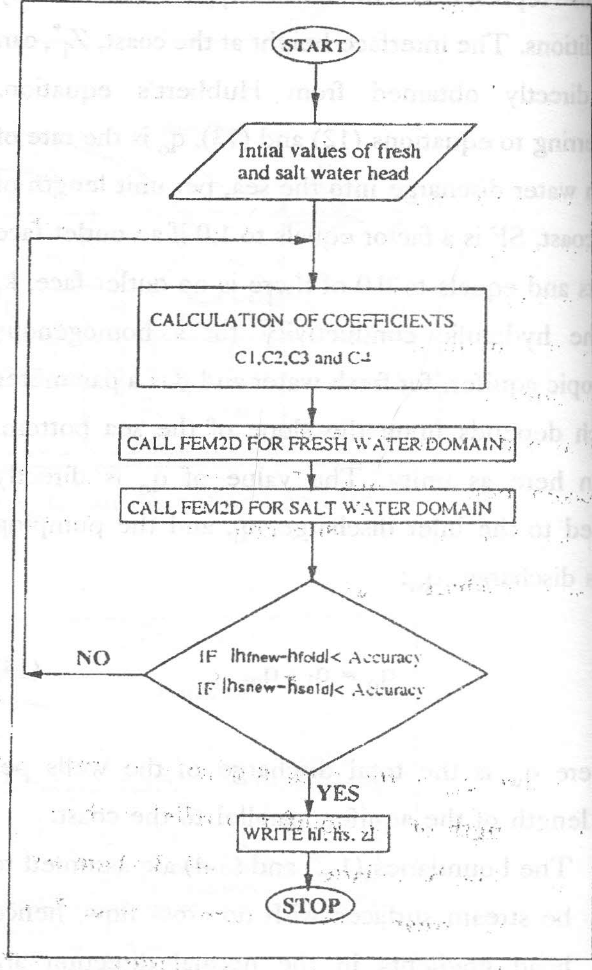


Figure 2. Flow chart of main programme.

*a- Fresh water flow to the sea (no wells):*

The following values have been used in the comparison:

$Z_B = 0.0$ , $H_o = 100.0$ m, $Z_T = 60.0$ m,

$k_f = 60.0$ m/d, $k_s = 61.5$ m/d (homogeneous, isotropic),

$\rho_f = 1000$ kg/m$^3$, $\rho_s = 1025$ kg/m$^3$,

$q_i = 10.0$ m$^2$/d (unconfined aquifer),

$q_i = 5.0$ m$^2$/d (confined aquifer).

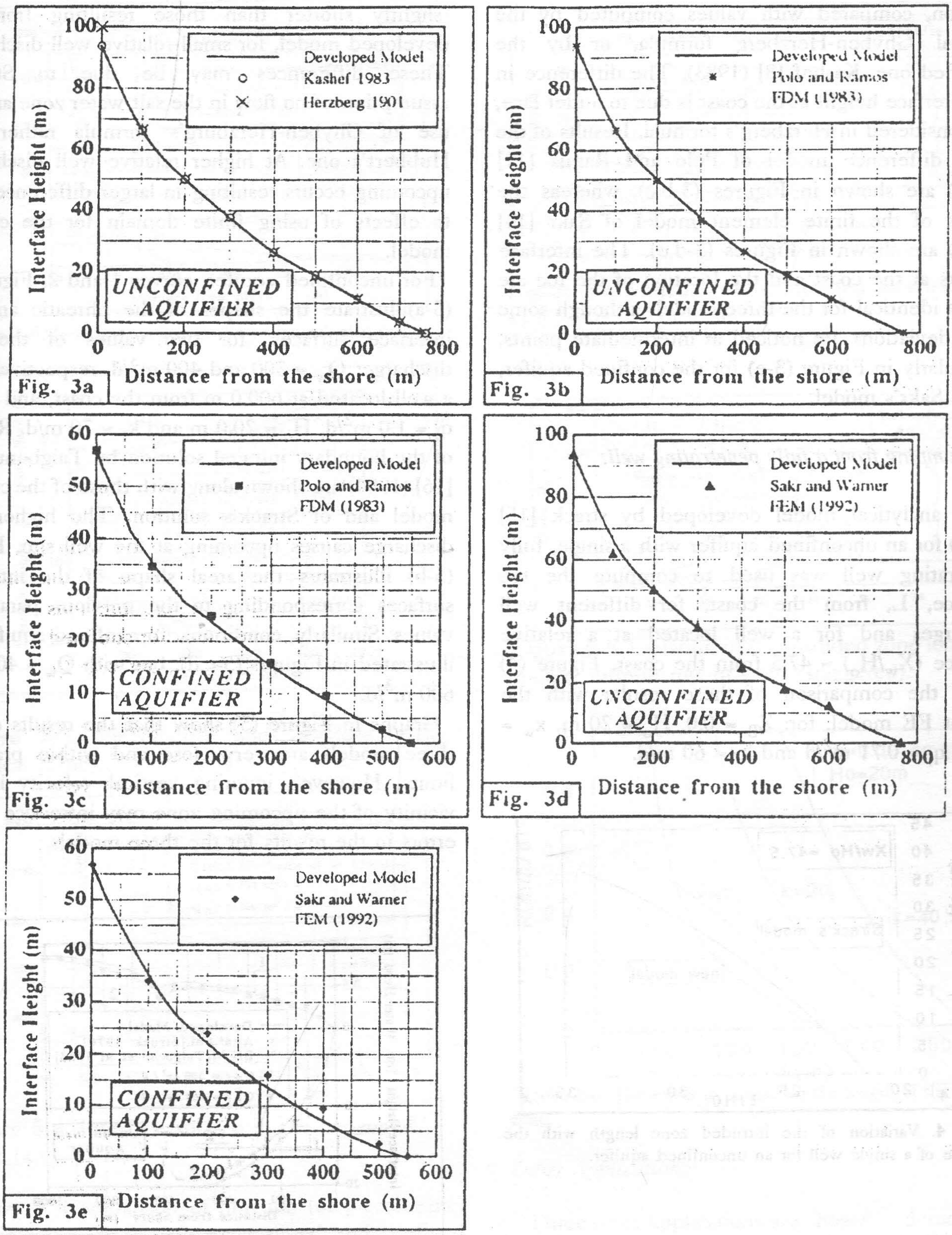


**Figure 3.** Interface shapes for unconfined and confined aquifers without wells.

Figure (3-a) shows the calculated interface and toe location, compared with values computed by the original Ghyben-Herzberg formula, or by the modified one, Kashef [8] (1983). The difference in the interface height at the coast is due to outlet face, not considered in Herzberg's formula. Results of the finite difference model of Polo and Ramis [11] (1983) are shown in Figures (3-b,c), whereas the results of the finite element model of Sakr [14] (1992) are shown in Figures (3-d,e). The interface heights at the coast and the location of the toe are almost identical for the three models although some small deviations are noticed at intermediate points, particularly in Figure (3-e) for the confined aquifer, using Sakr's model.

*b- Pumping from a fully penetrating well:*

The analytical model developed by strack [15] (1976) for an unconfined aquifer with a single, fully penetrating well was used to compute the toe distance, L, from the coast, for different well discharges and for a well located at a relative distance ($X_w/H_o$) = 47.5 from the coast. Figure (4) shows the comparison of these results with the current FE model, for: $Z_B$ = 0.0, $H_o$ = 20 m, $x_w$ = 950 m,$q_i$ = 0.71 $m^2/d$ and $k_f$ = 60 m/d.

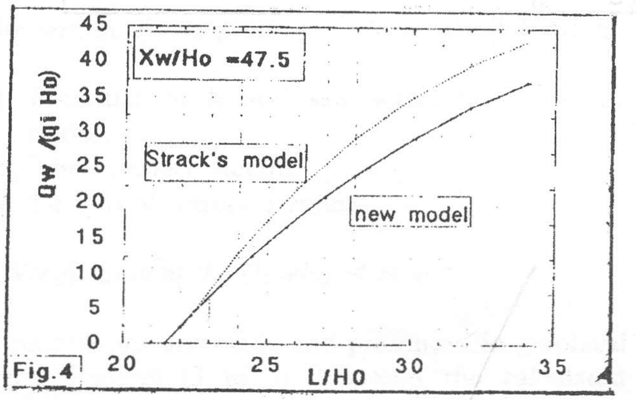


**Figure 4**. Variation of the intruded zone length with the discharge of a single well for an unconfined aquifer.

Toe distances computed from Strak's formula are slightly shorter than those resulting from the developed model, for small relative well discharges. These differences may be due to Strack's assumption of no flow in the salt water zone and the use of Ghyben-Herzburg's formula rather than Hubbert's one. At higher relative well discharges, upcoming occurs resulting in larger differences due to effects of using finite domain for the current model.

For unconfined aquifer, curves 1 and 2, Figure (5-a)illustrate the shapes of the phreatic and the interface surfaces, for two values of the well discharge: $Q_w$ = 300 and 400 $m^3/d$, respectively, for a well located at 600.0 m from the coast, and for $q_i$ = 1.0 $m^2/d$, $H_o$ = 20.0 m and $k_f$ = 70 m/d. Results of the boundary integral solution by Taigbenu et al [16] (1984) are shown along with those of the current model and of Strack's solution. The higher well discharge causes upcoming at the well site. Figure (5-b) illustrates the areal shape of the interface surfaces corresponding to the previous parameter values. Similarly, conditions for confined aquifer are illustrated in Figures (5-c,d), but with $Q_w$ = 400 and 600 $m^3/d$.

Graphs in Figure (5) show that the results of the three models are very close and within practical limits. However, ignoring vertical velocity in the vicinity of the upcoming zone may introduce some errors in the results for the three models.

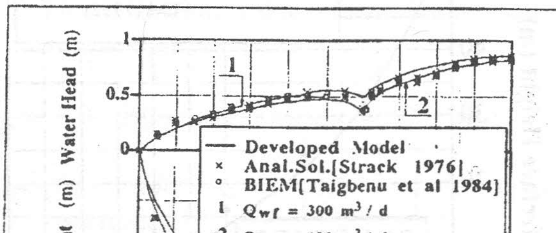


Figure 5-a. Phreatic and interface surface (vertical plane).

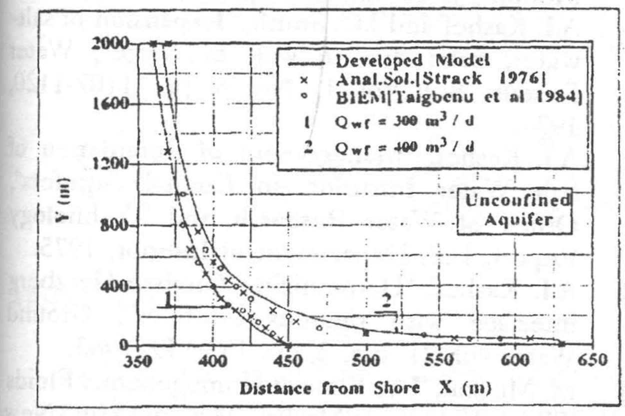


Figure 5-b. Interface surface in the horizontal plan (x,y).

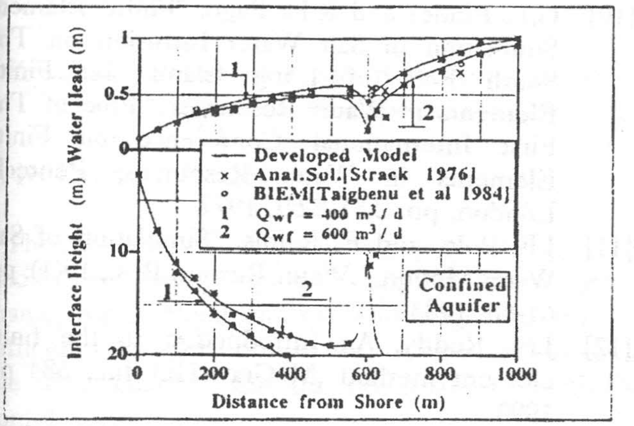


Figure 5-c. Piezometric and interface surface (vertical plane).

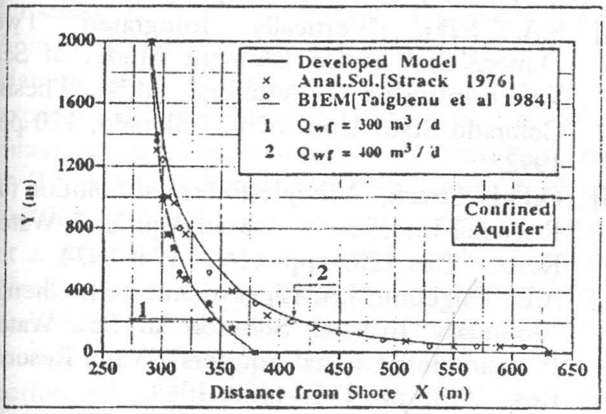


Figure 5-d. Interface surface in the horizontal plane (x,y)

**Figure 5.** Comparison of phreatic (or piezometric) and interface surface for unconfined and confined aquifers with a single well.

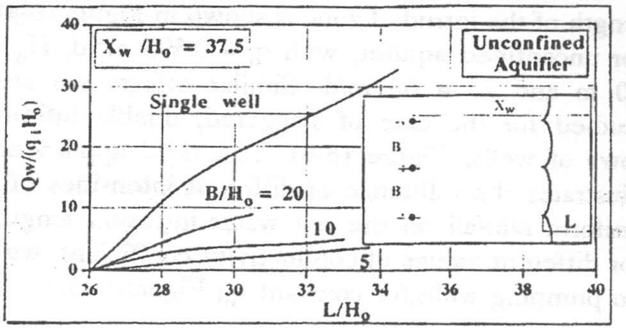


Figure 6-a. Variation of the intruded zone length for an infinite row of wells.

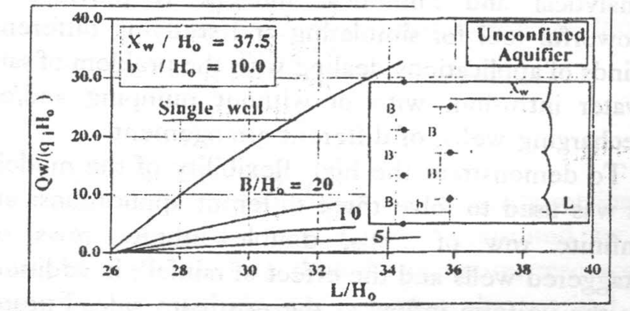


Figure 6-b. Variation of the intruded zone length for double, infinite row of wells (staggered).

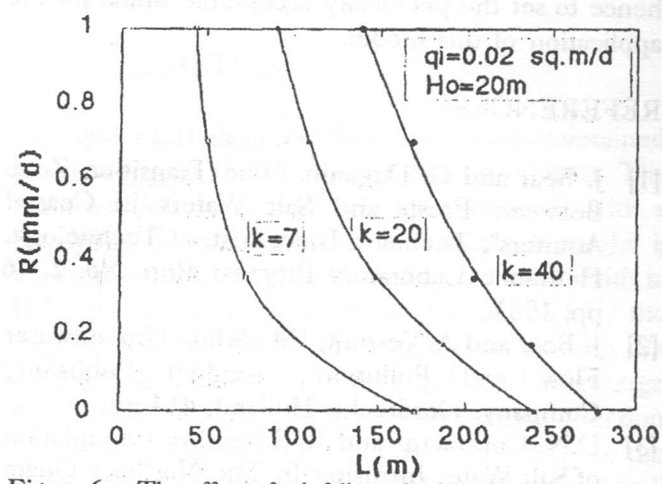


Figure 6-c. The effect of rainfall on the length of the intruded zone.

### *Other Applications:*

Three more applications are chosen to demonstrate the versatility of the developed model. Effects of varying the discharge and the separating distance, B, for an infinite row of equal-discharge wells on the length of the intruded zone is shown in Figure (6-a), for unconfined aquifer, with $q_i = 0.595\ m^2/d$, $H_o = 20$ m and $k_f = 60$ m/d. Similar effects are also studied for the case of staggered, double infinite rows of wells, Figure (6-b). The third application illustrates the influence of different intensities of a uniform rainfall on the salt water intrusion length, for different values of conductivity coefficient, with no pumping wells,for constant $q_i$, Figure(6-c).

## CONCLUSIONS

The developed, quasi-3D, areal, finite element model has been compared and verified with other analytical and numerical models. It provides a powerful tool for simulating and studying different kinds of applications dealing with the problem of salt water intrusion, with or without pumping and/or recharging wells, of different arrangements.

To demonstrate the high flexibility of the model, it was used to solve three different applications: an infinite row of wells, double infinite rows of staggered wells and the effect of rainfall, in addition to the uniform inflow at the upstream side. Future research should be made to evaluate the effect of using vertically integrated head values at the nodes, hence to set the practically acceptable limits for the application of this model.

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