ANALYSIS AND DESIGN OF CENTER PIVOT LATERALS

H.M. Hathoot*
Dept of Agric. Engrg., Col. of Agric.,
King Saudi Univ., Riyadh, Saudi Arabia.

H.M. Abo-Ghobar, A.I. Amoud
and F.S. Mohammad
Dept. of Agric. Engrg., Col. of Agric.,
King Saud Univ., P.O. Box 2460, Riyadh.

ABSTRACT

The analysis of a center pivot lateral of uniform diameter and equally spaced sprinklers is presented in this paper. The friction head loss as well as head losses at individual sprinklers are taken into account. In estimating friction losses the Darcy-Weisbach equation with a friction factor equation which practically covers the turbulent zone on Moody diagram are used. A computer program for analysing and designing center pivot laterals is presented. The calculations by this technique are compared with well known techniques and published experimental data and the results were found in good agreement. A numerical example for estimating lateral pipe diameter and estimating the total head loss is presented and the results are compared with other design technique i.e. Hazen-Williams method using a friction correction factor. Graphs for discharge, pressure and Reynolds' number distributions along the lateral are provided.

Keywords: Center pivot, Laterals, Sprinklers-friction, Head loss

INTRODUCTION

In the last thirty years the use of center pivot systems in some countries has grown rapidly primarily because of their low labor requirements and their ability to irrigate large areas.

Among the earliest investigators who analysed center pivot laterals were Kincaid and Heermann (1970). They used numerical techniques based on the Darcy-Weisbach equation to calculate the pressure distribution on a center pivot system.

An analytical approximation for calculating head loss in a center pivot lateral was derived by Chu and Moe (1972). They assumed an infinite number of tiny sprinklers evenly spaced along the lateral with the discharge proportional to the radius for a uniform irrigation. The most common method for analysing sprinkler laterals is the Hazen-Williams method using a friction correction factor accounting for decrease in flow rate along the lateral (Cuenca, 1989). Reddy and Apolayo (1988) provided the values of friction correction factors for center pivot irrigation systems of equally spaced sprinklers.

Extensive studies were carried out on analyzing the center pivot irrigation system and evaluating the application rate, von Bernuth (1983), Johnson et al. (1987), Mohamoud et al. (1992), James and Blair (1984), while Johnson et al. (1986) presented a study on the economics of center pivot irrigation system design.

The objective of this paper is to provide a design technique for center pivot laterals that counts for actual friction losses in addition to minor losses.

THEORY

In center pivot laterals where sprinklers have a common spacing, s, the first sprinkler usually has a

* On Leave from Faculty of Engineering, Alexandria University, Alexandria, Egypt.

different spacing from the pivot, s_1. Thus the distance from the pivot to the end sprinkler is given by:

$$L = s_1 + (n-1)s \qquad (1)$$

in which n is the number of sprinklers. To design a center pivot lateral the pressure at the end sprinkler, P_e, is to be assumed so as to produce the required discharge of the end nozzle, Kincaid and Heermann (1970). Starting from the outer end of a horizontal lateral, Figure (1), pressure heads H_{n-1} and H_n at the last two sprinklers are interrelated by:

$$H_{n-1} + \frac{Q_{n-1}^2}{2gA^2} = H_n + \frac{Q_n^2}{2gA^2} + h_n \qquad (2)$$

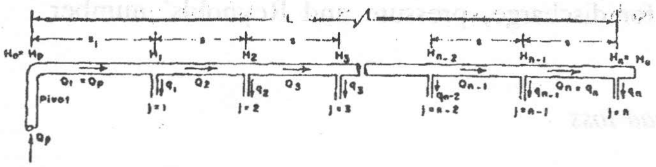

Figure 1. Schematic sketch of center pivot lateral.

in which A is the cross-sectional area of lateral, h_n is the total head loss within the last reach of lateral considering the pressure rise due to change of momentum, Q_{n-1}, Q_n, are lateral discharges upstream from points n-1 and n, respectively and g is the acceleration due to gravity.

The friction head loss is estimated herein according to the Darcy-Weisbach equation (Streeter and Wylie, 1983) since it is the most universally accepted one, von Bernuth (1990). In terms of the discharge, the Darcy-Weisbach equation takes the form:

$$h_f = \frac{8 f_n \, s \, Q_n^2}{\pi^2 g \, D^5} \qquad (3)$$

in which f_n is the friction coefficient for the last reach of lateral and D is the lateral diameter. Combining Eqs. 2 and 3 and solving for H_{n-1}:

$$H_{n-1} = H_n + \frac{Q_n^2}{2gA^2} - \frac{Q_{n-1}^2}{2gA^2} + \frac{8 f_n s Q_n^2}{\pi^2 g D^5} + h_n^{`} \qquad (4)$$

in which $h_n^{`}$ is h_n minus the friction head loss within the last reach. The lateral discharges in the last two reaches of the lateral are interrelated by:

$$Q_{n-1} = Q_n + q_{n-1} \qquad (5)$$

in which q_{n-1} is the discharge of sprinkler n-1, Figure (1).

From Eqs. 4 and 5, we have:

$$H_{n-1} = H_n - B[Q_{n-1}^2 - (Q_{n-1} - q_{n-1})^2] + E f_n (Q_{n-1} - q_{n-1})^2 + h_n^{`} \qquad (6)$$

in which B and E are given by:

$$B = \frac{1}{2gA^2} \qquad (7)$$

and

$$E = \frac{8s}{\pi^2 g \, D^5} \qquad (8)$$

For any two successive interior sprinklers we have the following general equation:

$$H_j = H_{j+1} - B[Q_j^2 - (Q_j - q_j)^2] + E f_{j+1} (Q_j - q_j)^2 + h_{j+1}^{`} \qquad (9)$$

The pressure head at the pivot H_p can be estimated in terms of the pressure head at the first sprinkler as follows:

$$H_p = H_1 + E_1 f_1 Q_p^2 + h^{`} \qquad (10)$$

in which f_1 is the friction coefficient for the first reach, Q_p is the total discharge of the pivot, $h^{`}$ is the head loss in fittings in the first reach and E_1 is given by:

$$E_1 = \frac{8 s_1}{\pi^2 g \, D^5} \qquad (11)$$

SPRINKLER DISCHARGE

For center pivot laterals with equally spaced sprinklers, sprinkler discharge is theoretically proportional to its radial distance from the center pivot as given by the following equation (Cuenca, 1989):

$$q_j = \frac{2Q_p\, s\, r_j}{R_o^2} \qquad (12)$$

in which r_j is the radial distance from the pivot to sprinkler j and R_o is the radius of the global wetted circular area. The discharge of the end sprinkler is thus given by:

$$q_n = \frac{2Q_p\, s\, L}{R_o^2} \qquad (13)$$

in which L is the total length of lateral. It is worthy to note that the above theoretical discharges can be reasonably attained in practice by using pressure regulators and choosing the proper size of sprinkler.

COEFFICIENT OF FRICTION

Center pivot laterals are steel or galvanized steel pipes, with diameters ranging between 101.6 mm (4") and 254 mm (10"), (James, 1988). Fur turbulent flow, which is common in center pivot laterals, the following equation, which was presented by Swamee and Jain (1976) can be successfully used:

$$f = \frac{1.325}{[\ln(\frac{\epsilon}{3.7D} + \frac{5.74}{R^{0.9}})]^2} \qquad (14)$$

in which ϵ is the absolute roughness of pipe and R is the Reynolds' number, which may be written as

in which ν is the kinematic viscosity of irrigation water.

$$R = \frac{4Q}{\pi D \nu} \qquad (15)$$

From Eqs. 14 and 15 we obtain the following general equation:

$$f_j = \frac{1.325}{\{\ln[\frac{\epsilon}{3.7D} + \frac{5.74}{(\frac{4Q_j}{\pi D \nu})^{0.9}}]\}^2} \qquad (16)$$

Equation 16 yields errors less than $\pm 1\%$ in the range of $10^{-6} \leq \epsilon/D \leq 10^{-2}$ and $5x10^3 \leq R \leq 10^8$. This range covers a significant portion of the turbulent zone on Moody diagram. The uncovered portion, $\epsilon/D > 10^{-2}$, is practically contained in the wholly rough flow zone and is not to be considered since the minimum pipe diameter is 101.6 mm and the absolute roughness is in the order of 0.045 mm.

HEAD LOSS AT RISER CONNECTION

The resultant head loss between two points upstream and downstream from the riser j may be put in the form:

$$h_{ud} = K_{ud} \frac{Q_j}{2gA^2} \qquad (17)$$

in which Q_j is the lateral discharge in the reach upstream from the riser and K_{ud} in a coefficient given through Gardel's equation (Miller 1978) as:

$$K_{ud} = 0.03(1-\frac{Q_{j+1}}{Q_j})^2 + 0.35(\frac{Q_{j+1}}{Q_j})^2 - 0.2(\frac{Q_{j+1}}{Q_j})[1-\frac{Q_{j+1}}{Q_j}] \qquad (18)$$

therefore the head loss at riser j is given by:

$$h_{ud} = \frac{Q_j^2}{2gA^2}\{0.03(1-\frac{Q_{j+1}}{Q_j})^2 + 0.35(\frac{Q_{j+1}}{Q_j})^2 - 0.2(\frac{Q_{j+1}}{Q_j})[1-\frac{Q_{j+1}}{Q_j}]\} \qquad (19)$$

The term h_{ud} essentially includes the pressure rise due to change of momentum at the riser, (Featherstone and Nalluri, 1982) and Hathoot et al. (1991). Therefore the term $h`_{j+1}$ of Eq. 9 may be put in the form:

$$h`_{j+1} = h_{ud} + h_{fit} \qquad (20)$$

in which h_{fit} is the head loss in fittings in the reach j+1.

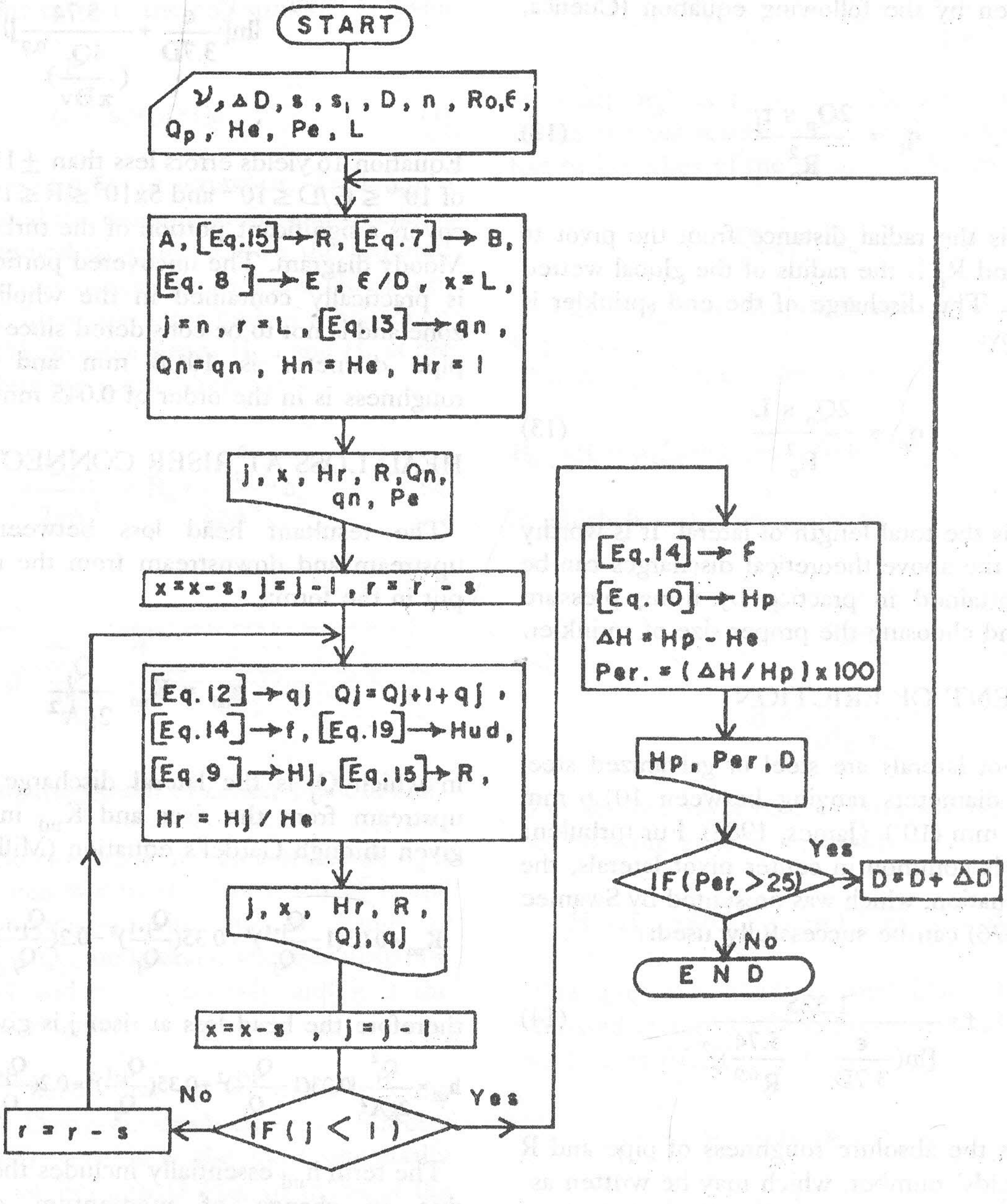

Figure 2. Flow chart for the computer program.

DESIGN CRITERIA

The pressure head at the outer end of lateral is initially assumed to satisfy the required outer sprinkler discharge, Kincaid and Heermann (1970). Addition of head losses at successive reaches along the lateral to the pressure head at the end sprinkler provides the pressure head at the pivot. In choosing the proper diameter of lateral the total head loss along the lateral should not exceed 25% to 30% of the pressure head at the pivot (Rolland, 1982).

COMPUTER PROGRAM

The flow chart of the computer program is presented for designing and analysing center pivot laterals. To apply this program, the quantities $\nu, s, s_1, n, \epsilon, Q_p, H_e, R_o$ and L should be available in advance. The following is the algorithm used in developing the computer program:

1- a small lateral diameter is first assumed.
2- the discharge of the outer end sprinkler, which is also the discharge of the outer reach of lateral, is estimated.
3- the Reynolds' number for the outer reach is estimated and accordingly the friction head loss is calculated.
4- the head loss at sprinkler n-1 is evaluated and added to the friction head loss and the outer end sprinkler head to give the head at sprinkler n-1.
5- the procedure followed in steps 3 and 4 are repeated stepwise in the pivot direction until the pressure head at the first sprinkler near the pivot, H_1, is evaluated.
6- the pressure head at the pivot, H_p, is obtained by adding the friction head loss in the first reach of lateral to the first sprinkler head H_1.
7- the percentage head loss, $100(H_p-H_e)/H_p$ is then calculated. If the percentage is less than or equal to 25% the assumed diameter is the proper one, otherwise the next practical bigger diameter is assumed and steps 1 through 7 are repeated.

To verify the adequacy and accuracy of the present technique the results are compared with experimental data presented by Kincaid and Heerman (1970) as shown in Figure (3). In this figure pressure head ratio p/p_o is plotted versus the relative distance from the pivot, x/L, for the present technique, Kincaid and Heermann (1970), together with experimental data.

The present technique provides better results than those of Kincaid and Heermann (1970) in the range $0<x/L<0.4$. This may be attributed to the fact that the present technique accounts for losses at sprinklers in addition to the friction losses. In the range $0.4<x/L<1.0$ the two curves are nearly coincident. For the above range the existence of experimental points below the curves was attributed to the improper adjustment of pressure regulator at that range, Kincaid and Heermann (1970).

Numerical Example 1:

It is required to design a center pivot lateral diameter for the following data: 20 sprinklers are equally spaced at 10.0m, and the first sprinkler is located 10.0m from the pivot. The lateral pipe is of steel, $\in$=0.04572 mm (0.00015 ft) and the kinematic viscosity of irrigation water at 22°C is 10^{-6} m^2/s. The pressure at the last sprinkler, p_e = 414.47 k.Pa and the total pivot discharge is 42.17 ℓ/s.

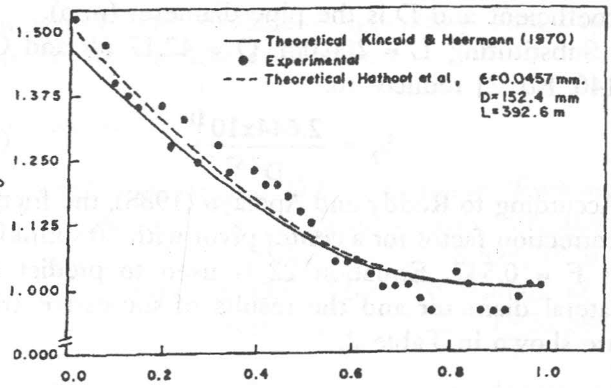

Figure 3. Experimental verification of the computer program.

Solution:

Trials were made through the computer program to choose the proper lateral diameter and the results are listed in Table (1).

Table 1. Results of trials according to the present technique. based on Darcy-Weisbach equation.

D(mm)	Pp (kPa)	Hp(m)	Percentage head loss between pivot and outer sprinkler
101.6	703.08	71.67	41.05%
127.0	510.12	52.0	18.75%

In Table (1) the required lateral diameter is 127 mm (5") with a percentage head loss of about 18.8%. The head loss at sprinklers constitutes about 14% of the total head loss and about 2.6% of the pressure head at pivot. For verification these results can be compared to those obtained from the Hazen-Williams equation with a friction correction factor. The Hazen-Williams equation may be put in the form:

$$h_f = KL\left[\frac{\left(\frac{Q}{C}\right)^{1.852}}{D^{4.87}}\right] \quad (21)$$

in which h_f is the friction head loss (m), K is a coefficient = (1.22×10^{10}), L is the pipe length (m), Q is the discharge (ℓ/s), C is the Hazen-Williams coefficient and D is the pipe diameter (mm).

Substituting L = 200.0m, Q = 42.17 ℓ/s and C = 140, Eq. 21 reduces to:

$$h_f = \frac{2.644 \times 10^{11}}{D^{4.87}} \quad (22)$$

According to Reddy and Apolayo (1988), the friction correction factor for a center pivot with 20 sprinklers is F = 0.547. Equation 22 is used to predict the lateral diameter and the results of successive trials are shown in Table 2.

Table 2. Results of trials based on Hazen-Williams equation.

D(mm)	h_f (m)	Fh_f (m)	H_p (m)	(Fh_f/H_p)x100
101.06	44.53	24.36	66.61	36.57%
127	15.02	8.22	50.47	16.29%

It is evident that the method that applies the Hazen-Williams equation and the present technique provides the same lateral diameter of 127 mm. However, the present technique yields head loss and pivot pressure head of 15.7% and 2.9% higher than those of the other method, respectively. This may be attributed to the different natures of Darcy-Wisbach and Hazen-Williams equations in addition to considering losses at sprinklers in the present technique.

The results of the computer program based on the data from example 1 with D=127 mm are plotted in Figures (4) and (5). The lateral discharge ratio Q/Q_p and the sprinkler discharge ratio, q/Q_p are plotted versus the lateral length ratio x/L. As expected, the sprinkler discharge increases linearly with the distance from the center pivot, Eq. 12. About 21% of the lateral discharge is consumed through the upstream half of the lateral and half of the discharge through 0.75 L. In Figure (4-b), the pressure ratio p/p_e is plotted versus the lateral length ratio, x/L, where p_e is the pressure at the outer end sprinkler. About 15.03% of the head loss, which is mainly due to friction, occurs within the upstream half of the lateral and half the head loss occurs at 0.28L. Reynolds' number for the lateral pipe versus the lateral length ratio, x/L is plotted in Figure (5). As shown in the figure, Reynolds' number ranges from 4.23×10^5 at the pivot to 4.02×10^4 at the outer end and is 3.32×10^5 at the midpoint of the lateral. It is evident that the range of Reynolds' number lies within the limits of validity of the Swamee and Jain's friction factor equation. In addition the relative roughness ε/D is about 3.6×10^{-4}, which is also within the permissible range.

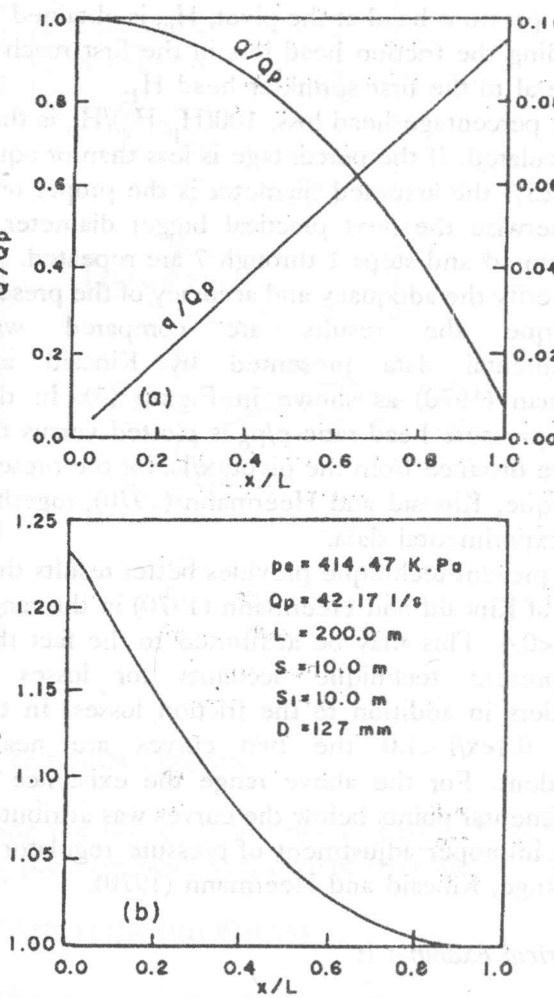

Figure 4. Discharge and pressure variation along the Lateral, Example 1.

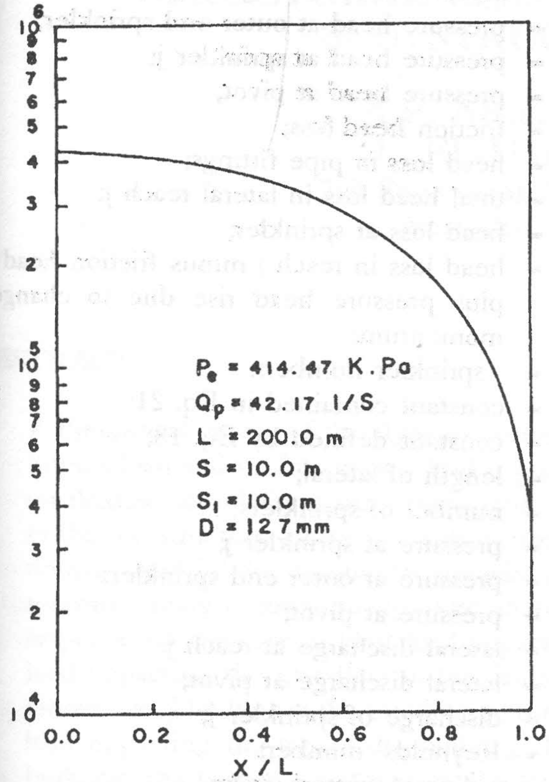

Figure 5. Reynolds' number variation along the Lateral, Example 1.

CONCLUSIONS

Calculations according to the present technique are consistent with experimental data and are more accurate than those of Kincaid and Heermann (1970). A numerical example is presented for designing the lateral diameter and the pressure head at the pivot. The numerical example is also solved by applying a method which considers the Hazen-Williams friction loss equation and a friction correction factor. Though the two methods provide the same pipe diameter, yet the present technique yields a head loss which is about 16% higher than that in the other method. In studying discharge and pressure distributions along the lateral it is found that 21% of the lateral discharge is consumed and 79% of the head loss occurs at the upstream half of the lateral. It is also proved that half the discharge and half the head loss occur at 0.75 L and 0.28 L, respectively. The value of Reynolds' number and the relative roughness of the lateral are found within the range of validity of the Swamee and Jain's friction factor equation. This insures fairly accurate results in estimating friction losses by the Darcy-Weisbach equation.

ACKNOWLEDGEMENT

The authors are indebted to Mr. Saleh Al-Hadary, Computer Center, College of Agriculture, King Saud Univ. for his valuable assistance.

REFERENCES

[1] D.C. Kincaid and D.F. Heermann "Pressure distribution on a center pivot irrigation system," Trans. ASAE, 13(5), 556-558, 1970.

[2] S.T. Chu and D.L. Moe "Hydraulics of a center pivot system," Trans. ASAE, 15(5). 894-896, 1972.

[3] R.H. Cuenca. "Irrigation system design, an engineering approach," Prentice Hall, Englewood Cliffs, New Jersey 07632, 1989.

[4] J.M. Reddy and H. Apolayo, "Friction correction factor for center pivot irrigation systems," *J. Irrig. and Drain. Engrg., ASCE*, 114(1), 183-185, 1988.

[5] R.D. von Bernuth, *Nozzeling considerations for center pivots with end guns*, Trans. ASAE, 419-422,. 1983.

[6] G.C. Johnson, W.W. Rochester, L.U. Hatch, L.M. Curtis and K.H. Yoo, *Analysis of center pivot irrigation systems in a humid-area environment*," Trans. ASAE, 1720-1725, 1987.

[7] Y. Mohamoud, T.R. McCarty and L.K. Ewing. "Optimum center pivot irrigation system design with tillage effects," *J. Irrig. and Drain. Engrg.*, ASCE, 118(2) 291-305, 1992.

[8] L.G. James and S.K. Blair. *Performance of low pressure center pivot systems*," Trans. ASAE, 27(6), 1753-1762, 1984.

[9] G.C. Johnson, E.W. Rochester, L.U. Hatch and L. Curtis. *A method for determining optimal center pivot design," Summer Meeting*, ASAE, Calif. Poly. Inst. No. 86-2094, 1986.

[10] V.L. Streeter and B.E. Wylie. *Fluid mechanics*, Int. Stud. Ed., McGraw-HillCo., New York, N.Y., 1983.

[11] R.D. von Bernuth. "Simple and accurate friction loss equation for plastic pipes," *J. Irrig. & Drain. Engrg., ASCE*, 116(2), pp. 294-

298, 1990.

[12] L.G. James. *Principles of farm irrigation system design*, John Wiley & Sons, 1988.

[13] P. Swamee and A. Jain. "Explicit equations for pipe flow problems," *J. Hydr. Div., ASCE, May*, pp. 1976.

[14] D.S. Miller. "Internal flow systems," The British Hydrodynamics Res. Assoc., *Fluid Engrg., Cranfield, Bedford, England*, 1978.

[15] R.E. Featherstone and C. Nalluri. *Civil Engineering hydraulics, essential theory and worked examples*, Granada Publishing, New York, 1982.

[16] H.M. Hathoot, A.I. Al-Amoud and F.S. Mohammad. "Analysis of a pipe with uniform lateral flow," *Alex. Engrg. J.*, 30, (1), pp. C49-C54, 1991.

[17] L. Rolland. Mechanized sprinkler irrigation, FAO Irrigation and Drainage Paper 35, FAO, Rome, 1982.

NOTATIONS

The following symbols are used in this paper:

A = cross-sectional area of lateral pipe;
B = constant defined by Eq. 7;
C = Hazen-Williams friction coefficient;
D = lateral pipe diameter;
E = constant defined by Eq. 8;
E_1 = constant defined by Eq. 11;
F = friction correction factor;
f_j = Coefficient of friction for lateral reach j.
g = acceleration due to gravity;
H_e = pressure head at outer end sprinkler;
H_j = pressure head at sprinkler j;
H_p = pressure head at pivot;
h_f = friction head loss;
h_{fit} = head loss in pipe fittings;
h_j = total head loss in lateral reach j;
h_{ud} = head loss at sprinkler;
$h_j`$ = head loss in reach j minus friction head loss plus pressure head rise due to change of momentum;
j = sprinkler number;
K = constant contained in Eq. 21;
K_{ud} = constant defined by Eq. 18;
L = length of lateral;
n = number of sprinklers;
p_j = pressure at sprinkler j;
p_e = pressure at outer end sprinkler;
p_p = pressure at pivot;
Q_j = lateral discharge at reach j;
Q_p = lateral discharge at pivot;
q_j = discharge of sprinkler j;
R = Reynolds' number;
R_o = radius of wetted circle;
r_j = distance from pivot to sprinkler j;
s = spacing between sprinklers;
s_1 = distance from pivot to first sprinkler;
x = distance from sprinkler to a general point on lateral;
ε = absolute roughness of lateral pipe material; and
ν = kinematic viscosity of water.