THE BEHAVIOR OF AXIALLY LOADED PILES

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ABSTRACT

A closed form analytical method is introduced to predict the behavior of axially loaded single pile. The versatility of the method is checked versus two field tests together with their finite element solutions. The method is shown to be capable of producing accurate results. It is valid for floating "friction", end bearing, and piles supported on compressible bearing stratum . It is also valid for the case of layered soil and piles with varying cross section. The relation between the pile movement and the soil resistance " skin friction" is introduced through using the subgrade reaction or the t-z method, while the relation between the tip movement and the soil reaction is introduced through using the p-z method. In general, the relation between the soil resistance and the displacement may be introduced using any suitable form, analytical or tabulated. The soil reaction, the axial load and the displacements along the pile is obtained using a single formula. The share of the load transmitted to the bearing stratum, the displacement at the tip and the top of the pile are obtained using the same formula. Finally, the method takes care of the cases with local yielding of soil along the pile and yielding of the underlaying stratum. It treats both linear and non-linear cases. No matrix inversion is needed. Also, the memory requirements is independent of the number of layers.

Keywords: Piles, Axial loading, Linear, Nonlinear Layered.

1. INTRODUCTION

In general, finding the ultimate load of a pile is not always sufficient to ensure functional operation of the supported structures. The load-settlement relation must be calculated to ensure adequate control of the allowable deformations. Field and laboratory loading tests are often used to determine load carrying capacity and-load-settlement relation with the restriction that; results apply for one site, one pile length, beside they are expensive. Therefore, there is a demand to introduce general procedures, numerical and analytical, that permit computing the load-settlement relation and the bearing capacity of the piles.

Generally, the behavior of axially loaded piles is predicted using on of the following methods (Poulos,1977):

1. Load transfer method; which employs measured relationship between pile movement and soil resistance (Coyle and Reese, 1966),(Kiousis and Elansary 1987).
2. Theory of elasticity; which employs the equations of Mindlin (Poulos and Mattes, 1969), (Poulos and Davis, 1968), Butterfield and Banerjee, 1971). This approach provides solutions to many practical problems but it becomes difficult if such factors as nonlinear soil behavior and layered soil must be included in the analysis.
3. Numerical methods; such as the finite difference method (Meyer, Holmquist and Matlock 1987),the finite element method (Desai,1974), (Randolph and Worth 1966), the boundary element method, and the finite layer analysis (Lee and Small, 1991).

In this paper, an analytical method that can handle layered soil and different conditions of the pile; floating "friction", end bearing, and piles supported on compressible bearing stratum, is introduced.

2. EQUATION OF MOTION

Select a coordinate system so that the positive direction of the vertical axis z is directed upward. The differential equation for equilibrium of a vertical pile segment coinciding with the vertical axis, and subjected to axial load at its top and embedded in a soil mass with tangential subgrade reaction k, force per unit length of the pile needed to produce unit displacement, is given by

$$k u + \frac{\partial N}{\partial z} = 0 \qquad (1)$$

where,

z distance along the pile

u axial displacement of the pile,

N axial force, and

K force per unit length of the pile needed to produce unit displacement with

$$N = - E A_p \frac{\partial u}{\partial z} \qquad (2)$$

where,

EA_p the axial rigidity of the pile.

From (1) and (2)

$$\frac{\partial^2 u}{\partial^2 z} - \lambda^2 u = 0 \qquad (3)$$

where

$$\lambda^2 = \frac{k}{EA_p}$$

The general solution of (3) is

$$u = A \sinh(\lambda z) + B \cosh(\lambda z) \qquad (4)$$

where

A and B constants of integrations.

It seems natural to mention the following:

1. While developing equation(1), it is assumed that the movement in the pile at any point is related only to the shear stress at that point and is independent of the shear stresses elsewhere along the pile. This limitation is inherent in load transfer methods utilizing tangential subgrade reaction model.
2. In obtaining equation (4), it is assumed that K is constant. In reality K is a function of, among others, displacement level, distance measured from ground surface, method used to drive the pile, lateral earth pressure. The present method, by its nature, takes care of this limitation.

3. LAYERED SOIL

To solve for the case of layered media, consider n layers with thicknesses $L_1, L_2, ..., L_n$ with L_1 next to the ground surface, and subgrade reactions $k_1, k_2, ..., k_n$, respectively. The axial rigidity of the pile at layer i is EA_{pi}. The subgrade reaction "spring constant" for the underlaying stratum is K_S, (defined as the force required to produce unit displacement at the pile tip).

Each layer has its independent coordinate system with the origin at the bottom of the layer and the positive direction of the vertical axis z_i directed upward.

The behavior of the pile in each layer is governed by an equation similar to (3), with a general solution similar to (4)

$$u_i = A_i \sinh(\lambda_i z_i) + B_i \cosh(\lambda_i z_i) \qquad (5)$$

where

A_i and B_i constants of integration corresponding to the i^{th} layer

z_i running coordinate along the i^{th} layer, and

$$\lambda_i^2 = \frac{k_i}{EA_{P_i}}$$

The relations connecting the 2n constants ,A_1 through B_n, must be established. The continuity at the interface between any two layers, i and i+1 requires

$$B_i = A_{i+1} \sinh(\lambda_{i+1} L_{i+1}) + B_{i+1} \cosh(\lambda_{i+1} L_{i+1}) \qquad (6)$$

Although, the displacements are measured in terms of local coordinates z_i, the global continuity is satisfied by (6).

The equilibrium condition demands

$$A_i = \alpha_{i+1} A_{i+1} \cosh(\lambda_{i+1} L_{i+1}) + \alpha_{i+1} B_{i+1} \sinh(\lambda_{i+1} L_{i+1}) \qquad (7)$$

with

$$\alpha_i = \frac{E_{i+1}\, A_{p_{i+1}}\, \lambda_{i+1}}{E_i\, A_{p_i}\, \lambda_i} \tag{8}$$

in a matrix form (6) and (7) can be written as

$$\begin{Bmatrix} A_i \\ B_i \end{Bmatrix} = \begin{bmatrix} C_i \end{bmatrix} \begin{Bmatrix} A_{i+1} \\ B_{i+1} \end{Bmatrix} \tag{9}$$

where the transfer matrix $[C_i]$ is given by

$$[C_i] = \begin{bmatrix} \alpha_{i+1} \cosh(\lambda_{i+1} L_{i+1}) & \alpha_{i+1} \sinh(\lambda_{i+1} L_{i+1}) \\ \sinh(\lambda_{i+1} L_{i+1}) & \cosh(\lambda_{i+1} L_{i+1}) \end{bmatrix} \tag{10}$$

From (9), it can be concluded that

$$\begin{Bmatrix} A_1 \\ B_1 \end{Bmatrix} = \begin{bmatrix} C_G \end{bmatrix} \begin{Bmatrix} A_n \\ B_n \end{Bmatrix} \tag{11}$$

where the global transfer matrix $[C_G]$ is given by

$$[C_G] = [C_2]\,[C_3]\,.....[C_{N-1}]\,[C_N] \tag{12}$$

4. BOUNDARY CONDITIONS

To find A_i and B_i, (i=1,2,...,n), the constants of integration, first consider the equilibrium at the pile's head. The axial force in the pile is given by (2). From which, the axial force at the pile's head N_o is given by

$$N_o = -E A_{p1} \lambda_1 \left[A_1 \cosh(\lambda_1 L_1) + B_1 \sinh(\lambda_1 L_1) \right] \tag{13}$$

At the pile's tip, three cases may occur. Firstly,the case of compressible bearing stratum ,which requires that the force developed at the pile's tip is equal to the reaction of the underlaying stratum. Thus

$$K_s B_n = E A_{pn} \lambda_n A_n \tag{14-a}$$

where,

K_s the subgrade reaction of the bearing stratum.

Secondly, the case of rigid bearing stratum, which leads to zero displacement of the pile tip and accordingly,

$$B_n = 0 \tag{14-b}$$

And finally, the case of "fully" floating pile, which requires zero force at the pile tip and demands

$$A_n = 0 \tag{14-c}$$

Using (11) and (13) with either (14-a), (14-b) or (14-c) depending on the tip's conditions, and solve for A_1 and B_1 or A_n and B_n. Using the values obtained and relationship (9), all the coefficient A_i and B_i can be obtained.

With all the integration constants at hand, we can find all the required quantities using relationship (5) and its derivative.

4. NONLINEAR ANALYSIS

To treat the cases of soil's nonlinearity and yielding of soil, the following steps has to be followed:

1. Divide the soil into n layers with each layer having its own force-displacement curve, similar to the one shown in Figure (1) with additional curve for the bearing stratum, if needed. In case of soil having constant properties with depth, of course, all layers would share the same curve.
2. Replace each curve by a broken line as shown in Figure (1). The slope of any line segment represents the secant subgrade reaction for the level of displacement shown on the horizontal axis below that segment. The lengths Δu_i may be taken arbitrary and need not be equal. However in practice, they depend on the degree of accuracy needed in the analysis.
3. For all layers, 1 through n, find the subgrade reactions $K_{1,i}$ where the first subscript refers to the displacement level and the second one refers to the layer.
4. For an applied load N_o=1 find the coefficients A_1 through B_n, using the procedures described before.
5. Find the displacement at the mid point of each layer, corresponding to N_o= 1. From those values,find the force N_o that causes the mid point

displacement in one layer, layer i, to reach Δu_1.

Note:- "here after, call the load calculated in step 5 above $N_{o,1}$ and the displacements, at mid points, associated with it by $D_{1,i}$ where for $D_{1,i}$ the first subscript refers to the stage of loading and the second one refers to the layer. For the load $N_{o,1}$ the second subscript refers to the stage of loading and the first one, always o, to remined us that the load is applied at the pile head."

6. Find the integration constants A_1 through B_n as in step 4 above. With the exception that, for layer i, mentioned in step 5, the subgrade reaction corresponds to the second displacement level.
7. Working, exactly, as in step 5, but now the displacement at the mid point of each layer, layer j, is given as the sum of the displacements $D_{k,j}$, where k varies from 1 to the present loading stage. The displacements in this stage are calculated using integration constants found in step 6.
8. Find the smallest incremental force $N_{o,2}$ that causes the displacement at mid point of any layer, except for layer i mentioned in step 5, to reach the displacement level Δu_1 <u>OR</u> $N_{o,2}$ that cause the displacement level in layer i to reach the displacement level $\Delta u_1+\Delta u_2$.
9. A total applied force $N_{o,1}+N_{o,2}$ corresponding to displacement $D_{1,j} + D_{2,j}$, where j=1,2,...,n, are obtained.
10. Steps 3 through 9 are repeated, with the required modification, to attain the required level of loading or displacement.

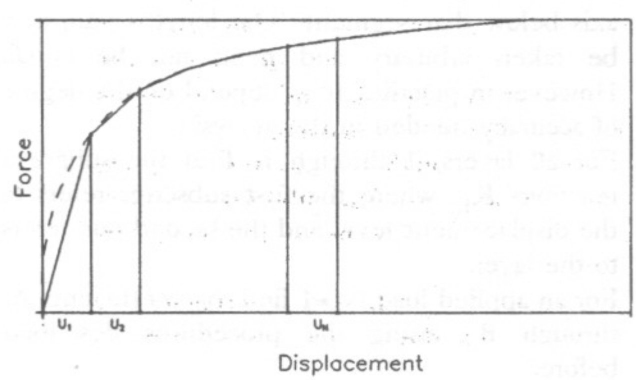

Figure 1. General (t-z) or (p-z) curve.

5. THEORY VERIFICATION

A field test is reported by Mccammon and Golder (1970), and analyzed by Meyer et al, (1975). This test was performed on 61 cm. (24 inch) diameter circular steel pile with wall thickness 12.7 mm. (0.5 inch) with embedded length of 48.15 m (158 feet) in cohesive material. The pile was closed at the tip. The field load test was performed after the pile had remained undisturbed for a period of 170 days.

Table (1) (Meyer et al,1975) shows peak side shear strength and length of each layer, together with the peak end bearing strength of the underlaying stratum.

Figure (2) shows the assumed force-displacement shape (Meyer et al,1975) for the layers and the bearing stratum. The bearing stratum force-displacement curve remains unchanged. For the layers force-displacement relations, two curves are used; the first curve A with yield displacement 1.27 mm. (0.5 inch) and curve B with yield displacement 7.62 mm. (0.3 inch).

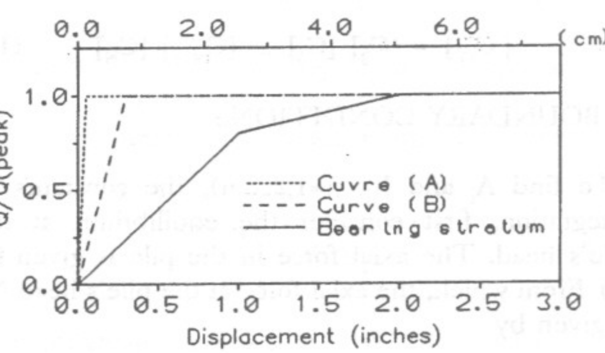

Figure 2. Force-displacement relation for side shear and end bearing.

Figure (3) shows the relationship between the load applied at the pile head and the head displacement. Three cases are tried; firstly using curve A, secondly using curve B and finally using curve B with half peak yield strength.

From figure (3), it can be shown that a very close agreement between the results obtained using the present method and both the finite element method and the field test.

Table 1. Properties of layers and bearing stratum, after Meyer et al, (1975)

	Length		Shear Strength	
	m	feet	KN/m^2	psf
Layer I	7.62	25.00	2.885	300.0
Layer II	18.28	60.00	9.619	1000.0
Layer III	22.25	73.00	19.239	2000.0
			KN	kips
Bearing stratum			505.00	56.5

A number of field pile load tests were conducted by the United States Army Corps of Engineers, in Arkansas Lock and Dam No. 4 on the Arkansas River in Arkansas, (Mansur and Hunter, 1970). Analysis of the data was carried out by (Desai, 1974). The pile used in this test is a steel pile with outer diameter 410 mm (16-in) and wall thickness 7.9 mm (.312-in) and total length of 16.1 m (52.8 ft). The site consists essentially of three major strata.

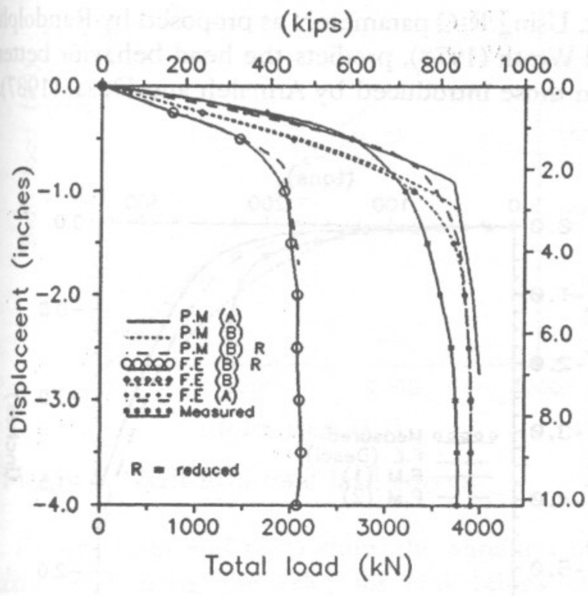

Figure 3. Head displacement-total load curves.

The force-displacement relation of each layer together with that of the underlaying stratum are simulated using a generalized Ramberg-Osgood model as proposed by Armaleh and Desai (1987).

Desai (1974) stated that " Adequate triaxial test data were not available for the soil at the LD4 site. Therefore, the required parameters where adopted on the basis of similar alluvial sands at other locations".

The general Ramberg-Osgood curve is given by

$$\tau_s = \frac{(k_{os} - k_{fs})\, z_s}{\left(1 + \left|\frac{(k_{os} - k_{fs}) z_s}{P_{fs}}\right|^m\right)^{1/m}} + k_{fs} z_s$$

where,

τ_S shear resistance along the shaft

u_S displacement

k_{OS} initial spring stiffness

k_{fS} final spring stiffness

P_{fS} load corresponding to the yield point, and

m the order of the curve, taken unity in this paper.

At each depth z, the value of k_{OS} is found by measuring the initial slope of the t-z curve at that depth. The final slope at any given depth z, k_{fS} is to be given by

$k_{fS} = 0.005\, k_{OS}$

P_{fS} is taken equal τ_{max}, the maximum shear strength.

The nonlinear p-z curve is also simulated using the R-O model as

$$p_t = \frac{(k_{os} - k_{fs})\, z_s}{\left(1 + \left|\frac{(k_{os} - k_{fs}) z_s}{P_{fs}}\right|^m\right)^{1/m}} + k_{fs} z_s$$

where

p_t pile tip resistance, and

k_{Ot}, k_{ft}, p_{ft}, and m are the R-O parameters for the pile tip.

Armaleh et al (1987) proposed that the expression introduced by Randolph and Wroth (1978) to predict the tip load in terms of the tip displacement is to be modified such that the resulting tip load be increased by a factor of 2.7 . This is done, for their model to simulate the tip behavior. Table (2) shows the properties of each layer together with the properties of the bearing stratum as proposed by Armaleh and Desai (1987).

Table 2. Parameters for t-z and p-z curves used in analysis, after Armaleh and Desai (1987).

Layer	Length		k_{os}		k_{fs}		p_{fs}	
	feet	m	t/in	kN/cm	t/in	kN/cm	t	kN
Layer I	24.0	7.315	2.63	9.2	.013	.045	.031	0.27
Layer II	23.0	7.010	5.81	20.3	.029	.1	0.17	1.5
Layer III	8.0	2.438	11.16	39.1	.056	.2	.032	2.85
Bearing stratum			k_{ot}		k_{ft}		p_{ft}	
			t/in.	kN/cm	t/in.	kN/cm	t	kN
			764.0	2676.3	3.82	13.4	70.0	623.0

In the present method it is found that accepting the expression introduced by Randolph and Wroth (1978) simulates the actual behavior of the pile's head and tip better than that introduced by Armaleh and Desai (1987). In the two cases considered it is found that; the proposed model simulates the actual response of the pile,nearly, up to top settlement equal 20 mm.(0.75 inch).

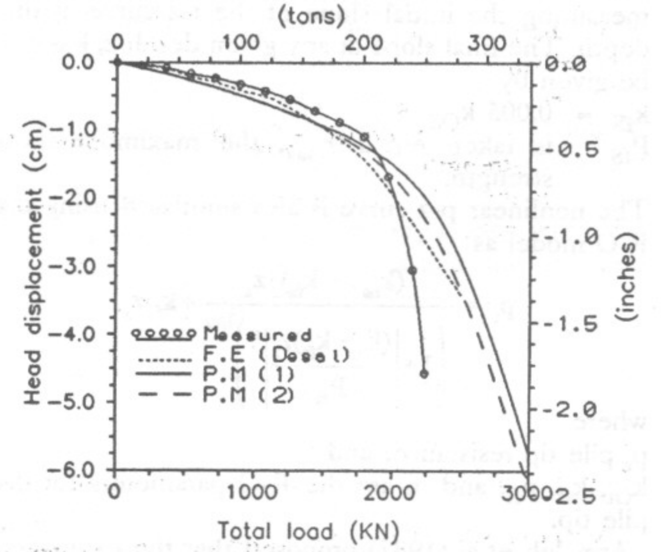

Figure 4. Head displacement-total load curves.

In Figures (4) through (8-b), the curves obtained using the present method and the R-O parameters proposed by Armaleh and Desai (1987) are refer to by P.M (1), while the curves obtained using the present method and the R-O parameters calculating using the approach of Randolph and Wroth (1978) are referred to by P.M(2).

Figure (4) shows the relationship between head displacement and the load applied at the pile head. From figure (4) it can be seen that the present method is conservative, and produces results very close to the finite element solution and the field test. Using R-O parameters as proposed by Randolph and Wroth (1978), predicts the head behavior better than those introduced by Armaleh and Desai (1987).

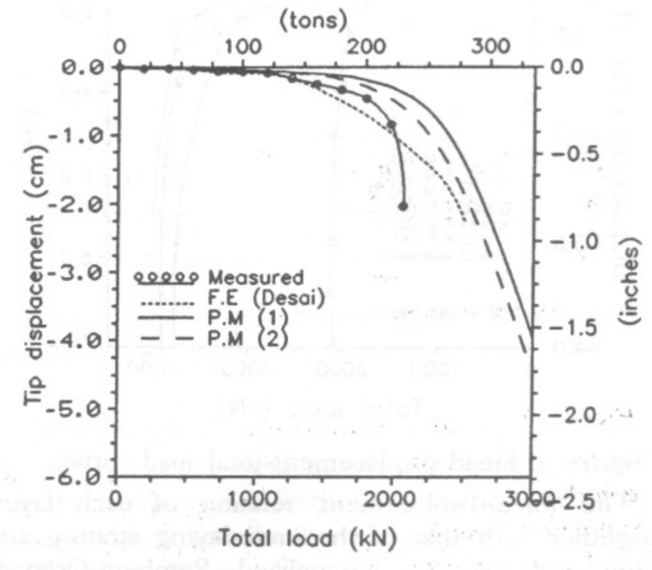

Figure 5. Tip displacement-total load curves.

The deviation between the two curves appears at displacements higher than 10 mm. (0.4 inch). This is expected, since changing the bearing resistance of the bearing stratum affects the force-displacement relation at higher displacements only.

Figure (5) shows the relationship between tip displacement and the load applied at the pile head. Again, from figure (5) it can be seen that, using R-O parameters as proposed by Randolph and Wroth (1978), predicts the tip behavior better than those introduced by Armaleh and Desai (1987).

Figures (6) and (7) show the relationship between shaft load and total load and the relation between tip load and total load, respectively. The difference between the results obtained using R-O parameters as proposed by Randolph and Wroth (1978), and those obtained using R-O parameters as proposed by Armaleh and Desai (1987) is negligible. Both of them simulate the finite element solution very well.

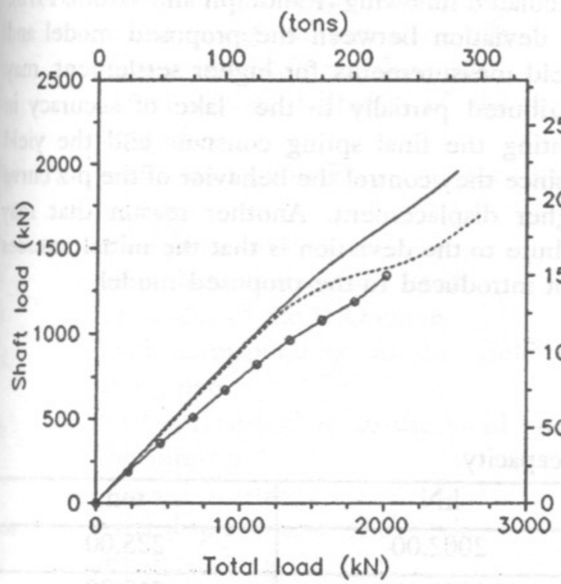

Figure 6. Shaft load-total load curves.

Figures (8-a) and (8-b) show the variation of the axial load along the pile, for two values of the applied load 1072.5 kN (120 ton) and 1787.5 (200 ton), respectively. From the figures, it is clear that the present method predicts the distribution in the axial load within the pile better than the finite element method. Beside, the distribution of the axial load is insensitive to the variation in R-O parameters representing the bearing stratum.

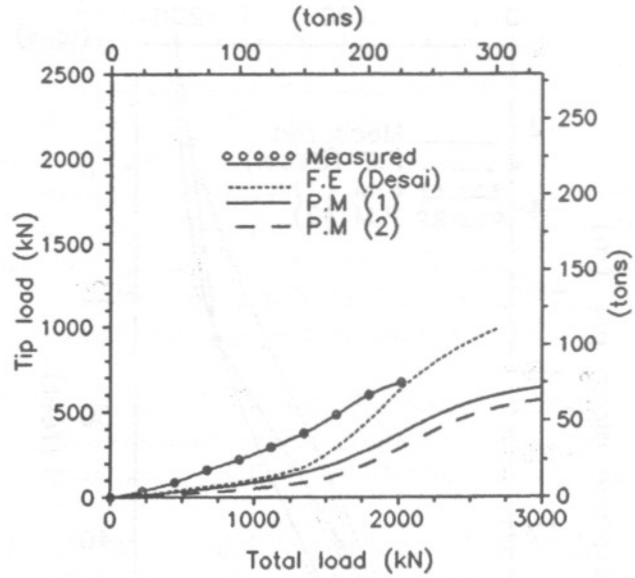

Figure 7. Tip load - total load curves.

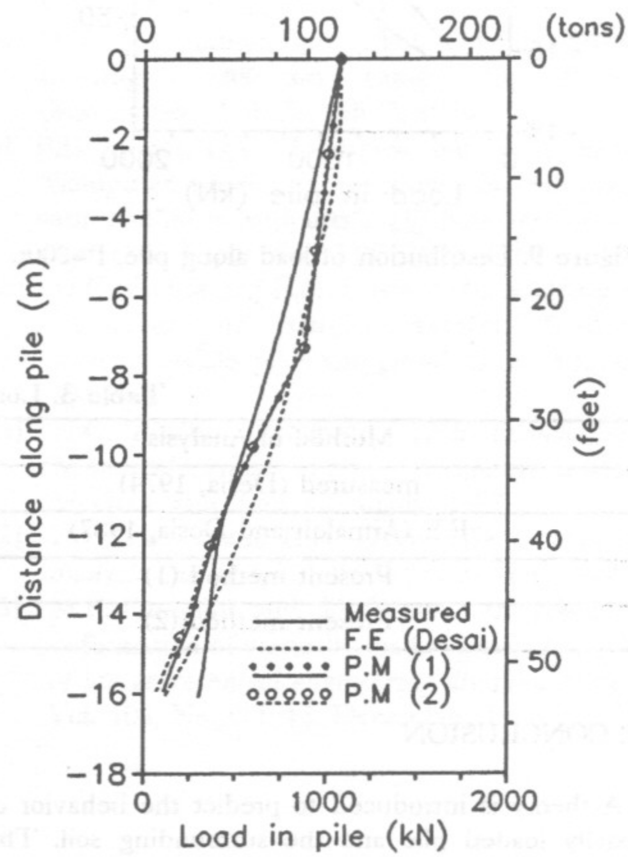

Figure 8. Distribution of load along pile, P =120 t.

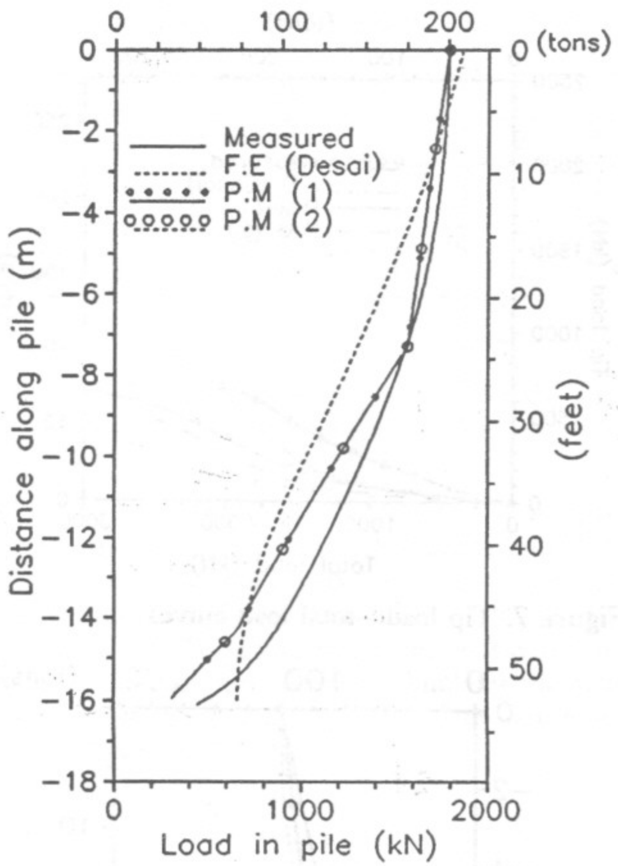

Figure 9. Distribution of load along pile, P=200t.

The load carrying capacity of the pile is measured from the field test (Desai, 1974) and evaluated using the finite element method (Armaleh and Desai, 1987). It is calculated using the present method twice; once using the R-O parameters as introduced by (Armaleh and Desai (1987) and once using the R-O parameters as proposed by Randolph and Wroth (1978). In all cases, the tangent method are used. In this method, the intersection of tangents to the initial and final portions of the pile head displacement-total load curve gives the value of the bearing capacity.

Table (3) gives the results of the comparison. From table (3), it can be seen that the proposed method succeeds in predicting the carrying capacity of the pile, especially if the parameters of the R-O curve are calculated following (Randolph and Wroth, 1978).

The deviation between the proposed model and the field measurements for higher settlement, may be attributed partially to the lake of accuracy in calculating the final spring constant and the yield load, since they control the behavior of the p-z curve at higher displacement. Another reason that may contribute to the deviation is that the initial stresses are not introduced to the proposed model.

Table 3. Load carrying capacity.

Method of Analysis	kN	ton
measured (Desia, 1974)	2002.00	225.00
F.E (Armaleh and Desia, 1987)	1940.00	218.00
Present method (1)	2200.00	247.00
Present method (2)	2100.00	235.90

7. CONCLUSION

A theory is introduced to predict the behavior of axially loaded pile and the surrounding soil. The theory takes into account the effect of variation of pile and soil properties with depth. It is valid for both linear and nonlinear analysis, using the same procedures. All possible conditions at the pile's tip is incorporated in the analysis.

The method is analytical and requires minimal computing time and memory space. Comparison with field tests and the finite element solution shows very close agreement. In case of linear analysis the solution is straight forward, while in case of non-linear analysis the solution is incremental, not iterative, which means higher accuracy and less computing time. The introduced method uses the secant modulus of the subgrade reaction, but the tangent subgrade reaction can be used with the same ease.

Appendix Notation

A_i and B_i	constants of integration
$[C_i]$	transfer matrix
$[C_G]$	global transfer matrix
EA_p	the axial rigidity of the pile
K	subgrade reaction along pile, "force per unit length of the pile needed to produce unit displacement"
K_S	subgrade reaction of the bearing stratum, "force needed to produce unit displacement at the pile tip"
k_{OS}	initial spring stiffness along pile
k_{fS}	final spring stiffness along pile
k_{Ot}	initial spring stiffness of the bearing stratum
k_{ft}	final spring stiffness of the bearing stratum
N	axial force in the pile
N_O	axial force at the pile's head
m	the order of the R-O curve
P_{fS}	load corresponding to the yield point along pile
P_{ft}	load corresponding to the yield point of the pile tip
P_t	pile tip resistance
u or u^S	axial displacement of the pile,
z	distance along the pile
τ_S	shear resistance along the shaft

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