STRUCTURAL STABILITY OF Z TYPE PURLINS, CLADDING AND BRACING SYSTEM OF LIGHT GAUGE STEEL FRAMES

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ABSTRACT

The behaviour of the Z type purlin section is studied together with the steel sheet cladding contribution. Steel sheeting is considered as elastic support to the purlins. Therefore the stability of the structural elements and the integrity of the whole structure is investigated. The study presented in this paper examines the stability of steel roofs while Z purlins are loaded eccentrically. The outcome of this research has proved the fact that light gauge steel roofs can't assure its stability unless there are adequate bracing and integration of different structural members of the roofing system. A case study is presented herein after historical back ground of failure and distortional events. The analytical study and the prescribed remarks provide a sound tool for design of light gauge steel members.

INTRODUCTION

Cold formed steel members, such as channels and z-sections provide substantial savings due to there high strength to weight ratio. As a result, they have become very popular in the construction of metal walls and roof systems in industrial, commercial, and agricultural buildings. In the meanwhile, the cross sectional configuration of a cold formed section, however, gives rise to behavioural phenomenon, that local or distortional buckling leads to a substantial reduction in their load bearing capacity. Sections like (C,Z) are prone to failure by local buckling.

In an experimental investigation encountered between 1976 and 1981 [1] involving z-section girts, it was observed that the main mode of failure was due to that corresponding to yielding in the design procedure.

For hot rolled sections in compression, the width to thickness ratio is limited on the base, that the different elements of the section will undergo plastic deformations before local buckling has taken place. But for cold formed sections, the elements are so thin such that the resistance of the section is based on local post buckling behaviour which will take place before overall plastification of the section.

The interaction between local and distortional or lateral buckling is treated as a reduction in the normal axial strength of the section by using the effective area of the cross section which is evaluated at the ultimate stress instead of the full section. This provision is limited to the case of doubly or single symmetrical sections.

Z-section are point symmetric and when subjected to direct compression, most specifications require that their allowable load be computed either through rational analysis or through testing. Rational analysis indicates that the critical stress of a z-section which is not subjected to local buckling is the smallest of the three critical values about the principal axes [2].

When the critical stress exceeds the proportional limit, then the behaviour is assumed to be inelastic and the ultimate stress has to account for the residual stresses and initial deformations. Therefore mathematical models are proposed to demonstrate the interaction between the Z type section and the attached sheeting.

Mathematical Models of Z Type Section and Attached Sheeting:

Two mathematical models are suggested (presented) herein to demonstrate the interaction between the cold formed z-type section and the sheet covering.

Model A

At the moment of buckling, the sheet acts as an elastic support to the purlin, and the displacements are only vertical and rotation.

Model B

The upper lip is fixed to the sheet which is rigidly fixed and at the moment of buckling the free part rotates about the intersection of the fixed and free part.

Mathematical Treatment of Model A

A bar with an open thin walled section is studied under the effect of an eccentrically compressive load P_o, and uniformly distributed loads q_x and q_y in X and Y directions, as shown in Figure (1).

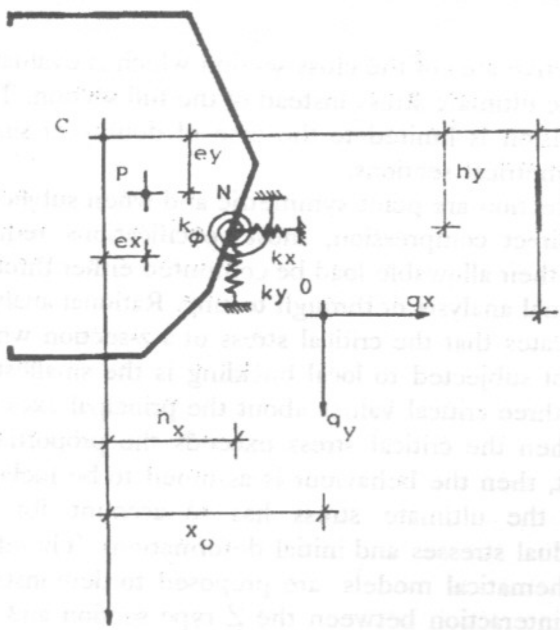


Figure 1. Geometrical notation.

The bar is considered elastically supported throughout its length at point N by three supports with stiffness k_x, K_y and K_ϕ in the x,y and about the z axes respectively. Denoting the components of the displacement of the shear center axis by u, v, and the angle of rotation of that axis by ϕ, the components of the displacement of the fiber at point N along which the reactions are developed, are:

$$U_N = u + (y_o - h_y)\phi \qquad V_N = v - (x_o - h_x)\phi \tag{1}$$

The corresponding reactions per unit length, assumed positive in the direction of the x and y and about z axes will be respectively:

$$- K_x [u + (y_o - h_y)] \phi \tag{2.a}$$

$$- K_y [v - (x_o - h_x)] \phi \tag{2.b}$$

and

$$- K_\phi \phi \tag{2.c}$$

Adding the lateral forces obtained from the lateral deflection and due to the end moments and the action of the initial compressive forces acting on the slightly rotated cross sections of the longitudinal fibers, to the elastic reactions, to have equilibrium in the x and y direction as follows:

$$EI_y u'''' + EI_{xy} v'''' + P u'' + P(y_o-e_y) \phi'' + K_x [u + (y_o - h_y) \phi] = q_x \tag{3}$$

$$EI_x v''''+ EI_{xy} u''''+ P v''- P(x_o - e_x) \phi'' + K_y [v - (x_o - h_x)\phi] = q_y \tag{4}$$

The intensity of the torque m_z distributed along the shear-center axis is equal to the couple developed by the elastic reactions and the eccentrically compressive load is:

$$\begin{aligned} m_z = & - (Py_o - Pe_y) u'' + (Px_o - Pe_x)v'' \\ & - (Pe_y \beta_1 + Pe_x\beta_2 + P I_o/A)\phi'' \\ & + k_x[u + (y_o - h_y)\phi](y_o - h_y) \\ & + k_y[v - (x_o - h_x)\phi](x_o - h_x) - k_\phi\phi = 0 \end{aligned} \tag{5}$$

where ' means differentiation once with respect to variable z_o and,

$$\begin{aligned} \beta_1 &= \frac{1}{I_x}\left(\int_A y^3 dA + \int x^2 y dA\right) - 2y_o \\ \beta_2 &= \frac{1}{I_y}\left(\int_A x^3 dA + \int y^2 x dA\right) - 2x_o \end{aligned} \tag{6}$$

Adding equation (5) to the internal torque, the equation of equilibrium about the axis passing by the shear-center will be:

$$EI_w\phi'''' - [GJ - Pe_y\beta_1 - Pe_xB_2 - P\frac{I_o}{A}]\phi''$$
$$-P(y_o - e_y)u'' - P(x_o - e_x)v'' + k_x[u + (y_o - h_y)\phi](y_o - h_y) \qquad (7)$$
$$-k_y[v - (x_o - h_x)\phi](x_o - h_x) + K_\phi\phi = 0$$

Bar With Prescribed Plane of Deflection

Due to the attachment of the z-section to the sheet covering in plane n-n Figure (2), the bar must be deflected in a known direction.

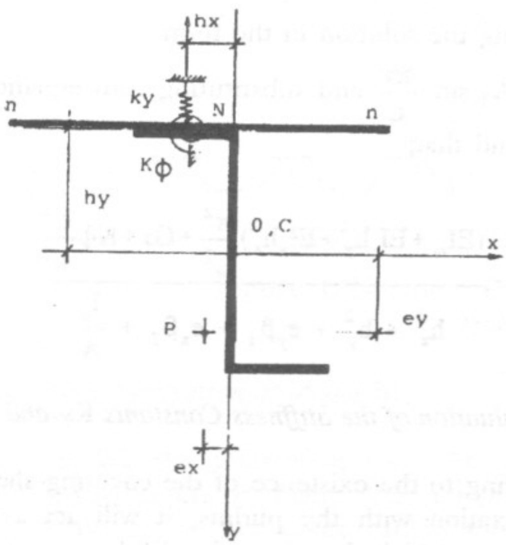


Figure 2. Section with prescribed displacement in (y-y) direction but not permitted in n-n direction.

The fibers of the bar in contact with the sheet cannot deflect in the plane of the sheet, but they are free to deflect in the perpendicular direction and rotate about the shear-center. The displacement components of fiber N are:

$$v_N = v + h_x\phi \qquad (d)$$
$$u_N = u - h_y\phi = 0 \qquad (e) \qquad (8)$$

which gives the horizontal displacement of the shear center as:

$$u = h_y\,\phi \qquad (9)$$

Owing to the restraint at fiber N, there will be reactions of intensity q_o which is distributed continuously throughout the fiber N, and acting in the direction parallel to the x-axis and equal to:

$$q_o = EI_y\,u'''' + EI_{xy}\,v'''' + Pu'' - P\,e_y\,\phi'' \qquad (10)$$

Also, the intensity of forces in the y direction will be:

$$q_y = EI_x\,v'''' + EI_{xy}\,u'''' + Pv'' + P.\,ex\,\phi'' + k_y\,(v + h_x\phi) \qquad (11)$$

By eliminating u from equation (11) by substitution from equation (9), equation (11) becomes:

$$EI_x\,v'''' + EI_{xy}\,u'''' + Pv'' + P.ex\,\phi'' + k_y(v + h_x\,\phi) - q_y = 0 \qquad (12)$$

Another equation for v and ϕ is obtained by considering the torsion of the bar about the axis passing through the shear-center. The uniformly distributed load transmitted to the section at the point N where the sheet is fixed to the Z-section produces an additional torque to that in equation (5). This additional torque is equal to:

$$-\,q_o\,h_y + q_y\,h_x \qquad (13)$$

adding this torque to that in equation (7), and substituting x_o and y_o equal to zero, then we obtain the equation of equilibrium about the axis passing through the shear center as follows:

$$[EI_\omega + EI_yh_y^2 + EI_{xy}h_xh_y]\phi''''$$
$$-[GJ - P(e_y\beta_1 + e_x\beta_2 + 2e_yh_y + e_xh_x - hy^2 + \frac{I_o}{A}]\phi'' + [2K_yh_x^2 + K_\phi]\phi$$
$$+[EI_xh_x - EI_{xy}h_y]v'''' - [P.ex + Ph_x]v'' + 2kyhx = 0 \qquad (14)$$

Taking the solution of equations (12) and (14) in the form:

$$V = A_1\sin\frac{\pi}{L}z \qquad (15)$$

$$\phi = A_2\sin\frac{\pi}{L}z \qquad (16)$$

$$q_x = q_y = 0 \qquad (17)$$

and substituting equations (15) and (16) into (12) then equation (14) becomes:

$$[\frac{\pi^4}{L^4}EI_x - \frac{\pi^2}{L^2}P + k_y]A_1 + [\frac{\pi^4}{L^4}EL_{xy}h_y - \frac{\pi^2}{L^2}P.e_x \qquad (18)$$
$$+ k_yh_x]A_2 = 0$$

$$\left\{(EI_x h_x - EI_{xy} h_y)\frac{\pi^4}{L^4} + [Pe_x + Ph_x]\frac{\pi^2}{L^2}\right\} A_1$$
$$+\{[EI_\omega - EI_y h_y^2 + EL_{xy} h_x h_y]\frac{\pi^4}{L^4} \qquad (19)$$
$$+[GJ - P(e_y\beta_1 + e_x\beta_2 + 2e_y h_y + e_x h_x - h_y^2 + \frac{I_o}{A})]$$
$$\frac{\pi^2}{L^2} + [2k_y h_x^2 + k_\phi]\} A_2 = 0$$

Solving this condition, which is represented by equations (18) and (19), we find the critical load P.

When the stiffness of the elastic vertical support K_y equals infinity ($Ky = \infty$), the bar sections rotate during buckling about the axis passing through N Figure (3) and will remain straight during buckling.

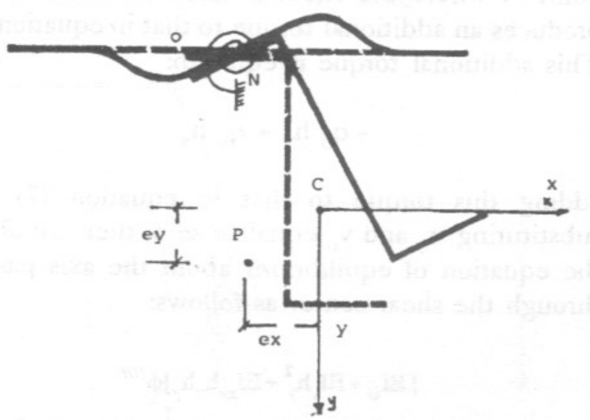


Figure 3. Displacement are not permitted in horizontal and vertical direction, and the section rotates only about fixed axis passing by point N.

For this case the displacement of point N will be

$$u_N = u + (y_o - h_y)\,\phi = 0, \qquad u = -(y_o - h_y)\phi$$
$$v_N = v - (x_o - h_x)\,\phi = 0, \qquad v = (x_o - h_x)\phi \qquad (20)$$

Differentiating these expressions yields the following,

$$u'' = -(y_o - h_y)\phi'' \quad \therefore \quad u'''' = -(y_o - h_y)\phi''''$$
$$v'' = (x_o - h_x)\phi'' \quad \therefore \quad v'''' = (x_o - h_x)\phi'''' \qquad (21)$$

Moreover Eqs (3) and (4) will result in the following relationships respectively.

$$k_x[u+(y_o-h_y)\phi] = -EI_y u'''' - EI_{xy} v'''' - Pu'' - P(y_o-e_y)\phi'' \qquad (22)$$

$$k_y[v-(x_o-h_x)\phi] = -EI_x v'''' - EI_{xy} u'''' - Pv'' + P(x_o-e_x)\phi'' \qquad (23)$$

An equation for the angle of rotation ϕ can be obtained by substituting equations (22) and (23) into (7), and putting $x_o=y_o=0$. The resulting equation is,

$$[EI_\omega + EI_y h_y^2 + EI_x h_x^2]\phi'''' - [GJ - P.e_y\beta_1 - P.e_x B_2 - P\frac{I_o}{A} - Ph_x^2 - Ph_y^2]\phi'' + K_\phi\phi = 0 \qquad (24)$$

Taking the solution in the form

$\phi = A_2 \sin \dfrac{\pi z}{L}$ and substituting into equation (24) we find that;

$$P_{cr} = \frac{(EI_\omega + EI_x h_y^2 + EI_x h_x^2)\dfrac{\pi^2}{L^2} + GJ + K\phi\dfrac{L^2}{\pi^2}}{h_x^2 + h_y^2 + e_y\beta_1 + e_x\beta_2 + \dfrac{I_o}{A}} \qquad (25)$$

Evaluation of the Stiffness Constants Ky *and* Kϕ

Owing to the existence of the covering sheet and its fixation with the purlins, it will act as elastic supports which decreases the displacements of the purlins. The covering sheet is fixed at the top and eave of the frame, which acts as a simply supported beam, Figure (4). It is easy to write the expressions for the stiffness K_y and K_ϕ, for the supporting purlin as:

$$K_y = \frac{EI_c}{H} \qquad (26)$$

and

$$K_\phi = \frac{EI_c}{C} \qquad (27)$$

where

$$H = \left[\frac{a^3b^2}{3d^2} + \frac{b^3a}{3d} - \frac{b^4a}{3d^2}\right]$$

$$C = \left[-\frac{a^3}{3d^2} + \frac{b^2}{2d^2}(d - \frac{2}{3}b) - \frac{b^2}{2d}\right]$$

and Ic is the moment of inertia of the covering sheet about the axis (c.c)

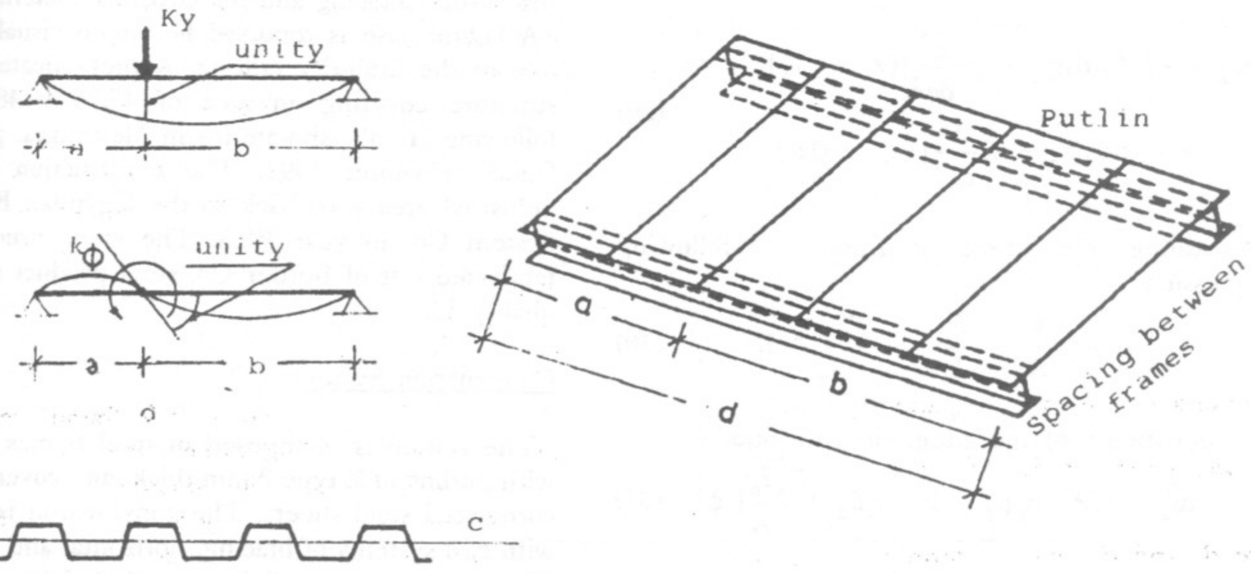


Figure 4. Notations for determining the torsional and vertival elastic supports stiffnesses.

Mathematical Derivation of Model B:

In this model, the interaction with the overall buckling mode in the xy plane does not include displacements in the xy plane. It is therefore assumed to be qual to zero.

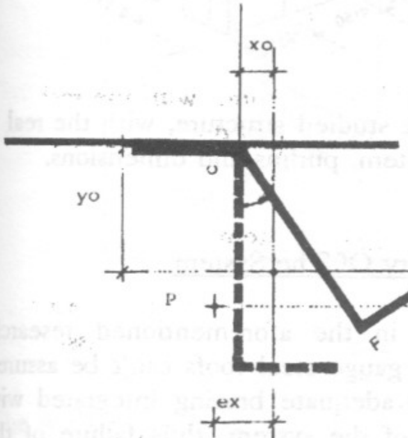


Figure 5. Notations and rotation of the section about 0.

Section OFG Figure (5). It is assumed to be elastically fixed along an axis passing through point 0. The elastic fixation is due to the covering sheet.

To find out the critical load, the member is assumed to act as a beam-column of open thin walled cross section, which at the critical situation will rotate about an axis passing through the line of intersection of the web and the flange of the z section. It is further assumed that the support conditions at z = 0 and z = L are of the hinged type. This assumption is a good approximation for the construction employed, and in any case is on the safe side.

To find the equation of equilibrium of the torque moment about the axis passing through 0, consider a fiber of the section with coordinates x and y where the normal stress is σ. The deflection components of this fiber of the bar, are:

$$u = (y_o - y)\,\phi, \qquad v = -(x_o - x)\,\phi \tag{28}$$

The intensities of the fictitious distributed torque obtained from the initial compressive force in the fiber acting on their slightly rotated cross sections is:

$$m_2 = -\int_A (\sigma t ds(y_o - y)\frac{d^2}{dz^2}[(y_o - y)\phi] + \int_A \sigma t ds(x_o - x)\frac{d^2}{dz^2}[(x_o - x)\phi] \tag{29}$$

substituting the stress σ from the following expression

$$\sigma = -\frac{P}{A} - \frac{Pe_y}{I_x}y - \frac{P \cdot e_x}{I_y} X \tag{30}$$

into equation (16) and integrating, we obtain:

$$m_z = -[P \cdot e_y \beta_1 + P \cdot e_x \beta_2 + P\frac{I_o}{A}] \phi'' \tag{31}$$

where β_1 and β_2 are as defined by equation (6). Adding the torque found by equation (31) to the internal torque, the equilibrium about the axis passing through O is obtained.

$$EI_\omega \phi'''' - [GJ - P.e_y\beta_1 - P.e_x\beta_2 - P\frac{I_o}{A}]\phi'' = 0 \tag{32}$$

Note that the constants, I_ω and I_o are calculated with respect to center of rotation 0:,
Assuming the solution of equation (32) of the form:

$$\phi = A_2 \sin\frac{\pi}{L}Z \tag{33}$$

Equation 32 becomes:

$$EI_w\frac{\pi^4}{L^4} + [GJ - P.e_y\beta_1 - P.e_x\beta_2 - P\frac{I_o}{A}]\frac{\pi^2}{L^2} = 0 \tag{34}$$

From which the critical load is equal to:

$$P_{cr} = \frac{EI_w\frac{\pi^2}{L^2} + GJ}{[ey\beta_1 + e_xB_2 + \frac{P_o}{A}]} \tag{35}$$

Case Study

Review of the Problem

Although occasional failures of steel structures were reported for various types of covering, it was not intended to describe the role of integrity between the vertical bracing and the covering material.

A failure case is reported herein to visualize the role of the integrity criteria. A prefabricated steel structure covering an area of 45,75 x 48,80 m, following Hien's laboratories in Alexandria, partially failed in winter 1992. The construction of this industrial area goes back to the Egyptian Building System Co. in year 1985. The steel structure is fabricated out of Buttler Co. steel product sections quality 52.

Construction System

The system is composed of steel frames tied up with purlins of Z type 2 mm thick, and covered with corrugated steel sheets. The construction is braced with two systems of bracing, horizontal and vertical. The diagonals are light rounded bars 16 mm diameter, Figure (6). No direct compression struts are accommodated. These missing members were the main reason of failure events.

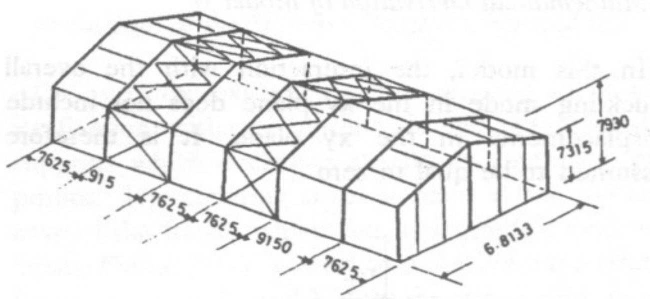


Figure 6. The studied structure, with the real bracing system, purlins and dimensions.

Structural Stability Of The System

As concluded in the aforementioned research, stability of light gauge steel roofs can't be assured unless there are adequate bracing integrated with other members of the system, thus failure of the system occurs due to loss of stability of the deck roof, which can't resist longitudinal wind loads. Wind action on the top roof is transferred directly through purlins to the transversal frames which's not fully braced with longitudinal compression struts to transmit wind load to the foundation. Moreover, the transversal columns of end gables weren't adequately connected to the foundation at the base level, because of the corrosion effect on the material of the columns and the bolts near the foundation. This made them easy to deform in the direction of wind and so they pulled out at their fixed ends when the structure moved under wind buffeting.

The results of such handicap of the system is that frames collapsed on each other in the longitudinal direction, end gable columns taken off, purlins twisted and the sheeting cover taken off and teared.

The attached pictures indicate failure events and consequences. The structural and bracing systems before collapse are shown in Figure (6).

Calculation Sheet

The variation of wind velocity is as shown in Figure (7)

$$v_z = v_g \, (Z/Z_g)^{\gamma}$$

where

- v_z is the mean wind speed at height Z above the ground
- v_g gradient wind speed assumed constant above the boundary layer
- Z building height above ground
- Z_g depth of boundary layer
- γ power law coefficient

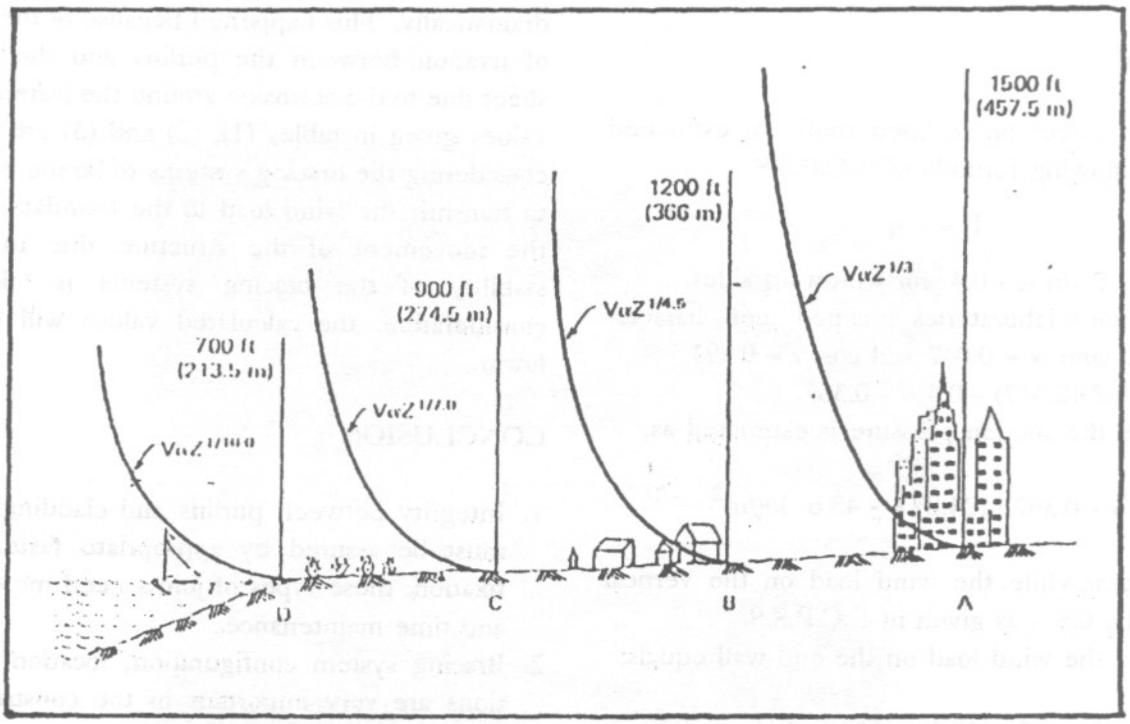


Figure 7. Variation of wind velocity with height.

The wind pressure is,

$$q = \tfrac{1}{2} \rho \, V_z^2$$

where ρ is the wind density.

The aerodynamical tests showed that the wind pressure depends on the geometrical dimension of structure, and its value is three times the calculated value from the previous equation, so Grashoff (6) suggested increasing the calculated value by 1.86. Then the wind pressure becomes

$$q = 0.122 \, v^2$$

For types of buildings near to the sea shore as the case of the Hien's laboratories where it lies on the shore of Mariout lake, the following aerodynamic variables for this kind of exposure is

V_g = 160 km/hr, Z_g = 213.5 mt, γ = 1/10

Therefore for a structure of height Z = 8 m., the estimated wind speed on that particular structure is

$$\text{wind velocity} = 160(\frac{8}{213.5})^{1/10}$$
$$= 115.2 \text{ km/hr}$$
$$= 31.98 \text{ m/sec}$$

Therefore $q = 0.122\,(31.98)^2 = 124.8$ kg/m^2

Wind Load

The wind forces on inclined roofs are estimated from the following formula of E.C.P.S.S.

$$P = c\,q$$

where $c = 1.2 \sin \alpha - 0.4$ for windward sides.
For the Hien's laboratories inclined roof characteristics with, sine α = 0.047 and cos α = 0.991.
Then, c = 1.2 (0.047) - 0.4 = - 0.367.
Therefore the suction pressure is estimated as,

$$P = -0.367\,(124.8) = -43.6 \text{ kg/m}^2$$

In the meanwhile the wind load on the vertical sides will be 0.8 q as given in E.C.P.S.S.
Therefore the wind load on the end wall equals:

$$P = 0.8\,(124.8) = 100 \text{ kg/m}^2$$

And the wind load per joint transmitted by bracing system $= \dfrac{100 \times 7.5 \times 7.4}{2} = 2.77$ ton

This load is applied directly on purlins as eccentric normal load.
Using equations (26) and (27) with the structure dimensions, the spring stiffnesses were found as:

$$K_y = \frac{48EI_c}{d^3} \quad \text{t/cm/cm}$$

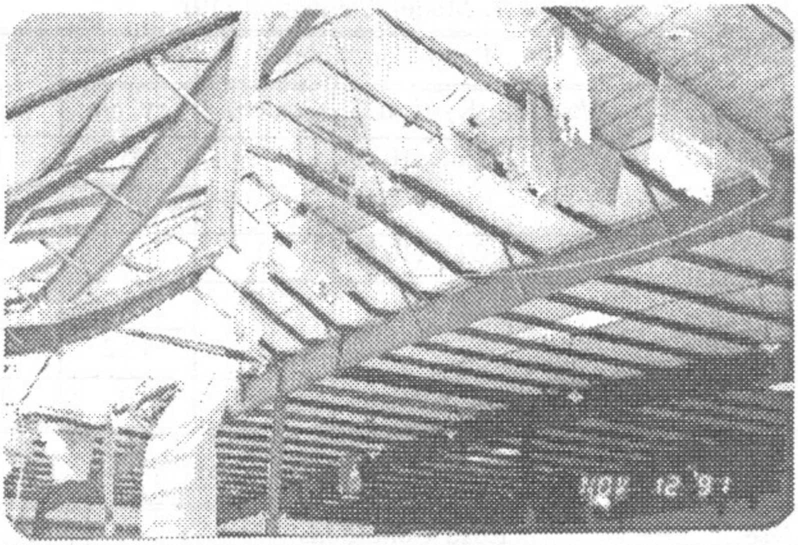
$$K_\phi = \frac{12EI_c}{d} \quad \text{t.cm/cm}$$

The critical axial loads were calculated based on models deformation A and B using equations (19), (25) and (35). The results are in Tables (1), (2) and (3). The critical values are to be compared with those calculated from the wind load.

From the collapsed structure and the results in tables (1), (2) and (3) it is easy to remark that the integrity between the covering sheet and purlins gives high values of critical loads while when the z section acts alone the capacity is reduced dramatically. This happened because of the absence of fixation between the purlins and the covering sheet due to the corrosion around the fasteners. The values given in tables (1), (2) and (3) are obtained considering the bracing systems to be more efficient to transmit the wind load to the foundation. When the movement of the structure due to lack of stability of the bracing systems is taken into consideration, the calculated values will be much lower.

CONCLUSION

1. Integrity between purlins and cladding material must be assured by appropriate fastening and fixation. these types of joints need more caution and time maintenance.
2. Bracing system configuration, location and sections are very important in the construction of light gauge steel structures.
3. The bracing must be located in the first and last panels (paneau) to save up the purlins from the axial load, otherwise the sections have to be designed to resist axial loads in addition to gravity and wind loads.
4. Check for local buckling is required for purlin sections. This has not been done in this study.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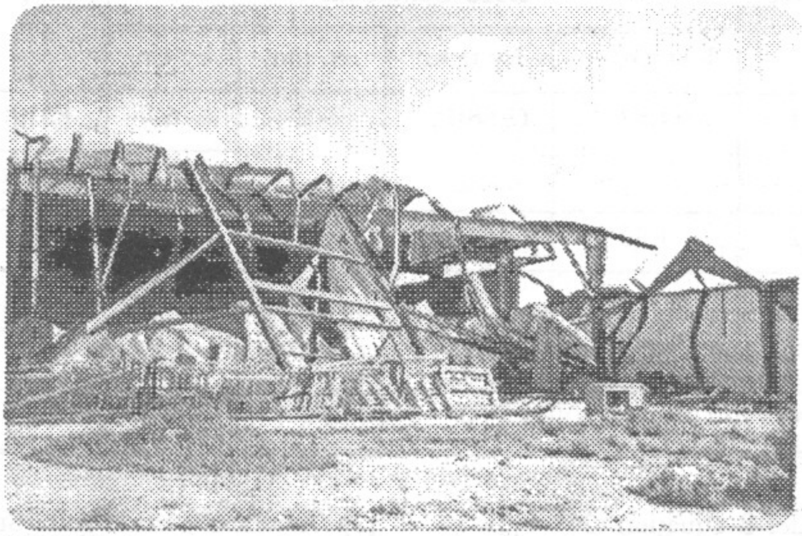

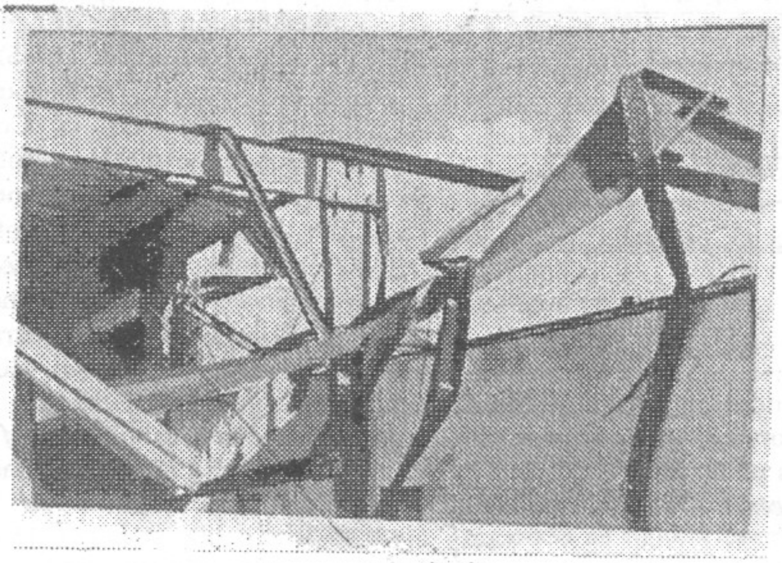

Table 1. Model A equation (19)

Case	Load Conditions				P_{cr} ton
	K_ϕt.cm/cm	Ky t/cm^2	ex.cm	ey. cm	
1	1.52	$2.68^*(10^{-6})$	- 2	6	17
				12.2	9
2	1.52	$2.68^*(10^{-6})$	0	0	20
3	0	0	- 2	6	1
				12.2	0.18
4	0	0	0	0	1.5

Table 2. Model B equation (25)

Case	Load Conditions				$P_{cr.}$ ton
	Kϕt.cm/cm	Ky t/cm^2	ex. mm	ey cm	
1	1.52	$(2.68)10^{-6}$	- 2	6	681,8
				12.2	681.8
2	1.52	$(2.68)10^{-6}$	0	0	681.8
3	0	0	- 2	6	2.45
				12.2	2.45
4	0	0	0	0	2.45

Table 3. Model B equation (35)

Case	$K\,\phi = Ky = \infty$		$P_{cr.}$ ton
	ex cm	ey cm	
1	- 3.2	3.1	1.32
		7.1	0.33
2	0	0	1.662

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