# STRUCTURAL RESPONSE OF SINGLE DEGREE OF FREEDOM SYSTEM TO AIRCRAFT IMPACT

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ABSTRACT

The maximum response of an elastic, damped, one DOF system to aircraft impact is studied. The results lead to the conclusion that using the simplified static analysis with a load equals to the peak impact load may be sufficient to predict the maximum response for structures whose natural frequencies ranges from 20 to 100 cycle/second.

*Keywords*: Dynamics, Impact, Aircraft strike, Structures, One DOF system.

## INTRODUCTION

Present estimations indicate that within ten years about 30% of the electric power generated in the world will be located in the vicinity of commercial or military airports, and therefore the need of providing adequate safeguards against an accidental aircraft strike may become a decisive factor in plant location and design. These safeguards imply developing the capability of bringing the plant to a safe shut down condition in case of a direct aircraft strike.

Nuclear regulations in several countries require that the reactor building and the equipment be designed to resist the impact of a commercial aircraft, hence, much work has been devoted recently to this subject (1 to 3). The impact of a Boeing 707-32 onto the dome of the secondary containment of a BWR reactor was investigated (1) using a finite element analysis of both linear and non linear response. A study concerning the effects of induced vibrations due to a Phantom RE-4R crash on the safety of secondary systems (equipments) of nuclear power plants was conducted (3). The influence of various parameters (time history of excitation direction and location of impact, soil damping etc) were discussed. Suggestions were made for developing suitable floor design spectra and use them to analyze multi-degree of freedom systems (3). Throughout the studies, the time history of the force used is corresponding to an aircraft impact against a rigid wall.

Since any structure can be represented as a single degree of freedom (SDF) system, which enables the analytical solution, then the nuclear station in our analysis is considered as one. In this paper the response of a single degree of freedom system to aircraft impact is investigated. The solution is conducted such that the equations describing the response at each interval of time are determined analytically, then these equations are computerized to get the maximum dynamic load factor at each frequency. The results obtained are identical to those of reference (2) which used a purely analytical solution for the case of undamped SDF system. The load description and the details of the procedure are presented.

## 2. CALCULATION OF FORCES DUE TO AIRCRAFT STRIKE

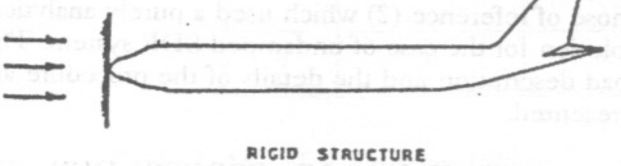
It appears that the most unfavorable angle of strike would in any case be normal to the surface under consideration (2). Consequently, only normal impact will be considered here. The reaction versus time curve will, of course, depend upon the characteristics of the target building. In most nuclear plant applications, however, the building could not undergo, without serious structural damage, i.e., deformations larger than a few inches. Such deformations are negligible in comparison with those of the collapsing aircraft which justify the assumption that in the computation of the total reaction versus time curve the building may be regarded as rigid. Consequently, once the reaction versus time relations has been determined for a given aircraft and for the velocity of its impact, it is directly applicable to any stable stiff structure.

### 2.1 *Impact velocity*

An impact velocity of 200 knots is used in the calculation. The reasons for selecting this value are: first, it may be expected that in the neighborhood of the airport the aircraft will not exceed normal takeoff or landing speeds; second, records of accidents of large commercial aircraft that occurred within a two and one-half mile radius of the end of a runway, the estimated or recorded impact velocity was 200 knots (2).

### 2.2 *Forcing Function*

The loading on structure produced by the impact of a commercial Boeing 707-320 aircraft is defined as a function of time (Figure 1-a). The peak force is $20x10^6$ lb and the duration is 330 milliseconds. The time history of the force, idealized by a polygonal curve in Figure (1-b) is the result of calculations in which the aircraft impact against a rigid wall was studied (2).



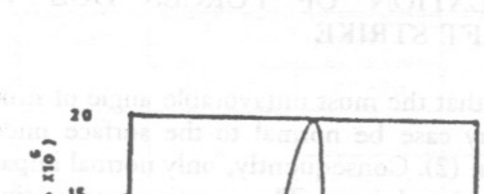




Figure 1-a. Reaction load for Boeing 707-320.

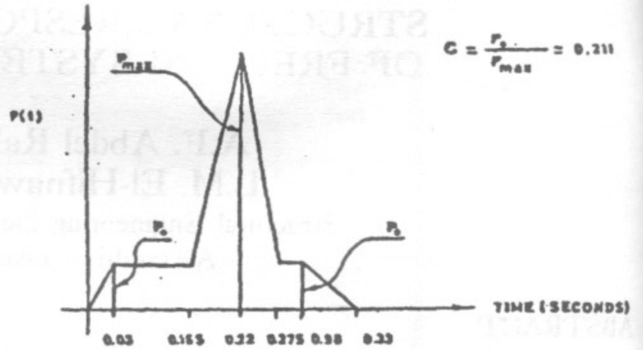



Figure 1-b. Idealized total reaction load for Boeing 707-320.

## 3. RESPONSE OF DAMPED LINEAR, ELASTIC ONE DEGREE OF FREEDOM SYSTEM

The equation of motion of any single degree of freedom, is entirely equivalent to the equation of motion of a simple spring-mass system (Figure 2) which may be written as

$$m\ddot{v}(t) + c\dot{v}(t) + kv(t) = p(t) \qquad (1)$$

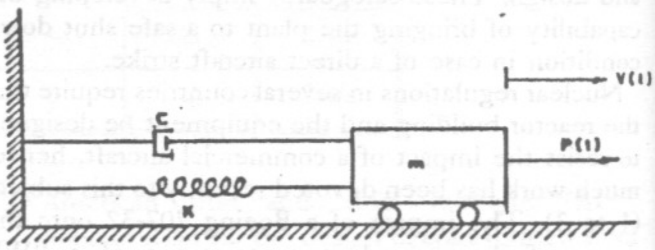



Figure 2. Basic SDF system.

Where m represents the mass of the system, c the damping of the dashpot, and k the stiffness of the spring.

The solution of equation 1 is obtained by considering first the homogeneous equation with the right side set equal to zero, i.e., the free vibration state

$$m\ddot{v}(t) + c\dot{v}(t) + kv(t) = 0 \qquad (2)$$

The solution of equation 2 is of the form (4)

$$v_c(t) = e^{-\xi\bar{\omega}t}\{A \sin \omega_D t + B \cos \omega_D t\} \quad (3)$$

in which $\xi$=damping ratio=$c/2m\omega$; $\omega=\sqrt{k/m}$; $\omega_D$=damped natural frequency=$\omega\sqrt{1-\xi^2}$; and the constants A and B may be expressed in terms of the initial conditions.

Since the total solution is the sum of the complementary solution obtained from equation 3 and the particular solution depending on the forcing function, the next step in our procedure will be to determine the particular solution of the different types of forcing functions illustrated in Figure (3).

## 4. PARTICULAR AND TOTAL SOLUTION OF DAMPED EQUATIONS OF MOTION (4 TO 7)

i- *For* $0 < t < t_1$:

The triangular load shown in Figure 3-a is expressed in the form

$$P(t) = P_o \cdot \frac{\tau}{t_1} \quad (4)$$

hence the equation of motion 1 takes the form

$$m\ddot{v}_p(t) + c\dot{v}_p(t) + kv_p(t) = p_o\frac{\tau}{t_1} \quad (5)$$

or

$$(D^2 + \frac{c}{m}D + \omega^2)v_p(t) = \frac{p_o\tau}{mt_1} \quad (6)$$

where D is the operator $\frac{d}{dt}$ and $\omega^2 = k/m$

then

$$v_p(t) = \frac{1}{D^2 + \frac{c}{m}D + \omega^2}\cdot\left\{\frac{P_o}{m}\cdot\frac{\tau}{t_1}\right\}$$

$$= \frac{1}{x+A}\left\{\frac{P_o}{m}\cdot\frac{\tau}{t_1}\right\} \quad (7)$$

where $x = D^2 + \frac{c}{m}D$ and $A = \omega^2$

Since the forcing function is a polynomial in time, t, then using the binomial theorem

$$\frac{1}{X+A} = \frac{1}{A\left(1+\frac{x}{A}\right)} = \frac{1}{A}\left(1+\frac{X}{A}\right)^{-1} = \frac{1}{A}\left(1-\frac{X}{A}+\ldots\right) \quad (8)$$

consequently, equation 7 may be written as

$$v_p(t) = \frac{1}{\omega^2}\left[1 - \frac{D^2+\left(\frac{c}{m}\right)D}{\omega^2} + \ldots\right]\cdot\left[\frac{P_o}{m}\cdot\frac{\tau}{t_1}\right] \quad (9)$$

noting that $m = k/\omega^2$, $\in = P_o/P_{max}$, and $\xi = c/2m\omega$

$$v_p(t) = \frac{\epsilon P_{max}}{k}\cdot\frac{t}{t_1} - \frac{2\epsilon\xi P_{max}}{k\omega t_1} \quad (10)$$

Then the total solution is the sum of equations 3 and 10 i.e.,

$$v(t) = \frac{\epsilon P_{max}}{k}\cdot\frac{t}{t_1} - \epsilon\frac{P_{max}}{k^2}\cdot\frac{c}{t_1} + e^{-\xi\omega t}[A_1 \sin\omega_D t + B_1 \cos\omega_D t] \quad (11)$$

where the constant $A_1$ and $B_1$ are determined from the initial conditions at $t = 0$ assuming the at rest condition [$v(0) = 0$, and $\dot{v}(0) = 0$].

ii- *For* $t_1 \le t \le t_2$

The particular solution for the case of rectangular load shown in Figure (3-b) is examined. Following the same steps as before,

$$v_p(t) = \frac{1}{D_2 + \frac{c}{m}D + \omega^2}\cdot\left[\frac{P_o}{m}\right] \quad (12)$$

Since $\frac{P_o}{m}$ is a constant

$$v_p(t) = \frac{P_o}{m}\cdot\frac{1}{(0)^2 + \frac{c}{m}(0) + \omega^2}e^{0t} = \frac{P_o}{m\omega^2} = \frac{P_o}{k} \quad (13)$$

Hence the total response for $t_1 \le t \le t_2$ is given by

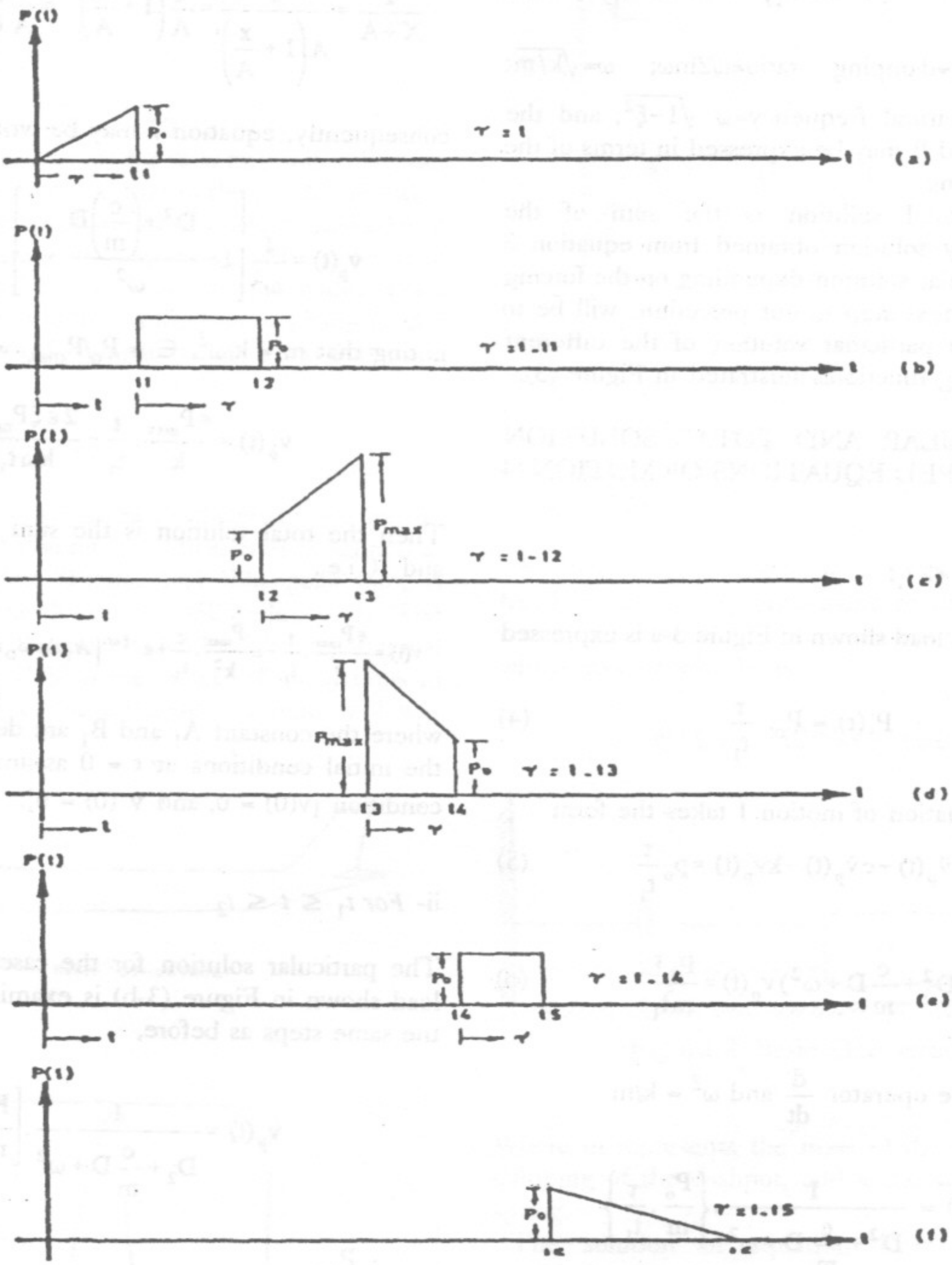


Figure 3. Main types of pulses of the aircraft impact load.

$$v(t)=\left[\frac{\epsilon P_{max}}{k}+e^{-\xi\omega(t-t_1)}\left\{A_2\sin\omega_D(t-t_1)+B_2\cos\omega_D(t-t_1)\right\}\right] \quad (14)$$

Where the constants $A_2$ and $B_2$ are evaluated from the initial conditions at $t = t_1$, i.e., at the end of the first pulse (equation 11).

iii- *For* $t_2 \leq t \leq t_3$

The trapezoidal load shown in Figure 3-c is expressed as

$$P(t)=P_o+(P_{max}-P_o).\frac{\tau}{(t_3-t_2)} \quad (15)$$

Since the forcing function is a polynomial in time; t, then the particular solution in this case can be evaluated as before from equation 9

$$v_p(t) = \frac{1}{\omega^2}\left[1 - \frac{D^2 + \frac{C}{m}D}{\omega^2} + \ldots\right]\left\{\frac{P_o}{m} + \frac{(P_{max} - P_o)}{m}\cdot\frac{\tau}{(t_3 - t_2)}\right\} \quad (16)$$

or

$$v_p(t) = \frac{\epsilon P_{max}}{k} + \left[\frac{P_{max}(1-\epsilon)}{k}\times\frac{(t-t_2)}{(t_3-t_2)}\right] - \frac{2\xi P_{max}(1-\epsilon)}{k\omega(t_3-t_2)} \quad (17)$$

Then, the total solution is given by

$$v(t) = \frac{\epsilon P_{max}}{k} + \frac{P_{max}(1-\epsilon)}{k}\cdot\frac{(t-t_2)}{(t_3-t_2)} - \frac{2\xi P_{max}(1-\epsilon)}{k\omega(t_3-t_2)} + e^{-\xi\omega(t-t_2)}\left\{A_3\sin\omega_D(t-t_2) + B_3\cos\omega_D(t-t_2)\right\} \quad (18)$$

The constants $A_3$ and $B_3$ are evaluated from the initial conditions at $t = t_2$ i.e., at the end of the second pulse (equation 14).

iv- *For* $t_3 \leq t \leq t_4$

The trapezoidal load shown in Figure 3-d is expressed as

$$P(t) = P_o + (P_{max} - P_o)\cdot\left(1 - \frac{\tau}{(t_4 - t_3)}\right) \quad (19)$$

Following the same steps as before, then the total solution is given by equation 18 after replacing $t_2$ and $t_3$ by $t_3$ and $t_4$:

$$v(t) = \epsilon\frac{P_{max}}{k} + \frac{2\xi}{\omega}\cdot\frac{P_{max}}{k}\cdot\frac{(1-\epsilon)}{(t_4-t_3)} + \frac{P_{max}}{k}(1-\epsilon)\left(1 - \frac{(t-t_3)}{(t_4-t_3)}\right)$$

$$+ e^{-\xi\omega(t-t_3)}\left[A_4\sin\omega_D(t-t_3) + B_4\sin\omega_D(t-t_3)\right] \quad (20)$$

Again the constants $A_4$ and $B_4$ are evaluated from the initial conditions at $t=t_3$, i.e., at the end of the previous pulse (equation 18).

v- *For* $t_4 \leq t \leq t_5$

The particular solution for the rectangular load shown in Figure (3-e) is determined from equation (14). Then the total solution takes the form

$$v(t) = \epsilon\frac{P_{max}}{k} + e^{-\xi\omega(t-t_4)}\left[A_5\sin\omega_D(t-t_4) + B_5\sin\omega_D(t-t_4)\right] \quad (21)$$

The constants $A_5$ and $B_5$ are evaluated from the initial conditions at $t = t_4$, i.e., from the conditions at the end of the previous pulse (equation 20).

vi- *For* $t_5 \leq t \leq t_6$

The triangular load shown in Figure 3-f is expressed as

$$P(t) = P_o\left(1 - \frac{\tau}{(t_6 - t_5)}\right) \quad (22)$$

Since the forcing function is a polynomial in time t, the particular solution is this case takes the form

$$v_p(t) = \frac{1}{\omega^2}\left[1 - \frac{D^2 + \left(\frac{c}{m}\right)D}{\omega^2} + \ldots\right]\times\left[\frac{P_o}{m}\left(1 - \frac{\tau}{(t_6-t_5)}\right)\right] \quad (23)$$

$$v_p(t) = \frac{\epsilon P_{max}}{k}\left(1 - \frac{(t-t_5)}{(t_6-t_5)}\right) + \frac{\epsilon P_{max}}{k(t_6-t_5)}\cdot\frac{2\xi}{\omega} \quad (24)$$

Then the total solution becomes

$$v_p(t) = \frac{\epsilon P_{max}}{k}\left[1 - \frac{(t-t_5)}{(t_6-t_5)}\right] + \frac{2\epsilon\xi P_{max}}{k(t_6-t_5)\omega}$$

$$+ e^{-\xi\omega(t-t_5)}\left[A_6\sin\omega_D(t-t_5) + B_6\sin\omega_D(t-t_5)\right] \quad (25)$$

Where the constants $A_6$ and $B_6$ are evaluated from the initial conditions at $t = t_5$, i.e., at the end of the previous pulse (equation 21).

The equations describing the response of an undamped SDF system can be obtained from those of a damped SDF system by putting the damping ratio $\xi$=0.

## 5. EXAMPLES AND CONCLUSIONS

The response of a one degree of freedom to the idealized total reaction versus-time curve of a Boeing 707-320 shown in Figure (1-b) is investigated for both undamped and damped cases. The results of the undamped system are compared with those of reference (2) and a full agreement is achieved. For the damped system a damping ratio $\xi$=5% is considered.

The dynamic load factor (8) (D.L.F) which is defined as the ratio between the dynamic response at any time t and the static response to the peak load $P_{max}$ is computed for frequency $f = 2\pi\omega$ larger than 1 cycle/ second (c.p.s). The maximum dynamic load factor is plotted for each frequency in Figure (4) for both undamped and damped cases.

The previous analysis suggest the following conclusions:

1- For structures with fundamental frequency of vibration $60 < f < 100$ c.p.s. the maximum D.L.F. for a normal aircraft strike is observed to be close to unity, for undamped cases. Therefore, a simplified static analysis with a load equal to the peak load appears to be adequate to predict the maximum displacements.

2- In the case of damped structures, the maximum dynamic load factor for normal aircraft strike is close to unity for frequencies $20 < f < 100$ c.p.s. Consequently, the simplified static analysis with a load equal to the peak impact load can be used to predict the maximum displacements.

3- The maximum dynamic load factor (D.L.F. max) for a damped system is smaller than that of the undamped system at any frequency of interest.

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Figure 4. Maximum response of one DOF system due to aircraft impact.
a) undamped system ($\epsilon = 0.0$) b) damped system ($\epsilon = 5\%$).