STATISTICAL ANALYSIS OF COLLECTED SEISMIC AND GEODETIC DATA FOR THE ESTIMATION OF PRELIMINARY DEGREE OF EARTHQUAKE HAZARD

M.M. Nassar
Faculty of Engineering,
Ain Shams University,
Cairo, Egypt.

A. Tealeb, M. Rabah
National Research Institute
of Astr. and Geophysics,
Helwan, Cairo, Egypt.

and **H.G. Elghazouly**
Faculty of Engineering,
Alexandria University,
Alexandria, Egypt.

ABSTRACT

The purpose of the current investigation is to apply the relevant statistical methodologies for determining the preliminary degree of earthquake hazard, with special emphasis on Aswan region. In this context, a comparative study of earthquake hazard analysis determined by seismic data, geodetic information or a combination between both sources will be performed. Since Aswan region is found to be one of the active seismic zones in Egypt and especially encompassing the vital engineering structure of the High Dam, it has been selected as a pilot project for the present investigation. Such a choice has been motivated by the fact that both seismic and geodetic data needed for the current investigation have been collected and become available over the Aswan region since 1982.

Keywords: Aswan region, Induced earthquake, seismic data, crustal movements, synthesized probability.

1. INTRODUCTION

The present experience with catastrophic earthquake consequences have shown the risk and the degree of catastrophic damage depend mainly on the built in material and the structural type. Human losses and injuries are caused by demolition and damage to buildings, due to their inadequate level of safety. Of course, the scale of catastrophic consequences in residential areas varies according to the magnitude of occurred earthquake. Therefore, to prevent future catastrophic consequences codes for seismic design and construction of buildings should be elaborated and put into effect. These codes are mainly based on seismic studies in the concerned regions.

The occurrence of earthquakes, especially, those that have caused causalities or damage, has been documented throughout historical time. From all different sources of such documentation, earthquake catalogues are usually determined. For instance in China, Middle East, and Japan earthquake catalogues span several thousand years. However, in some parts of the world like California, statistical useful catalogues extended only for a few decades [Wess, 1981]. The quality of statistical analysis of catalogued earthquakes improves with the acquisition of information. Unfortunately, the number of recorded earthquakes is insufficient to follow statistical evaluations for small areas and small time periods. However, it is possible to predict the probabilities of occurrence of large events in selected restricted intervals of both space and time (e.g., in a given area during a 5-years period) from a catalogue spanning a few hundred years. On the other hand, such a likelihood statement or "generalized prediction" is usually inadequate to permit taking specific preparatory measures in advance of any hazards [Vogel, 1981].

The distribution of the hypocenters of earthquakes recorded by the telemetric stations network from July 1, 1982 to December 31, 1990 has been studied using a developed software. The total number of earthquakes within the entire Aswan region was found to be about 1360. However, most of them (about 95%) are located in the area enclosed by latitudes (23°18′N, 23°54′N) and longitudes (32°18′E, 32°54′E) namely 1290 earthquakes of magnitudes ranging between 0.5 to 5.0 that occurred over the entire Aswan region during the above nine years of collected data. It is interesting to find out that the later number (1290 earthquake) comprises all significant earthquakes whose magnitudes ≥ 3.5 and hence such particular area (approximately 0.6°x0.6°) will be considered for the current analysis as the specific area of interest. The number of recorded earthquakes in this specific area of Aswan region within these nine years period is not sufficient and does not have homogeneous distribution to follow reliable statistical evaluations for such small time period. Nevertheless, such data may still be used to provide preliminary information about the approximate probability of occurrence of large events in Aswan region, for the purpose of indicating a preliminary degree of earthquake hazard. This is actually what we are trying to introduce in the present paper. This will also serve as a practical example of applying preliminary probability concepts on actual data of earthquake statistics, to be followed in the future as new data are collected.

It has been found that, the preliminary probabilities of a coming earthquake event based on seismic records only are not sufficient for reliable earthquake prediction analysis, and hence it must be supplemented by actually observed premonitory effects such as crustal movements [Rabah, 1992]. Nowadays, it is believed that the crustal movements precursor represents one of the most important precursors that can be observed either continuously or discontinuously over the active area of interest [Nassar et al., 1994]. First, simple statistics of earthquakes based on seismic data will be introduced. Then, statistical model of synthesized probabilities based on assumed crustal movements premonitory effect will be analyzed. Finally, appropriate conclusions and recommendations will be extracted.

2. SIMPLE STATISTICS OF EARTHQUAKES BASED ON SEISMIC DATA

Recall that the considered period of earthquakes based on Aswan catalogue in the current investigation extended only from July 1982 to December 1990 which may be considered a relatively small size of data. Nevertheless, such data may still be used to provide preliminary information about the approximate probability of occurrence of large events in the specified area for the purpose of indicating a preliminary degree of hazard. For this purpose, preliminary probabilities of earthquake magnitude and time of occurrence as well as the combined probability of both will be performed.

2.1. *Preliminary Probability of Earthquake Magnitude*

The basic statistical formulations needed for our computation here are given in detail by Rabah, 1992. Here, the preliminary probability of occurrence of the coming earthquake whose magnitude falls in a prespecified range of magnitude between M_i, M_{i+1} is expressed by the following equation based on seismic data alone in terms of the respective probability distribution function (PDF) [Rikitaki, 1976] as:

$$P_o(\Delta M) = P(M_i, M_{i+1}) = \frac{\left[\left(\frac{E_i}{E_s}\right)^{-\gamma} - \left(\frac{E_{i+1}}{E_s}\right)^{-\gamma}\right]}{\left[1 - \left(\frac{E_i}{E_s}\right)^{-\gamma}\right]} \quad (1)$$

Where: E is the energy released due to an earthquake event of magnitude M and is given by Gutenberg and Richter, 1956 as :

$$LOG_{10}\, E = \alpha + \beta M \quad (2)$$

In which α and β are empirical constants, whose values are found to be:

$$\alpha = 11.8 \text{ and } \beta = 1.5 \quad (3)$$

According to Gutenberg and Richter (1956), if E is measured in units of ergs, E_s and E_t are the released energies corresponding to earthquake event of M=4.5 and M=7.0 respectively, as may be expected for Aswan region. This can be obtained by direct substitution in equation (2) and we get :

$$\left.\begin{aligned} E_s &= 3.5481 \times 10^{16} \text{ erg} \\ E_t &= 1.9953 \times 10^{22} \text{ erg} \end{aligned}\right\} \quad (4)$$

Similarly, E_i, E_{i+1} are the released energies corresponding to M_i, M_{i+1}. And γ is defined as:

$$\gamma = b / \beta \tag{5}$$

In which β is the constant given by equation (2) and b is one of two constants of Gutenberg and Richter formula (1944). In this formula, the number of occurrence (N) for earthquakes of magnitude M in a region during a certain period is correlated to M, which reads [Bath, 1979]:

$$\mathrm{LOG}_{10}\, N = a - bM \tag{6}$$

The value of b has been evaluated through the least squares best fitting line for Aswan earthquake available catalogue presented in Table (1). Earthquakes of magnitude $M < 2$ have been excluded from our computations (whose number is 554) in order to eliminate the uncertainties effect in determining the number (N) for lower magnitudes on the estimated value of b [Gupta, 1984]. This means that the remaining number (736 earthquake) is used only for evaluating b, which represent about 57% of the total number of earthquakes in Aswan area. The result of the least square best fitting provides that:

$$b = 1.04 \tag{7}$$

Table 1. Frequency of earthquake occurrence in the specified area during 1982-1990 as classified according to magnitude $M \geq 2.0$.

M	Frequency
2.0	500
2.5	142
3.0	73
3.5	18
4.0	1
4.5	1
5.0	1
Total	736

By substituting in equation (1) for E_s and E_i from equation (4); γ from equation (5,7) and E_i, E_{i+1} from equation (3) for a specified value for M_i and M_{i+1}, the preliminary probability P_o (M_i, M_{i+1}) can be easily obtained for different values of M. Recall that usually M_i, M_{i+1} represent the lower and upper boundaries of an arbitrary intervals ΔM_j, which can be easily repeated for various successive intervals over the entire range of changing earthquake magnitude between M_{min} and M_{max}. Taking M_{min}=4.5 and M_{max} = 7.0 for Aswan region in our case as mentioned before, the obtained corresponding probability values for successive intervals of magnitude $\Delta M = 0.5$ can be evaluated in terms of PDF. the resulting PDF as well as corresponding CDF are given in Table (2). From this table, it can be concluded that the cumulative preliminary probabilities (CDF) for any magnitude value $\leq M_i$ range between 0% and 100% and inversely proportional with M_i, which is of course logic. On the other hand, such results provide a preliminary significant degree of risk which corresponds to a preliminary probability ≥ 50% of a coming earthquake whose magnitude will be $M \geq 5$ over the specified area under study.

Table 2. Probabilities of an earthquake occurring in the specified area to fall in a magnitude range between M_i and M_{i+1} (expressed in terms of a PDF).

M_i	M_{i+1}	[PDF] P (M_i,M_{i+1})	Descending [CDF]
4.0	4.5	0.000	1.000
4.5	5.0	0.700	0.300
5.0	5.5	0.211	0.089
5.5	6.0	0.064	0.025
6.0	6.5	0.019	0.006
6.5	7.0	0.006	0.000

2.2 Preliminary Probabilities of an Earthquake Occurrence Time

If earthquake occurrence is assumed to be stationary and random in the time domain, the occurrence frequency can be assumed to be governed by a Poisson distribution [Rikitaki, 1969]. Here, the mean frequency of earthquake events per unit time (usually one year) will be denoted by K, k equals the number of earthquake events of prespecified magnitude (or magnitude range) occurred within a certain period of time (usually the time span of earthquake catalogues) as averaged per year. This can be easily obtained for a specific M or ΔM using instrumentation earthquake catalogues for the special area of interest (or historical earthquake catalogues otherwise). Hence, the probability of having n earthquakes during a time interval Δt will be given, based on seismic data only, according to Rikitaki, (1969) in terms of PDF by:

$$P(\Delta t) = \frac{(K\ \Delta t)^n \cdot e^{-K\Delta t}}{n!} \qquad (8)$$

The probability of having no earthquake during the same time period T is obtained as:

$$P_{n=0}\,(\Delta t) = e^{-k\Delta t} \qquad (9)$$

Accordingly, the probability of having at least one earthquake between epochs t_1 and t_2 where $t_2 > t_1$ (as PDF) becomes:

$$P(\Delta t_j) = P(t_1, t_2) = 1 - e^{-k(t_2 - t_1)} \qquad (10)$$

For simplicity, denoting the initial epoch by 0 and the second epoch relative to the initial epoch by t equation (10) can be rewritten in terms of ascending CDF as:

$$P(0, t) = 1 - e^{-kt} \qquad (11)$$

Where the initial epoch (taken here as 0) is the epoch at which the data for earthquake catalogues used to compute k is terminated.

Once the time span of interest t has been specified, for instance t_j=10 days, one month, one year,..., etc., the corresponding cumulative probability of earthquake occurrence within this span can be computed directly from equation (11) as :

$$P_o(t_j) = P(t \le t_j) = 1 - e^{-kt_j} \qquad (12)$$

Which indicates that $P_o\,(t_j)$ is directly proportional to the time span t_j. This seems logical, since the probability of earthquake occurrence increases when the time span increases. Similar comments as has been stated earlier for predicting earthquake magnitude, concerning the degree of hazard hold true here again for the time of occurrence of earthquake events. In addition, the probabilities $P_o(t_j)$ as computed from equation (12) will be treated also as preliminary probabilities. The cumulative probability $P_o\,(t_j)$ can be easily repeated for different selected time span t_j from few days to tens or hundreds of years, providing a certain magnitude or magnitude interval has been specified, which in turn has direct influence on the computed value of k. Accepting the fact of earthquake occurrence in the specified area of Aswan region of 1981 ($M = 5.5$), and using the information given in table (1), only two earthquake with $M \ge 4.5$ occurred over the period of nine subsequent years (1982-1990) in this area. From this information the value of the mean frequency k can be obtained approximately as:

$$k = 3/10 = 0.3 \text{ earthquake/year} \quad \text{for } M \ge 4.5 \qquad (13)$$

Note here that small magnitude intervals can not be considered for determining the corresponding different values of k due to the relatively very small time span of catalogued data of the specified area, and hence only one interval ($M \ge 4.5$) was used.

Substituting from equation (13) into equation (12), the preliminary probabilities of at least one coming earthquake can be estimated for the different values of the period t_j. Table (3) outlines the obtained results for selected values of t_j (ranging from one day to 50 years) relative to the end of 1990 as the initial epoch (end of the used catalogued data for the specified area). From this table, one can see that the preliminary significant degree of hazard of a coming earthquake event with magnitude $M \ge 4.5$ is significant ($P_o(t) \ge 50\%$) will be encountered in Aswan region during the coming three years following the chosen initial epoch of 1990. In addition, the probability $P_o(t_j)$ is directly proportional with the time period t, and reaches its ultimate value of 100% within approximately 30 years for the specified area.

Table 3. Cumulative probabilities of having at least one earthquake event whose magnitude $M \geq 4.5$ in the specified area during the coming period of (t_j) relative to end of 1990 as zero epoch.

(t_j)	$P_o(t_j)$
1 Day	0.000
7 Days	0.006
30 Days	0.008
90 Days	0.072
0.5 Year	0.139
1 Years	0.259
2.5 Years	0.528
5 Years	0.777
10 Years	0.950
20 Years	0.998
30 Years	1.000

2.3 A Combined Probabilities of Earthquake Magnitude and Time

In hazard studies one may be interested to seek the probability of earthquake occurrence at certain magnitude interval simultaneously constrained by a certain time span. This can simply be obtained from the assumed individual probability distributions of both M and t via the well known statistical rules. When the distribution of magnitude and time of occurrence are assumed to be independent from one another (i.e. mutually exclusive). Therefore, the combined (simultaneous) probability of having at least one earthquake whose magnitude falls in an interval between M_1, M_2, within the period from t_1 to t_2 can be obtained by multiplying equation (1) and (12) and the result will be [Rikitaki, 1969]:

$$P_o(\Delta M,\Delta t)=P(M_1,M_2;t_1,t_2)=P_o(M_1,M_2).P_o(t_1,t_2) \quad (14)$$

Remembering that the magnitude of individual probabilities $P(M_1,M_2)$ and (t_1,t_2) is less than one, the result of equation (14) will be always less than the result of each one of equations (1) and (12), for a certain specified values of M_1, M_2 and t_1, t_2. Nevertheless, the combined probability may be significant or even strong, which implies similar treatment concerning the degree of hazard as stated above. It may be of interest to maintain here that, although equation (14) implies to work with PDF's, one can still apply the same equation with different combination of PDF's and/or CDF's. However, one must pay attention to the appropriate interpretation of the corresponding intervals of ΔM and Δt, For instance, equation (14) may be expressed as:

$$P_o(\Delta M_i,t_j) = P_o(\Delta M_i).P_o(t_j) \quad (15)$$

This equation is evaluated for different values of M_i intervals and t_j periods, and the obtained results are illustrated in Table (4) which is nothing else but the combination between the previous two tables, namely (2) and (3). From Table (4), the specified area is expected to be hit by significant earthquake event (combined probability $\geq$ 50%) with magnitude $5.0 \geq M \geq 4.5$ ($P_o \geq 50\%$) within the coming five years period and probability of approximately 100% within the coming 30 years.

Based on the above results and discussions, it can be seen that tables similar to (2), (3) and (4) can be helpful in examining the general tendency of earthquake occurrence in the area of interest. However, such probabilities, based on simple statistics of precursor time and magnitude deduced from seismic records only without taking any physical forerunners into consideration could provide misleading information in some cases. For instance, a small probability could be estimated today (less than 1% for an earthquake event of magnitude M>5.5) to occur within few days, in spite of the real fact that an earthquake event of $M = 6$ may hit the same area suddenly before that. Therefore, the probabilities estimated here must be dramatically modified when some premonitory effects closely connected with earthquake occurrence are actually observed.

3. STATISTICAL MODEL OF THE SYNTHESIZED PROBABILITIES BASED ON ASSUMED CRUSTAL MOVEMENTS PREMONITORY EFFECT

As mentioned before, preliminary probabilities of a coming earthquake event based on seismic records only are not sufficient for reliable earthquake prediction analysis, and hence it must be supplemented by actually observed premonitory effects such as crustal movements. Nowadays, it is believed that the crustal movements represent one of the most important precursors that can be observed either continuously or discontinuously over the active area of interest [Nassar et al., 1994]. In our case the mean radius "r" of the crustal deformed area associated with a coming expected earthquake will be assumed to be r=3 km. This value will be used in the statistical model based on crustal movements premonitory effect, as will be shown below.

3.1. Synthesized Probability of Earthquake Magnitude

The relation equation between magnitude M and mean radius of crustal deformation r for an earthquake is measured in units of cm is obtained empirically by Dambara (1964) as:

$$\mathrm{LOG}_{10}\, r^3 = 8.18 + 1.53\, M \qquad (16)$$

Which can be rewritten as :

$$M = 1.96\, \mathrm{LOG}_{10}\, r + 4.45 \qquad (17)$$

where r is measured in km.

Table 4. The combined preliminary probabilities of having at least one earthquake event in the specified area of a prespecified magnitude interval within various next time periods.

Selected Magnitude Intervals		The Next Time Period (T)						
M_i	M_{i+1}	t=90 Days	t=0.5 Year	t=1 Year	t=5 Years	t=10 Years	t=20 Years	t=30 Years
4.5	5.0	0.050	0.097	0.181	0.544	0.665	0.669	0.700
5.0	5.5	0.015	0.029	0.055	0.164	0.201	0.210	0.211
5.5	6.0	0.005	0.009	0.017	0.050	0.061	0.064	0.064
6.0	6.5	0.001	0.003	0.005	0.015	0.018	0.019	0.019
6.5	7.0	0.000	0.000	0.002	0.005	0.006	0.006	0.006

Let us now suppose that the mean radius r of the deformed area is determined, via repeated measurements using geodetic techniques. In that case a probable magnitude M_o of the coming earthquake can be preliminary estimated from equation (17), based on the observed r which represents the crustal deformation effect. Due to the scatter of $\mathrm{LOG}_{10}\, r^3$ vs. M plots as can be seen in Figure (1), M_o can not be obtained deterministically. If the difference between the actual magnitude M and M_o is assumed to follow a Gaussian distribution [Rikitaki, 1974], the premonitory crustal movements effect probability of the earthquake event whose magnitude takes a value between M and M+dM is in terms of PDF obtained as:

$$P_{GM}(M, M+dM) = \frac{h_M}{\sqrt{\pi}} \cdot e^{-h_M (M^2 - M_o^2)} \cdot dM \qquad (18)$$

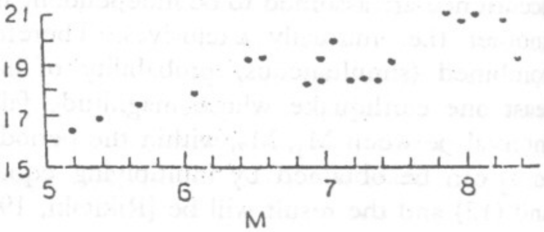

Figure 1. Log r vs. M, where r is the effective radius in cm of the area over which crustal deformation associated with an earthquake is observed. (After Dambara, 1964).

In which h_M is determined by the standard deviation σ_M of M_o as:

$$h_M = \frac{1}{\sqrt{2 \cdot \sigma_M}} \qquad (19)$$

Dambara (1964) obtained $\sigma_M = 0.8$ for a data set of 19 earthquakes. Although the scatter for earthquakes having a large magnitude is considerably large as can be seen in Figure (1), a fairly small standard deviation can be expected for the data for earthquakes of which the magnitude amounts to 6.6 or less. Depending on such result Rikitaki (1974) assumed $\sigma_M = 0.2$. Consequently, after substituting in equation (19) for $\sigma_M = 0.2$ we get h_M as:

$$h_M = 3.536 \qquad (20)$$

Strictly speaking, the assumption that a Gaussian distribution holds good is not true because we have no M's which are larger than 8.8 or so, which means that we can only get a small size sample of M from actual seismic observation. Nevertheless, the probability discussion may be approximately applicable to an estimate of probability.

The probability of an earthquake event for the magnitude to fall in a range between M_1 and M_2 is accordingly given by:

$$P_{GM}(M_1,M_2) = \frac{h_M}{\sqrt{\pi}} \int_{M_1}^{M_2} e^{-h_M^2 \, (M - M_o)^2} \, dM \qquad (21)$$

Using the basic definition of an error function erf=$\Phi(x)$, which can be easily integrated and tabulated to the required precision that is given by [e.g. Francis, 1979]:

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} \, du \qquad (22)$$

Equation (21) can be simply expressed as follows [Rikitaki, 1969]:

$$P_{GM}(M_1, M_2) = 0.5 \; \{\Phi\,[h_M (M_2 - M_o)] - \Phi\,[h_M (M_1 - M_o)]\} \qquad (23)$$

The probability as obtained from equation (24) can be evaluated for each one of the n interval of earthquake magnitude as illustrated before, namely:

$$P_1(\Delta M_i) = P_{GM}(M_1,M_2) = 1/2 \; \{\Phi\,[h_M (M_{i+1} - M_o)] - \Phi\,[h_M (M_i - M_o)]\} \quad \text{for } i = 1,2,\ldots\ldots, n \qquad (24)$$

Which can be rewritten as :

$$P_1(\Delta M_j) = 1/2 \; \{\Phi\,[x_2] - \Phi\,[x_1]\} \qquad (25)$$

In which :

$$\left.\begin{aligned} x_2 &= h_M \left(M_{i+1} - M_o\right) \\ x_1 &= h_M \left(M_i - M_o\right) \end{aligned}\right\} \qquad (26)$$

Let us now suppose that the mean radius of the deformed area "r=3 km" is determined via, for instance, repeated geodetic measurements of the Kalabsha geodetic network [Mahmoud, 1988]. By substituting r=3 km into equation (17), the probable magnitude M_o of the coming earthquake can be estimated as:

$$M_o = 5.3852 \qquad (27)$$

Therefore by direct substitution in equation (25) for h_M from equation (20); M_o from Equation (27); and the specified values for M_i and M_{i+1}, the values of x_1 and x_2 can be determined. Then from $\Phi(x)$ table, the values of $\Phi(x_1)$, $\Phi(x_2)$ can be extracted. Hence by substituting the values of $\Phi(x_1)$, $\Phi(x_2)$ in equation (25), the premonitory crustal movements effect probabilities of occurrence of a coming earthquake event in respective magnitude ranges can be obtained. Table (5) outlines the obtained results of the probabilities of occurrence of a coming earthquake $P_i(\Delta M_j)$ whose magnitude falls in a prespecified range ΔM_j between M_j, M_{i+1} starting with M=4.5 up to M=7 and taking ΔM=0.5. From this table, it can be seen that the probability of earthquake occurrence of magnitude falls between 5.0 and 5.5 is the largest (69 %) provided that a deformed area with mean radius r=3 km has been observed as crustal movements premonitory effect.

Recall that the preliminary probabilities $P_o(\Delta M_j)$ as determined from earthquake catalogue of the specified area for respective magnitude ranges are given in Table (2). In the case on hand, when a precursory land deformation having 3 km mean radius in the area under consideration occurs, the probabilities of occurrence of the coming earthquake $P_i(\Delta M_j)$ are presented in Table (5). Consequently, the synthesized probabilities can be computed by applying Bayes' theorem [Francis, 1979] as following:

$$P(\Delta M_j) = \frac{P_o(\Delta M_j)\ P_1(\Delta M_j)}{\sum_{i=1}^{5} P_o(\Delta M_i)\ P_1(\Delta M_i)} \quad , j=1,2,\ldots, 5 \tag{28}$$

Table 5. Probabilities of earthquake events (as PDF) with different magnitudes over the specified area based on observed crustal deformations with mean radius r= 3Km.

M_i	M_{i+1}	$P_i(M_i,M_{i+1})$
4.5	5.0	0.027
5.0	5.5	0.690
5.5	6.0	0.282
6.0	6.5	0.001
6.5	7.0	0.000

By substituting the values of P_o and P_1 from Tables (2) and (5) respectively into equation (28), the corresponding synthesized probabilities can be obtained as indicated in table (6). From this table, the synthesized probabilities, when an anomalous crustal deformation is assumed to be observed in the specified area, for an earthquake event whose magnitude ranges between 5.0 and 5.5 is still the largest (79%) and is considered to be a strong one.

Table 6. The synthesized probabilities (as PDF) of earthquake events within different magnitude intervals when earthquake catalogues are supplemented by observed crustal deformation in the specified area.

M_i	M_{i+1}	$P_i(M_i,M_{i+1})$
4.5	5.0	0.103
5.0	5.5	0.798
5.5	6.0	0.099
6.0	6.5	0.000
6.5	7.0	0.000

3.2. Synthesized Probability of Earthquake Occurrence Time

Tsubokawa (1973) was the first who pointed out a linear relation between the logarithmic precursor time T of an anomalous land deformation and earthquake magnitude M in the form as [Kasahara, 1981]:

$$\mathrm{LOG}_{10}\ T = 0.79\ M - 1.88 \tag{29}$$

In which T is measured in units of days.

For simplicity, put $\zeta = \mathrm{Log}_{10}\ T$, i.e. Equation (29) can be rewritten as:

$$\zeta = \mathrm{LOG}_{10}\ (T) = 0.79\ M - 1.88 \tag{30}$$

Recalling that the preliminary magnitude M_o of the coming earthquake is obtained from equation (27), while the corresponding actual magnitude is denoted by M, therefore, equation (29) can be evaluated twice for M_o and M resulting in two values of ζ_o and ζ respectively.

Assume $\zeta - \zeta_o$ follows the Gaussian distribution, same as has been assumed for the difference $M - M_o$ before, the probability for the precursor time to take a value between ζ_1 and ζ_2 can be obtained in terms of PDF as:

$$P_1(\Delta t_j) = P_{Gt}(t_1,t_2) = \zeta^{P_G(\zeta_1,\zeta_2)} = \frac{\zeta^h}{\sqrt{\pi}} \int_{\zeta_1}^{\zeta_2} e^{\zeta^{-h^2(\zeta_o-\zeta)^2}}\, d\zeta \tag{31}$$

Where: ζ_o is the logarithmic precursor time as obtained from equation (30) for $M = M_o$ and, M which M_o is to be estimated from equation (17), for the adopted value of r. h_ζ is defined by:

$$h_\zeta = \frac{1}{(\sqrt{2}.\sigma_\zeta)} \tag{32}$$

In which $\sigma_\zeta = 0.2$ is tentatively assumed in the following calculation [Rikitaki, 1969]. From which h_ζ can be obtained as:

$$h_\zeta = 3.536 \tag{33}$$

Evaluating the integration in equation (31), one find the following result similar to equation (25) as PDF:

$$P_1(\Delta t_j)=0.5\ \{\Phi\ [h_\zeta(\zeta_2-\zeta_o)]-\Phi\ [h_\zeta(\zeta_1-\zeta_o)\} \quad (34)$$

Where : Φ represents the usual known error function erf (x) expressed by equation (22) in which:

$$x = h_\zeta(\zeta_j-\zeta_o) \quad (35)$$

Again, the time interval Δt_j in equation (34) can be substituted by different consequent intervals for j=1,2,3,...,s. The computed probabilities $P_1(\Delta t_j)$ from the PDF given in equation (34) will be used later for the determination of the corresponding synthesized probability.

On the other hand, one is usually interested, from the practical view point, to perform earthquake prediction analysis using the precursor time which is referenced to the used initial epoch. In this case, the corresponding cumulative probability, computed as a CDF from the PDF of equation (34), should be used instead. Thus the cumulative premonitory crustal movements probability for an earthquake to occur during time interval 0-t can be obtained similar to equation (25) as:

$$P_1(t_j) = P(t_1 \le t_j) = P_{Gt}\ (0,t)$$

$$= 0.5\ \{\ 1+\Phi\ [h_\zeta\ (\zeta_j-\zeta_o)]\} \quad (36)$$

From equation (35) the probability of earthquake time occurrence $P_1(t_j)$ due to the observed crustal movements data as a prediction element (namely, the mean radius of deformed area r=3 km) is given in equation (36) in terms of CDF can be written as:

$$P_1\ (t_j) = P_1\ (t \le t_j) = 0.5\ \{\ 1+\Phi\ [x_j] \quad (37)$$

Where ζ_j can be determined from equation (30) as :

$$\zeta_j = \mathrm{LOG}_{10}\ t = \mathrm{LOG}_{10}\ (T) = 0.79\ M - 1.88 \quad (38)$$

and ζ_o corresponds to ζ when M = M_o as defined above in equation (27). Substituting from equation (28) into equation (38), ζ_o can be obtained as:

$$\zeta_o = 2.3743 \quad (39)$$

h_ζ is taken from equation (33). Substituting from equations (38), (39) and (33) into equation (35), x_j can be obtained for different precursor time periods t_j as:

$$x_j = 3.536\ .\ (\ \mathrm{LOG}_{10}\ t - 2.3743) \quad (40)$$

From the error function table [Francis, 1979], the different values of $\Phi\ (x_j)$ into equation (37), can be extracted. By substituting the different values of $\Phi\ (x_j)$ into equation (37), the corresponding probability of earthquake occurrence (as CDF) within a certain time period t_j can be obtained [when observed a deformed area of mean radius r as a premonitory effect].

Table (7) gives the obtained results after evaluating equation (37) for different values of t_j. From this table, it is easy to note that the probability of precursor time of earthquake event is very large within the period 1-5 years with magnitude approaching 5.5 (eq. 27). This has been done based on the assumption that a deformed area of mean radius r=3 km is observed in the specified area.

Table 7. Cumulative probability $P_1\ (t \le t_j)$ of earthquake events precursor time t according to observed crustal deformation as a premonitory effect with mean radius r = 3 km in the specified area.

(t_j)	$P_i(t \le t_j)$
1 Day	0.000
7 Days	0.000
30 Days	0.000
90 Days	0.018
0.5 Year	0.286
1 Years	0.827
2 Years	0.993
5 Years	0.999
10 Years	1.000
20 Years	1.000
30 Years	1.000

Remember that the given results of $P_o\ (t_j)$ and $P_1(t_j)$ in both tables (3) and (7) respectively are in terms of CDF's. Consequently, the corresponding PDF's namely $P_o\ (\Delta t_j)$ and $P_1\ (\Delta t_j)$ must be evaluated firstly, in order to be able to use equation

(28) for the corresponding synthesized probability computations. This can be done either by recomputation using equations (10) and (34); or from tables (3) and (7) through simple subtraction of the CDF's values over the desired time intervals. The later approach is of course much easier, and time saving, and hence is used here. The obtained results of the transformed PDF's $P_o(\Delta t_j)$ and $P_1(\Delta t_j)$ as well as the computed synthesized probability $P(\Delta t_j)$ from equation (28) expressed as PDF and then transformed to corresponding CDF $[P(t_j)]$ are arranged in Table (8). The reason for including the synthesized CDF will be for the purpose of combined probability computations, [similar to eq. (15)], and for precursor time t analysis analogously to our previous discussion. From Table (8), it can be seen that the probability in the specified area becomes significant (> 50%) during the first two years after the initial epoch, providing a crustal deformation premonitory effect of mean radius r=3km has been observed in the specified area.

3.3 A Synthesized Combined Probability of Earthquake Magnitude And Occurrence Time

One may be interested to seek the simultaneous probabilities of having at least one earthquake, whose magnitude falls in a prespecified interval ΔM_i between M_i and M_{i+1} in the specified area constrained by prespecified time period t_j (relative to the initial epoch). This can be computed using equation (15) for the premonitory crustal movements effect observational only, which can be simply obtained by multiplying the corresponding values of Tables (5) and (7) respectively. The obtained results are presented in Table (9). Moreover, the same concept of combined probabilities can be applied on the computed synthesized probabilities through equation similar to equation (16), the obtained results are illustrated in Table (10).

From Tables (9) and (10), one can easily see that the probability of occurrence of an earthquake event falling in magnitude range between 5.0 and 5.5 in the specified area especially, during the first year. Again, as mentioned before, one can conclude that the final decision concerning earthquake prediction analysis is governed mainly by the results of crustal movements premonitory effect, rather than the results of earthquake catalogues alone. This ensures the practical visibility of earthquake physical phenomena in earthquake prediction studies with respect to earthquake prediction analysis based on seismic statistics only.

4. SUMMARY AND CONCLUSIONS

Based on the above statistical analysis of collected seismic and geodetic data in the specified area of interest in Aswan region, the obtained results may be summarized as follows:

1. Based on the catalogued earthquakes in the specified area, the preliminary probability of having an earthquake event whose magnitude falls between 4.5 and 5.0 is the largest (70%) [Table (2)]. The cumulative preliminary probability of having at least one coming earthquake event whose magnitude > 4.5 becomes significant (> 50%) after the coming three years following 1990 [Table (3)]. Therefore, the specified area is expected to be hit by at least one significant earthquake event (combined probability > 50%) with magnitude $4.5 \leq M \leq 5.0$ within the coming five years period (54.4%) and corresponding approaching 100 % during the coming 30 years.
2. When a deformed area of mean radius r=3 km is observed in the specified area as a premonitory crustal movements effect, the probability of earthquake event of magnitude falling between 5.0 and 5.5 is the largest (69%) [Table (5)]. The corresponding probability of precursor time of earthquake event becomes significant (> 50%) within the period of the first year with magnitude approaching 5.5 [Table (7)]. Hence the specified area is expected to be hit by at least one significant earthquake whose magnitude falls between 5.0 and 5.5 within the coming first year (57.1%) and corresponding probability of approximately 100 % during the coming 20 years [Table (9)].
3. When the probabilities, which have been obtained from the earthquake catalogue, are merged with the probabilities, which were resulted from observing crustal deformed area with r=3 km in the specified area as a premonitory crustal movements effect, the synthesized probability will be the outcome. The results are shown in Tables (6), (8) and (10).

Their results lead to similar conclusions as given in the second item above. These synthesized probabilities are governed mainly by the results of crustal movements premonitory effect, rather than the results of earthquake catalogues. This result is an indication to the practical visibility of earthquake physical phenomena and their potential in earthquake studies when compared with the preliminary extrapolation based on seismic statistics.

Table 8. The synthesized probabilities of precursor time of earthquake events in the specified area (deformed radius = 3 km and nine years f earthquake catalogue) expressed in terms of both PDF and CDF

Time Interval	Transformed PDF's		Synthesized Probabilities	
Δt_j (year)	$P_o(\Delta t_j)$	$P_1(\Delta t_j)$	PDF: $P(\Delta t_j)$	CDF: $P(t_j)$
0.0 - 1/4	0.024	0.018	0.003	0.000
1/4 - 1/2	0.115	0.268	0.237	0.003
1/2 - 1.0	0.120	0.541	0.499	0.240
1.0 - 2.0	0.192	0.166	0.245	0.739
2.0 - 5.0	0.326	0.006	0.015	0.984
5.0 - 10.0	0.173	0.001	0.001	0.999
10.0 - 20.0	0.049	0.000	0.000	1.000
20.0 - 30.0	0.001	0.000	0.000	1.000

Table 9. The combined (simultaneously) probability of having at least one earthquake event of prespecified magnitude falling in the interval ΔM_i between M_i - M_{i+1} and constrained by a prespecified time period t_j (relative to the initial epoch) based on observing a deformed area of mean radius r = 3 km in the specified area as a premonitory crustal movements effect only.

ΔM_i		Time Period (t_j)						
M_i	M_{i+1}	t=90 Days	t=0.5 Year	t=1 Year	t=2 Years	t=5 Years	t=10 Years	t=20 Years
4.5	5.0	0.001	0.008	0.022	0.027	0.027	0.027	0.027
5.0	5.5	0.012	0.197	0.571	0.685	0.689	0.690	0.690
5.5	6.0	0.005	0.081	0.233	0.280	0.282	0.282	0.282
6.0	6.5	0.000	0.000	0.000	0.001	0.001	0.001	0.001
6.5	7.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 10. The combined synthesized probability of having at least one earthquake event of prespecified magnitude interval ΔM_i and constrained by a prespecified time period t_j based on observed crustal deformed area of mean radius r = 3 km (as crustal movements effect) and earthquake catalogue for the same area.

ΔM_i		Time Period (t_j)						
M_i	M_{i+1}	t=90 Days	t=0.5 Year	t=1 Year	t=2 Years	t=5 Years	t=10 Years	t=20 Years
4.5	5.0	0.000	0.025	0.076	0.101	0.103	0.103	0.103
5.0	5.5	0.002	0.192	0.590	0.785	0.797	0.798	0.798
5.5	6.0	0.000	0.024	0.073	0.097	0.099	0.099	0.099
6.0	6.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6.5	7.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000

According to the above results, one can conclude that the final decision concerning earthquake hazard analysis is governed mainly by the results of crustal movements premonitory effect, rather than the results of earthquake catalogues alone. This ensures the practical visibility of earthquake physical phenomena like crustal movements in earthquake hazard studies as integrated with the earthquake catalogue data. Consequently, for meaningful earthquake hazard analysis, it is recommended to supplement the available earthquake catalogue data with other physical precursors particularly the crustal movements. This, of course, calls for the establishment and optimization of relevant geodetic networks over the active seismic regions throughout the Egyptian territory.

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