SIMULATION AND DESIGN OF OPTIMAL CONTROLLER FOR A TWO JOINTS ARM ROBOT WITH PARAMETER UNCERTAINTIES

T. Awad

Mechanical Engineering Department, Faculty of Engineering, Alexandria University, Alexandria, Egypt.

ABSTRACT

A simulation program for two joints arm robot with uncertainty of the system description is developed using TUTSIM simulation language. The effect of the variation of the uncertainties on the time response of the robot is studied. Then feedback control loops are suggested. These loops are feeding back the position, the velocity and a function of the input signal of each link to each input of the arm magnified with certain proportional gain constants. An objective function is built including the maximum overshootings , the settling times and the steady state errors as a function of the twelve gain constants of the controller. A modified gradient method is used to obtain the optimum gain constants minimizing the objective function.

Keywords: Robot control, robust control, optimal control.

NOMENCLATURE

- A Transfer matrix [6x6]
- B Transfer matrix [6x2]
- C_i Coupling coefficients of the uncertain parameters, i=1,2,3
- X; State variables i=1,2,...6.
- K Controller matrix[6x2].
- U Matrix [2x1] of the input signals in the case of uncontrolled robot.
- R Matrix [2x1] of the input signals in the case of optimal controlled robot.
- θ_1 Response of the first moving link of the Robot.
- θ_2 Response of the second moving link of the Robot.
- Objective function.

INTRODUCTION

There are many instance when a system model is not known or partially unknown for the purposes of controller design.

At the present time, there appear to be three general philosophies providing possible solutions to the problem.

a) Identification of a low order approximate system model [1] from off-line analysis of input/output data obtained from system records. The resulting model is then used as the basis of controller design and the success of the approach assessed by on-line tuning at the commissioning stage or by extensive simulation of the controller using the real plant model.

- b) Self-tuning control of the partially unknown system[2] using a control strategy based on an assumed low order parametric system model and on line identification of the required controller parameters.
- c) Robust design of the control system in a manner ensuring that the performance is insensitive to the unknown components[3,4] or the nonlinear varying component.

In this work the two joints arm robot with partially unknown information about the friction in the joints is studied, a simulating model is built and an optimum controller insensitive for the unknown friction torques is designed.

Problem formulation

The two joints arm robot with position servo without feedback control loops shown in figure (1) [5] is taken as an example. The motion of the arm links can be represented by the block diagram of

Alexandria Engineering Journal, Vol. 33, No. 4, October 1994

figure (2). In this diagram the inductance and viscous damping of the electrical servo motors are ignored. The amplifier pole is also neglected because it occurs at high frequency.

The accuracy of the position and velocity of each link are affected by the friction in the joints which appears as coupling blocks in the block diagram. The friction between mating parts of a robot arm depends on the finishing of manufactured surfaces and their lubrications.

The measurements of the resulting friction torques at robot joints is not a simple task. Many friction models are applied in the literature [6] but still the friction torques in the joints are partially known i.e. for each one its nominal value and range are known. The state space presentation of the arm robot without control is:

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{U} \tag{1}$$

Which may be expressed as follows:

$$\begin{bmatrix} \dot{X}_{1} \\ \dot{X}_{2} \\ \dot{X}_{3} \\ \dot{X}_{4} \\ \dot{X}_{5} \\ \dot{X}_{6} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_{1}} & 0 & 0 & 0 & 0 & 0 \\ K_{1} & (C_{1} - K_{2}) & -K_{1} & 0 & C_{2} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{T_{2}} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{T_{2}} & 0 & 0 \\ 0 & C_{3} & 0 & K_{3} & -K_{4} & -K_{3} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_{1} \\ X_{2} \\ X_{3} \\ X_{4} \\ X_{5} \\ X_{6} \end{bmatrix} + \begin{bmatrix} \frac{1}{T_{1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_{2}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$
 (2)

To investigate the time responses of the two joints arm robot under the effect of these uncertainties the TUTSIM block language [7] is used to simulate the robot. This language is a high level computer language that needs a block diagram representing the system to be modelled. This block diagram is more like the analog computer wiring diagram. The easiness in writing the program from the blocks, the availability of nonlinear, discontinuous, special mathematical blocks and the facility of using up to 999 blocks in one program enables to apply this language in simulating large plants or systems with multi-input multi-output variables .

In the block diagram shown in Figure (2), the coefficients C_i , (i = 1,2,3) are the coupling coefficients representing the nonlinear uncertainties in the inner loops. This block diagram is translated to a TUTSIM block diagram as shown in figure (3). Given the gains, the time delays and the nonlinearities the following nominal values .

$$K_1 = K_2 = 10$$

 $K_3 = K_4 = 2$
 $C_1 = C_2 = C_3 = 0.2$





Alexandria Engineering Journal, Vol. 33, No. 4, October 1994

to many to shade



Figure 2. Block diagram representation for the arm robot without control.



Figure 3. TUTSIM block diagram representation for the arm robot.

Suppose now that C_1 , C_2 , C_3 vary up to $\pm 100\%$ of their nominal value.

Two step inputs equal to 1 radians are given to the model. Different values are given to to each C_i while keeping the others with constant values and recording the response of the system. These values are chosen lying in the range of the suggested variation of these nonlinearities (i.e. $\pm 100\%$) of its nominal values.

The curves of Figure (4) are the position time step response curves of the two links of the arm robot for different values of the coupling coefficients. The figure shows the band of the response of the two links in the range of variation of the coupling coefficients between 0.and 0.4.

Figures (5,6,7) show the variation of the settling time based on $\pm 2\%$ for the different values of C₁, C₂, C₃.



Figure 4. Step response band for the two moving links of the arm robot for values of C_1 , C_2 and C_3 between 0, 0.4.

Figure (5) shows that for zero values of C_2 , C_3 the settling time for θ_1 increases and for θ_2 decreases by the increase of C_1 .

Figure (6) shows that for $C_1 = 0.2$, $C_3 = 0$ the settling time for θ_1 increasing by the increase of C_2 from 0 to 0.4, but the settling time of θ_2 decreases for values of C_2 between 0 and 0.2 and then increases for the greater values of C_2 . For values of $C_1 = 0.2$, $C_2 = 0.2$ the settling time of θ_1 slightly decreases for values of C_3 between 0 and 0.1 then increase while that of θ_2 increases for the whole range of C_3 between 0 and 0.4.



Figure 5. Settling time against the coupling coefficient C_1 .



Figure 6. Settling time against the coupling coefficient C_2 .



From the above curves it can be concluded that the longer settling time is the result of the values of $C_1 = C_2 = C_3 = 0.4$.

Control Loop

It is suggested to multiply each of the states X_i (i = 1,2,...6) by gain constants and feedback them into the input signal R [2x1] representing the required positions of the robot links.

The position state equation will be:

$$\dot{X} = AX + BU$$
 (3)
 $U = R - KX$

Where K is the [2x6] matrix representing the controller.

The block diagram representing the above equations is shown in Figure (8).

Optimum Controller Design

It is required to find the twelve values of the gains in the controller matrix K that minimize the overshooting and the settling time of the position step response of the arm links with minimum sensitivity to the changes in the values of the uncertainties.

The following objective function is built:

$$\psi = \sum_{i=1}^{i=m} ABS(1 - \theta_1)_i + ABS(1 - \theta_2)_i \quad (4)$$

Where θ_1 and θ_2 are the time response of the first and second moving links of the robot. This objective function is directly proportional to the overshooting, the steady state error and indirectly to the settling time.

The gain constants are changed by the gradient strategy of Zettle [8] to minimize these objective functions. For the robot model the values of the uncertainties C_1, C_2, C_3 are taken 0.4. These values are concluded to give the worst condition for the settling time and consequently the maximum overshooting. Equations (3) are solved using Rungakutta method of the fourth order with time interval 0.001 second. The optimization technique of Zettle [8] is started with an initial guess for all the twelve gains of the controller equals to 0.1. The optimum values of the controller obtained from the minimization of the objective function are $K_{11}=0.0386, K_{21}=0.1394, K_{31}=0.03619, K_{41}=0.0547, K_{51}=0.0797$

 $K_{41} = -0.0931$, $K_{12} = 0.1369$, $K_{22} = 0.1298$, $K_{32} = 0.1476$, $K_{42} = 0.1725$ $K_{52} = 0.2188$, $K_{62} = 4755$ AWAD: Simulation and Design of Optimal Controller...



Figure 8. Block diagram of the controlled arm robot.

The objective function is minimized to 6.9% of its value with the initial guess gains. Testing the optimum controller for the changes of the uncertainties C_i for values between 0 and 0.4 gives a maximum increase in the minimum objective function of 0.46% which has no sensible effect on the response of the robot.

Figure (9) Shows the position step response of the links of the arm robot without control and with optimum controller obtained from the objective function.

CONCLUSION

The choice of optimal values of controller gains in control loops that feedback the state variables, minimizes the sensitivity of the system to the uncertainties, the steady state errors and the settling time.



Figure 9. Step responses of the robot without and with optimum controller.

Alexandria Engineering Journal, Vol. 33, No. 4, October 1994

AWAD: Simulation and Design of Optimal Controller...

REFERENCES

- [1] P. Eykhoff, System Identification, Wiley. 1974.
- [2] U. Borisson, "Self tuning regulators for a class of multivariable systems", *Automatica*, 15, 1979.
- [3] D.H. Owen and A. Chotai, A., "Robust control of unknown or large scale systems using transient data only", Research report No 134, Dept Control Eng. Univ. Sheffield UK, 1980.
- [4] J. Pentinnen and H.N. Koive, "Multivariable tuning regulators for unknown systems", *Automatica*, 16, 1989.
- [5] Rich and Klafter, *Robotic Engineering an integrated approach*, Prentice-Hall, 1989.
- [6] S. Megahed, "Contribution a la modalisation geometric et dynamique des robots manipulateurs a structure de chaine cinamatique simple ou complexe-application a leur commande", *These d'Etat*, Universite Paul Sabatier, Toulouse, 1984.
- [7] TUTSIM Simulation Block languauge, Applied I, USA, 1982.
- [8] G. Zettle, "Ein Verfahren zum Minimiern einer Function bei eingeshranktem Varitionsbereich der Parameter", Num. Math. 1970, 15, 415-432.

Alexandria Engineering Journal, Vol. 33, No. 4, October 1994