NORMAL INCIDENCE REFLECTION OF AMBIENT-TRANSPARENT FILM-ABSORBING SUBSTRATE SYSTEM

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ABSTRACT

The normal incidence reflectance is discussed and obtained for a system contained of ambient, transparent coating film, and absorbing substrate. We formulated and calculated the phase angle of the complex amplitude reflection coefficient. The conditions for minimum and maximum intensity reflectance are discussed and formulated at normal incidence of a monochromatic wave. We calculated the two limiting reflectances for twenty one metals coated by a transparent film of refractive index equals 1.5. Families of iso-optical thickness-contours are discovered in the complex N_3-plane, for minimum or maximum normal incidence reflectance.

Keywords: Optical, Incidence, Reflectance, Contours, Thickness, Transparent, Absorbing.

I. INTRODUCTION

The subject of dielectric and metallic thin plane-parallel films, situated between two homogeneous media is of practical interest and considerable importance in optics. Such films may be produced with the help of high-vacuum evaporation techniques and their thickness may be controlled with very high accuracy. They have many useful applications such as coatings which reduce the reflectivity of a given surface. Thin film coatings have a variety of applications within the optics industry. They are used as filters, reflectors, and planer wave-guides.

Determination of optical constant such as complex refractive index of absorbing materials is based on reflectance measured at normal incidence as a function of incident photon wavelength.

In oblique incidence, each of the reflected and transmitted wave has two components, parallel and perpendicular to the plane of incidence. The distinction between them is disappeared in the case of normal incidence. Stationary property of normal-incidence reflection from isotropic surfaces has been treated by Azzam [1].

The refractive index of a transparent layer of a quarter-wave optical thickness coating a transparent substrate can be chosen to produce half-wave retardation in reflection and no change of polarization in refraction at any angle of incidence [2]. In [2] it was shown that the layer required to produce half-wave retardation near normal incidence reduces the substrate reflectance to its square.

In this paper, we discuss the intensity (power) reflectance and the phase angle of the complex amplitude reflection coefficient for a transparent film coating an absorbing substrate at normal incidence. It was done for solid sodium and massive silver as metallic substrates coated with transparent films of different refractive indices and different optical thicknesses. The conditions of minimum and maximum intensity reflectance are deduced carefully. We have calculated these two limiting reflectances for many metals coated with a transparent film of refractive index equals 1.5 . The corresponding limiting optical thicknesses of the film at wavelength of 5893 °A are calculated. Contours of constant optical thickness in the complex refractive index plane are discovered and discussed for maximum or minimum normal incidence reflectance.

II. THEORY

a- *Normal incidence reflectance*

Figure (1) shows a transparent homogeneous dielectric film of thickness h and refractive index n_2 to be situated between a transparent homogeneous medium of refractive index n_1 and an absorbing substrate of a complex refractive index N_3, $N_3=n_3+ik_3$ where n_3 is the refractive index and k_3 is the extinction coefficient of the absorbing medium.

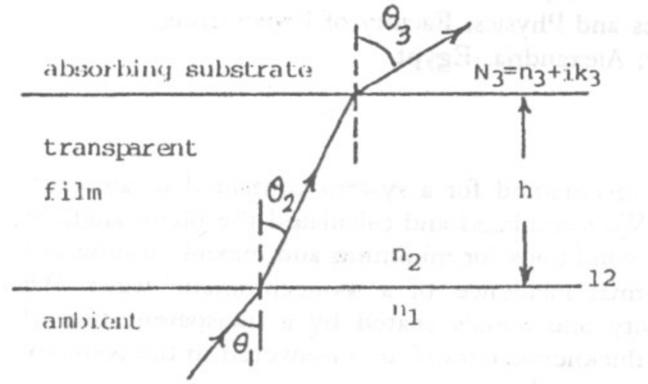

Figure 1. Ambient-transparent film-absorbing substrate system.

Consider a plane monochromatic wave of wavelength λ incident on the interface 12 with an angle of incidence Θ_1 so that it emerges with an angle Θ_3 through the absorbing substrate. The complex amplitude reflection coefficient of the substrate is given by [3] :

$$r=\frac{(m_{11}+m_{12}P_3)P_1-(m_{21}+m_{22}P_3)}{(m_{11}+m_{12}P_3)P_1+(m_{21}+m_{22}P_3)}=|r|e^{i\delta}, \quad (1)$$

where

$$m_{11}=m_{22}=\cos\beta,\ m_{12}=-i\sin\beta/P_2,$$
$$m_{21}=-iP_2\sin\beta,\ \beta=(2\pi/\lambda)P_2 h\cos\theta_2,$$

and

$$P_j=n_j\cos\theta_j, \quad (j=1,2,3). \quad (2)$$

In the case of normal incidence, $\theta_j=0$, so that:

$$P_1=n_1,\ P_2=n_2,\ P_3=n_3+ik_3,$$

and

$$\beta=(2\pi/\lambda)n_2 h=(2\pi/\lambda)H, \quad (3)$$

where H is the optical thickness of the film ($H=n_2h$). Substituting these expressions into Eq. (1), one can get the following normal incidence reflectance R as a function of β:

$$R(\beta)=|r|^2=\frac{(A\cos\beta+n_1k_3\sin\beta)^2+(B\sin\beta-n_2k_3\cos\beta)^2}{(C\cos\beta+n_1k_3\sin\beta)^2+(D\sin\beta-n_2k_3\cos\beta)^2}, \quad (4)$$

where

$$A=n_2(n_1-n_3),\ B=n_2^2-n_1n_3,$$
$$C=n_2(n_1+n_3),\ D=n_2^2+n_1n_3. \quad (5)$$

In the absence of the film, the normal incidence reflectance has the well-known formula [3] :

$$R(0)=\frac{(n_1-n_3)^2+k_3^2}{(n_1+n_3)^2+k_3^2}. \quad (6)$$

Eq. (4) can be rearranged to the following form :

$$R(\beta)=\frac{(A^2+n_2^2k_3^2)\cos^2\beta+(B^2+n_1^2k_3^2)\sin^2\beta-n_2k_3(n_2^2-n_1^2)\sin2\beta}{(C^2+n_2^2k_3^2)\cos^2\beta+(D^2+n_1^2k_3^2)\sin^2\beta-n_2k_3(n_2^2-n_1^2)\sin2\beta} \quad (7)$$

b- *Conditions of minimum and maximum normal incidence reflectance*

At minimum or maximum normal incidence reflectance $R(\beta_m)$, $\frac{dR(\beta)}{d\beta}|_{\beta_m}=0$, where $\beta_m=\beta_{min}$, corresponds to the optical thickness H_{min} of the film required for minimum reflectance $R_{min}\langle R(0)$. Also, $\beta_m=\beta_{max}$, corresponds to the optical thickness H_{max} of the film required for maximum reflectance $R_{max}\rangle R(0)$.

Derivations showed that differentiation of Eq.(7) w.r.t. β equals to zero under the condition,

$$\tan(2\beta_m)=\frac{\sin(2\beta_m)}{\cos(2\beta_m)}=\frac{2n_2k_3}{n_2^2-|N_3|^2}. \quad (8)$$

From which

$$\sin(2\beta_m)=\pm\frac{2n_2k_3}{\sqrt{(n_2^2-|N_3|^2)^2+4n_2^2k_3^2}}, \quad (9)$$

and

$$\cos(2\beta_m)=\pm\frac{n_2^2-|N_3|^2}{\sqrt{(n_2^2-|N_3|^2)^2+4n_2^2k_3^2}}. \quad (10)$$

Eq. (9) or (10) helps in calculating β_m which is independent of the ambient refractive index n_1. To evaluate R_{min} and R_{max}, we have to test the condition $\frac{d^2R(\beta)}{d\beta^2}|_{\beta_m}\gtrless 0$, using Eq.(7). We deduced that, the condition $\frac{d^2R(\beta)}{d\beta^2}|_{\beta_m}\gtrless 0$ is achieved when $\sin(2\beta_m)\gtrless 0$. Thus, we have three distinguished cases using Eq.(8):

I- $n_2 > |N_3|$; i.e. $\sin(2\beta_m)$ and $\cos(2\beta_m)$ have the same sign. In this case :

i) $R = R_{max}$, when $\sin(2\beta_m) < 0$ and $\cos(2\beta_m) < 0$. Hence,

$$\beta_{max} = (2\beta_m + \pi)/2, \qquad 3\pi/2 > 2\beta_{max} > \pi \tag{11}$$

ii) $R = R_{min}$, when $\sin(2\beta_m) > 0$ and $\cos(2\beta_m) > 0$. Hence,

$$\beta_{min} = \beta_m, \qquad \pi/2 > 2\beta_{min} > 0 \tag{12}$$

II- $n_2 < |N_3|$; i.e. $\sin(2\beta_m)$ and $\cos(2\beta_m)$ have different signs. In this case:

i) $R = R_{max}$, when $\sin(2\beta_m) < 0$ and $\cos(2\beta_m) > 0$. Hence,

$$\beta_{max} = (\pi - \beta_m)/2, \qquad 2\pi > 2\beta_{max} > 3\pi/2 \tag{13}$$

ii) $R = R_{min}$, when $\sin(2\beta_m) > 0$ and $\cos(2\beta_m) < 0$. Hence,

$$\beta_{min} = (\pi - 2\beta_m)/2, \qquad \pi > 2\beta_{min} > \pi/2 \tag{14}$$

III- $n_2 = |N_3|$; In this case, and according to Eqs. (9) and (10),

$$\beta_m = m\pi/4; \quad m = 1,3,5,7,....$$
$$\beta_{max} = \beta_m \text{ where } m = 3+4n, \text{ and}$$
$$\beta_{min} = \beta_m \text{ where } m = 1+4n, \qquad n = 0,1,2,.... \tag{15}$$

It is clear, in all the previous cases, that the difference between β_{min} and β_{max} is $\pi/2$. Due to the periodicity of the normal incidence reflectance function $R(\beta)$, we found that there is a value, $0 < \beta_o < \pi$, at which the normal incidence reflectance $R(\beta_o)$ has the same value $R(o)$ at the absence of the film as shown in Figure (2), so that:

$$\beta_o = 2\beta_{min} = \tan^{-1}\frac{2n_2k_3}{n_2^2 - |N_3|^2}. \tag{16}$$

Hence,

$$R(o) = R(m\pi) = R(\beta_o + m\pi); \quad m = 0,1,2,.... \tag{17}$$

This result shows that a film of optical thickness that satisfies the condition $\beta = \beta_o + m\pi$, has no influence on the intensity of the reflected radiation. In the special case, $n_2 = |N_3|$, the value of β_o equals $m\pi/2$, m=1,3,5,...

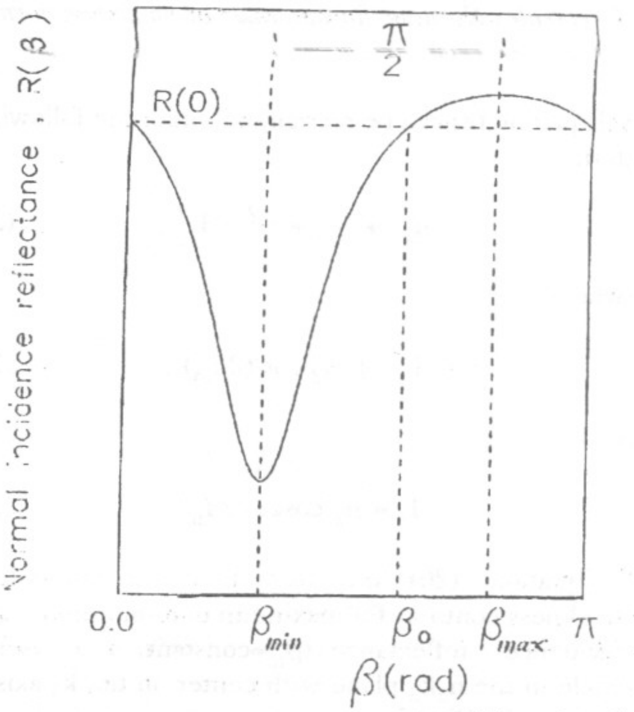

Figure 2. Periodic reflectance function $R(\beta)$ against β.

c- *Phase angle of the complex amplitude reflection coefficient*

The complex amplitude reflection coefficient, using Eq.(1), can be given by:

$$r = \frac{|P|e^{i\alpha}}{|q|e^{i\gamma}} = |r|e^{i(\alpha-\gamma)} = |r|e^{i\delta_r}, \tag{18}$$

where,

$$\tan\alpha = \frac{B\sin\beta - n_2k_3\cos\beta}{A\cos\beta + n_1k_3\sin\beta},$$

and

$$\tan\gamma = \frac{D\sin\beta - n_2k_3\cos\beta}{C\cos\beta + n_1k_3\sin\beta}.$$

Hence,

$$\tan\delta_r = \tan(\alpha - \gamma)$$
$$= 2n_3\frac{n_1^2k_3\tan_\beta^2 - n_2(n_2^2 - n_1^2)\tan\beta + n_2^2k_3}{[n_1^2(n_3^2 - k_3^2) - n_2^4]\tan_\beta^2 + 2n_2k_3(n_2^2 - n_1^2)\tan\beta + n_2^2(n_3^2 - k_3^2 - n_1^2)}. \tag{19}$$

According to the numerator of Eq.(19), one can find that there are two values of β at which $\tan\delta_r = 0$, i.e. the amplitude reflection coefficient is real.

d- *Contours of constant optical thickness in the complex N_3 plane for minimum or maximum normal incidence reflectance*:

Equation (8) can be rearranged under the following form:

$$n_3^2 + (k_3 + J)^2 = L^2 \ , \qquad (20)$$

where

$$J = \pm\, n_2 \cot (2\beta_m) \ , \qquad (21)$$

and

$$L = n_2 \operatorname{cosec} (2\beta_m). \qquad (22)$$

Equation (20) indicates that the iso-optical thickness contour- for maximum or minimum normal incidence reflectance (β_m=constant)-is a (semi) circle in the n_3k_3 plane with center on the k_3 axis at (0,-J) and radius L.

III- RESULTS AND DISCUSSION

The normal incidence reflectance is calculated for solid sodium and massive silver as absorbing substrates coated with different transparent films. It's clear, from figures (3) and (4), that through the periodic range $0\langle\beta\langle\pi$, there are minimum and maximum values for the normal incidence reflectance. It's obvious that when n_2 increases, both of β_{min} and β_{max} decreases (the difference between them is still constant and equals π).In this case the minimum reflectance R_{min} has a lower value, while the maximum reflectance R_{max} has a larger value. In other words, films of more refractive index n_2 and less thickness h contribute to less minimum reflectance and more maximum reflectance.

The special case, in which the reflectance has either a minimum or a maximum value through the periodic interval $0 \langle \beta \langle \pi$, can be achieved when k_3=0. That is for a transparent substrate. The reflectance in this case can be maximized or minimized according to the value of the refractive index n_2 of the coating transparent film.

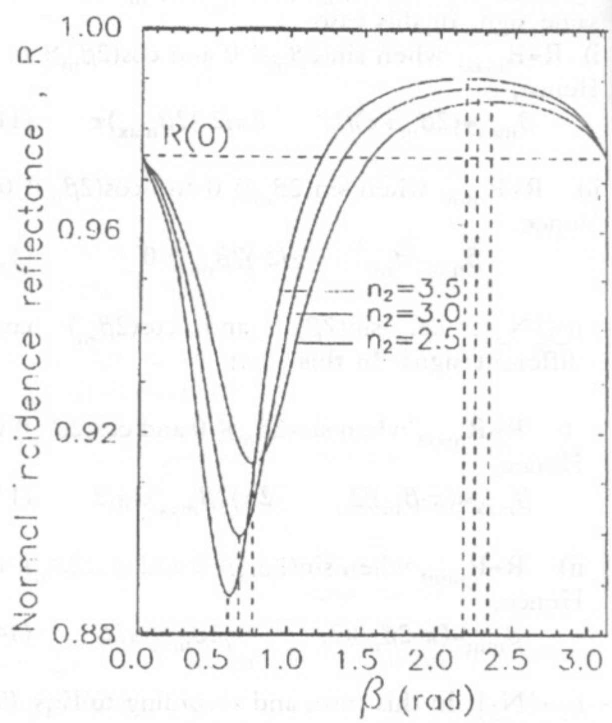

Figure 3. Normal incidence reflectance against β for solid sodium. n_1=1, n_3=0.044, k_3=2.42.

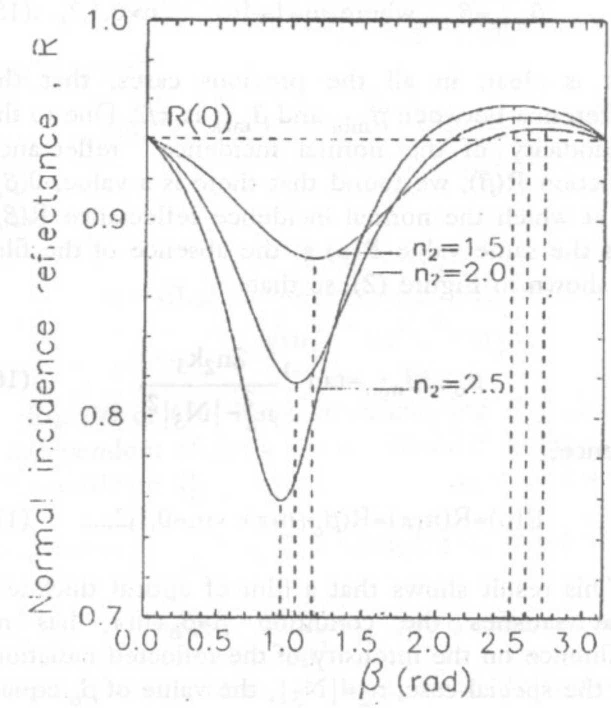

Figure 4. Normal incidence reflectance against β for massive silver. n_1=1, n_3=0.2, k_3=3.44.

Table 1. Optical constants (n_3,k_3) of some metals at λ=5893 °A, and the optical thicknesses of their coating film (H_{min},H_{max}) corresponding to the minimum and maximum normal incidence reflectances (R_{min},R_{max}). (n_2=1.5).

Metal	n_3	k_3	H_{min} (°A)	R_{min}	H_{max} (°A)	R_{max}	R(0)
solid sodium	0.044	2.42	953.0	0.952	2426.	0.979	0.975
massive silver	0.2	3.44	1089.	0.880	2561.	0.945	0.940
massive magnesium	0.37	4.42	1169.	0.859	2641.	0.935	0.931
molten potassium	0.084	1.81	824.0	0.872	2298.	0.941	0.925
massive cadmium	1.13	5.01	1212.	0.699	2685.	0.853	0.847
massive aluminium	1.44	5.23	1228.	0.662	2701.	0.838	0.827
electrolytic gold	1.47	2.83	1024.	0.592	2498.	0.835	0.815
liquid mercury	1.60	4.80	1214.	0.666	2687.	0.793	0.785
massive zinc	1.93	4.66	1219.	0.527	2692.	0.745	0.745
massive copper	0.62	2.57	995.0	0.541	2469.	0.762	0.731
crystal gallium	3.69	5.43	1295.	0.471	2769.	0.718	0.713
massive antimony	3.04	4.94	1266.	0.456	2740.	0.708	0.701
massive cobalt	2.12	0.04	1201.	0.425	2675.	0.687	0.675
electrolytic nickel	1.58	3.42	1140.	0.407	2614.	0.674	0.656
massive manganese	2.41	3.88	1211.	0.375	2684.	0.651	0.639
massive lead	2.01	3.48	1171.	0.356	2645.	0.637	0.620
electrolytic platinum	2.63	3.54	1214.	0.313	2687.	0.604	0.591
massive rhenium	3.00	3.44	1235.	0.284	2709.	0.580	0.569
massive tungsten	3.46	3.25	1263.	0.255	2737.	0.555	0.546
massive bismuth	1.78	2.80	1115.	0.271	2588.	0.569	0.543
evaporated iron	1.51	1.63	973.0	0.093	2445.	0.381	0.326

Table (1) gives the optical thicknesses (H_{min},H_{max}) in °A of a film (n_2=1.5) required for minimum and maximum reflectances (R_{min},R_{max}) respectively, for twenty one metals used as absorbing substrates. Calculations showed that a great percentage reduction in reflectance can be obtained when any absorbing substrate, having the same optical constants (n_3, k_3) as the evaporated iron, is coated by 973 °A of a film of refractive index equals 1.5 . In this case when the surface 12 is illuminated normally by a monochromatic wave (λ=5893 °A), we have:

$$\frac{R(0)-R_{min}}{R(0)}=71.5\%.$$

Figures (5) and (6) show the behaviour of the phase angle of the complex amplitude reflection coefficient for the solid sodium and massive silver against the optical thickness of different coating films. From the intersections between each curve and the horizontal zero -tan δ_r -line, it is clear, as an interesting result, that the complex amplitude reflection coefficient has two real values for two limiting optical thicknesses of films having large refractive indices.

Figure (7) gives the families of iso-β_m-contours in the n_3k_3 plane for glass layer (n_2=1.5) of optical thickness n_2h. According to Eq.(20), it is obvious that the iso-β_m-contour passes through the origin (0,0) when J=L. This case cannot be achieved practically at any finite value of β_m.

All semi-circles intersect at a point of (n_3=n_2,0), i.e. the case in which the medium 3 is the same as medium 2. In other words, any finite iso-β_m-contour must pass through (n_2,0) point.

Each semi-circle lies partially in [n_3,+k_3] and partially in [n_3,-k_3] plane. Since J 〈 L at any finite value of β_m, no iso-β_m-contour can lie totally in [n_3,+k_3] or [n_3,-k_3] plane.

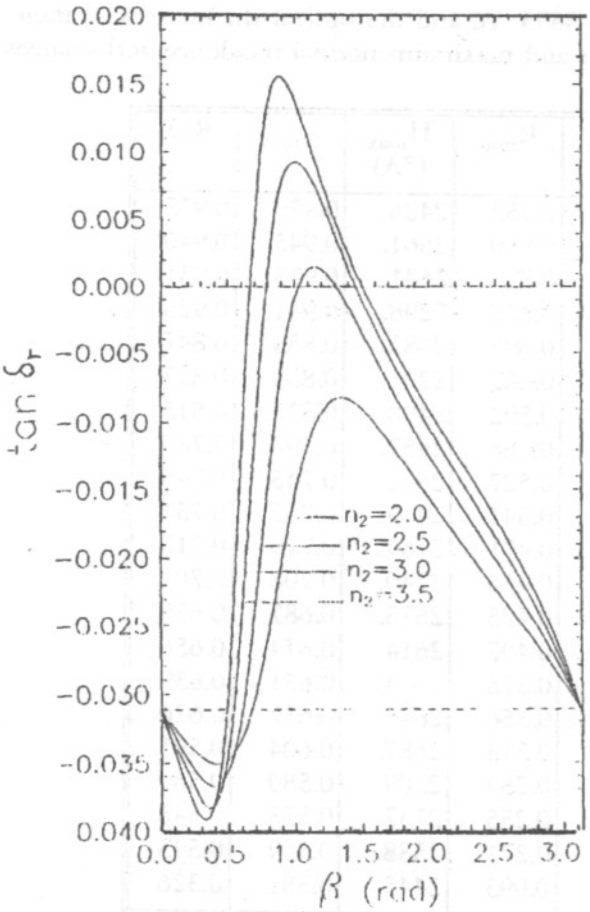

Figure 5. tan δ_r against β for solid sodium, $n_1 = 1$, $n_3 = 0.044$, $k_3 = 2.42$.

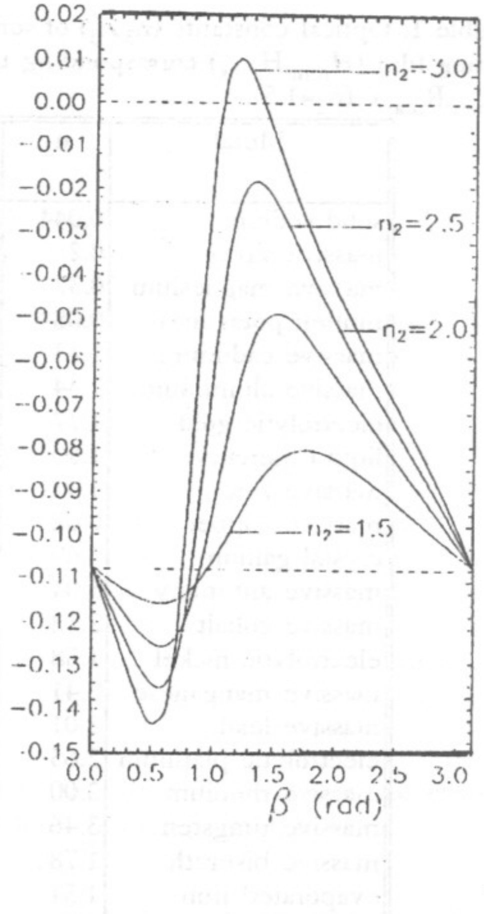

Figure 6. tan δ_r against β for massive silver, $n_1 = 1$, $n_3 = 0.2$, $k_3 = 3.44$.

I-V CONCLUSION

Thin transparent films on absorbing substrates have many practical uses. They are employed, for example, to protect metallic mirrors and to increase their reflectivity. They may also be used to reduce the reflectivity of a metal surface. For a non-absorbing film, the reflectance is a periodic function of the film thickness, with a period of a half-wavelength. The normal incidence reflectance has two limiting values, minimum and maximum, through the period of the film thickness, when the substrate is made of an absorbing material. On the other hand, the normal incidence reflectance has either a minimum or a maximum value through the same period when the substrate is made of a transparent material.

As an important conclusion, for a coating film of large refractive index, a small thickness can reduce the minimum normal incidence reflectance to an appreciable value, while it increases the maximum reflectance with a small amount. Another important investigation is that the large values of the refractive index of the film can contribute to a real amplitude reflection coefficient for two different thicknesses of the film.

Families of iso-optical thickness-contours are discovered as semi-circles in the complex N_3-plane. These contours may be of particular importance in some treatments of our special case. For example, when the knowledge of the optical constants of the absorbing substrate is essentially required to get minimum or maximum normal incidence reflectance with a given optical thickness of the film. This can be done using the conditions of minimum and maximum reflectance mentioned above.

It's obvious, according to the shape of the iso-β_m-contour, that for the same value of the real refractive index of the absorbing substrate, there are two values of the extinction coefficient that contribute to the same minimum or the same maximum normal incidence reflectance.

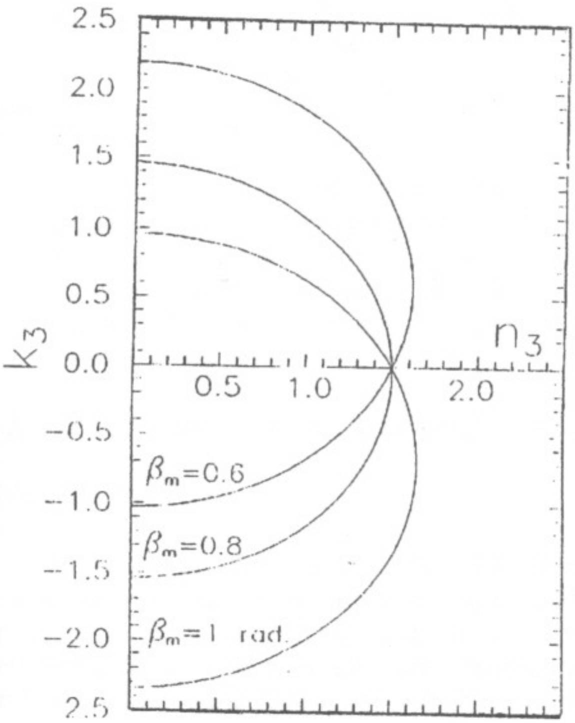

Figure 7. Families of iso-β_m contours in the complex N_3 plane. n_2= 1.5, for glass layer of optical thickness n_2h.

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