GEOMETRICAL ANALYSIS OF TRAVERSE MISCLOSURE

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ABSTRACT

It is already established that the Standard Deviation values(SD) OF angles and lengths in Traverse' nets can be obtained by different methods, among them are mean values and the distribution of misclosure. In this paper a contribution is made by introducing another technique , where polygon or link Traverse can be drawn on one line. The standard deviation in this method reflects the effect of all sources of errors that may arise during measurement. Moreover the initial error will be taken into consideration in addition to its applicability by using personal computers or desk calculators.

Keywords: Survey control, Traverse misclosure, polygon link, geometrical analysis, measuring accuracy.

1. INTRODUCTION

Traversing is one of the most indispensable forms of survey control, specially in town and detail topographical surveying. It is aimed in this paper to perform an accurate geometrical analysis of the error of misclosure leading to unbiased estimate of both angle and distance measuring accuracy. In this way, an insight for a homogeneous accuracy of these two kinds of measurements would be attained.

A detailed explanation for this method is presented. The derivation of the standard deviation for all different measurements (angles, directions, distances) depending on longitudinal and perpendicular displacement are given for different cases. At the end, numerical examples are selected to check over the precision and accuracy of the suggested method.

2. METHODOLOGY

According to the classical definition of longitudinal and perpendicular displacement [Lazaref, 1988], the relation between them and the components of the closing error will be described.

Longitudinal displacement (f_l): is the value of the displacement of the last point from its first position in the direction of the traverse line. It results from errors in distance measurements.

Perpendicular displacement (f_u): it is the value of the displacement of the same points in the direction perpendicular to the direction of the traverse line. It results from errors in the angular measurements.

These displacement may be obtained by either graphical or analytical methods. This paper discusses how to get the longitudinal and the perpendicular displacements analytically. This goal is achieved by using the components (f_x) and (f_y) of the total closing error value (f_s) in the direction of basic axes.

In Figure (1),point 1 represents the first point of the traverse and point N is the last one which will take the position N′ (the traverse circumference is represented by line 1-N). The closing error (f_s), the components of the closing errors (f_y) and (f_x), and the components of displacements (f_l) and (f_u) are shown in the same figure.

The relationship between the components of the closing error (f_y) and (f_x) and the displacements (f_u) and (f_l) may obtained with reference to Figure (2). From the geometry of the figure, the following relations are directly obtained:

$$f_l = \acute{N}A + AB = f_y \sin\theta + f_x \cos\theta \tag{1.a}$$

$$f_u = AD - CD = f_y \cos\theta - f_x \sin\theta \tag{1.b}$$

where,

$$\sin\theta = \frac{[\Delta Y]}{[S]} \text{ and } \cos\theta = \frac{[\Delta X]}{[S]}$$

[ΔX], [ΔY] and [S] are the sums of the component of the traverse's length and total length respectively. Substituting sin Θ and cos Θ into Eqs. (1.a) and (1.b) we get:

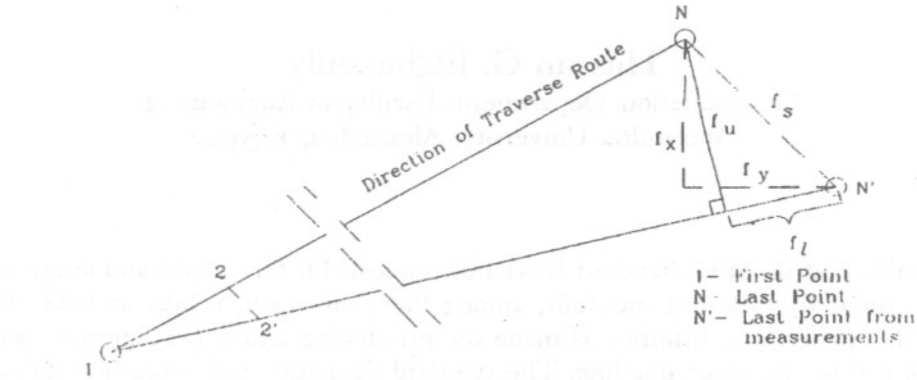

Figure 1.

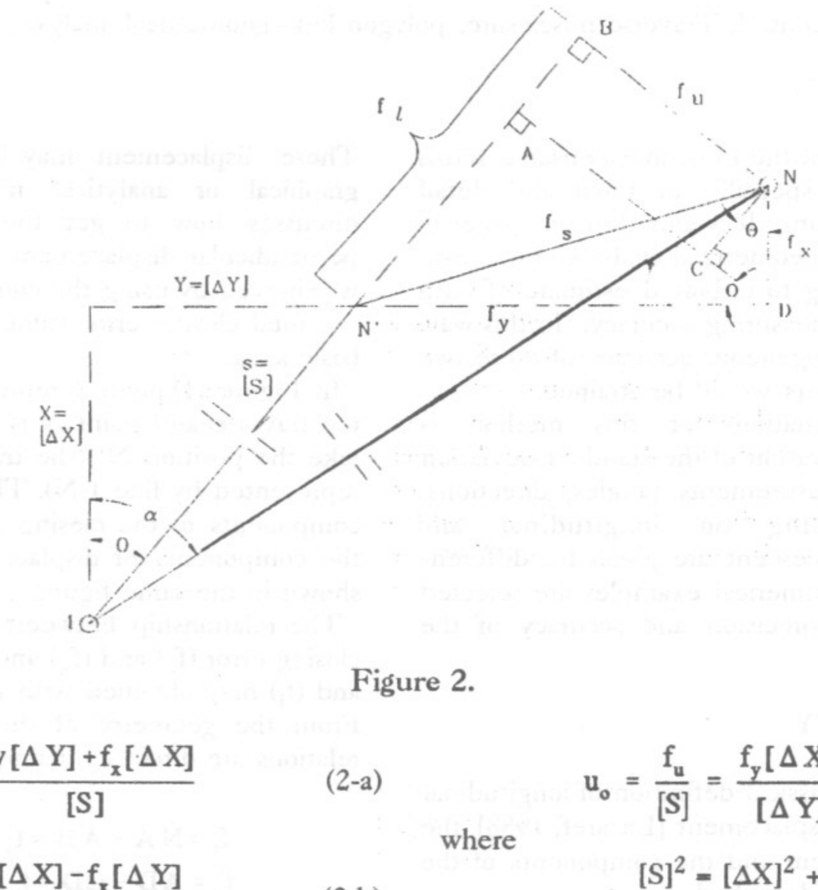

Figure 2.

$$f_l = \frac{f_y[\Delta Y]+f_x[\Delta X]}{[S]} \tag{2-a}$$

$$f_u = \frac{f_y[\Delta X]-f_x[\Delta Y]}{[S]} \tag{2-b}$$

The displacements per unit length (l_o and u_o for longitudinal and perpendicular) can be given as follows:

$$l_o = \frac{f_l}{[S]} = \frac{f_y[\Delta Y]+f_x[\Delta X]}{[\Delta Y]^2+[\Delta X]^2} \tag{3-a}$$

$$u_o = \frac{f_u}{[S]} = \frac{f_y[\Delta X]-f_x[\Delta Y]}{[\Delta Y]^2+[\Delta X]^2} \tag{3-b}$$

where

$$[S]^2 = [\Delta X]^2 + [\Delta Y]^2$$

Referring to Figure (3), the components of closing error f_x and f_y can be derived from the components of their displacement as follows:-

$$f_x = AC-AB = f_l\cos\theta - f_u\sin\theta \tag{4-a}$$

$$f_y = N'C+CD = f_l\sin\theta + f_u\cos\theta \tag{4-b}$$

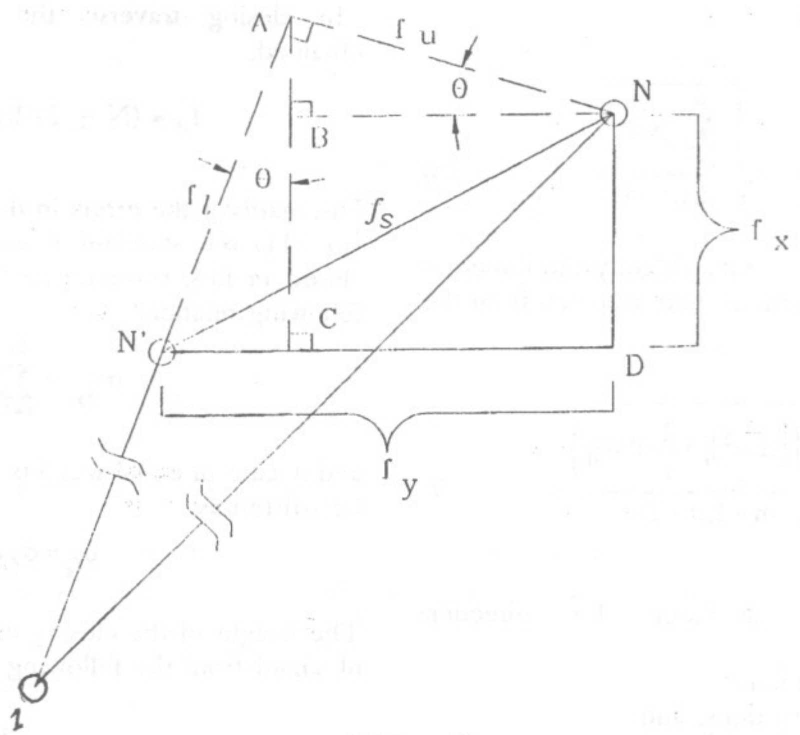

Figure 3.

3. ACCURACY OF CALCULATIONS FOR ANGLES

The accuracy of calculation of angles can be expressed by many methods that relate there parameters. As combining angles and its mean, angles and the closing error of the traverse, and angles and the longitudinal and perpendicular displacements of the traverse. The suggested method of accuracy estimation is explained in the next sections.

3.1 The standard deviation of angle

It is well known that, in case of calculating an angle several times, we can easily estimate the mean. Also we can judge the accuracy of the angles by calculating the standard deviation (σ). The well known standard deviation in this case is given as, [Hazay, 1970; Ewing and Mitchell, 1970 and Mikhail, 1976]:

$$\sigma = \sqrt{\frac{[vv]}{(n-1)}} \tag{5}$$

where v's are the residuals, n is the number of repeated measurements (number of observation).

In order to improve accuracy, the number of measurements has to be increased. Therefore Eq. (5) reveal the accuracy and for getting more preciseness a large number of points in net are chosen [Myhkailofyh, 1984]. From the following relation the standard deviation could be estimated as:-

$$\sigma^2 = \frac{1}{s}\sum_{i=1}^{s} \sigma_i^2 \tag{6}$$

where s is the number of stations (points).

Substituting Eq. (6) into Eq (5) one can get the standard deviation for only one observation as follows:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} [vv]_i}{s(n-1)}} \tag{7}$$

And, the standard deviation for the mean of n observations is given by,

$$\sigma_n = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{\sum_{i=1}^{n}[vv]_i}{ns(n-1)}} \qquad (8)$$

It will be convenient to judge directly the direction measurements and estimate their accuracy from the following equation:-

$$\sigma_d = \sqrt{\frac{\sum_{i=1}^{n}\left([\Delta\alpha^2]_i - \frac{1}{r}[\Delta\alpha]_i^2\right)}{(n-1)(r-1)}} \qquad (9)$$

where:

- σd The standard deviation for direction measurements
- r number of directions
- n number of observations, and
- Δ_α the residual for directions

3.2 The standard deviation from closing error

The standard deviation of angle calculated from the angular closing error of the net or the route can obtained consequently from the following formulation: In the traverse (closing net) or in the link traverse the standard deviation (σ) is:

$$\sigma = \sqrt{\frac{[pf_\beta^2]}{R}} \qquad (10)$$

where:

- f_β the angular closing error obtained from the net
- p the weight of the angular closing error, and
- R number of traverse or routes.

From Figure (4) one can get the previous values as in the following derivation,

$$f_\beta = (\alpha_{NB} \pm N \times 180^\circ) - (\alpha_{A1} + \sum_{i=1}^{N} B_i) \qquad (11)$$

where

$$\sum_{i=1}^{N} \beta_i = \beta_1 + \beta_2 + ... + \beta_N$$

and N number of angles.

In closing traverse the following formula i[s] obtained,

$$f_\beta = (N \pm 2)\, 180^\circ - \sum_{i=1}^{N} B_i \qquad (12)$$

Disregarding the errors in direction A-1 and N-B i[n] Eq. (11) the standard deviation of angles for th[e] closed or link traverse can be estimated from th[e] following relation,

$$\sigma^2_{f_\beta} = \sum_{i=1}^{N} \sigma^2_{\beta_i} \qquad (13)$$

and in case of equal weights for angles, Eq. (13) wil[l] be written as:

$$\sigma_{f_\beta} = \sigma_\beta \sqrt{N} \qquad (14)$$

The weight of the closing error in the route can be obtained from the following relation accordingly,

$$P = \frac{k}{\sigma^2_{f_\beta}} \qquad (15)$$

where k is a constant of proportionality and here k is equal to σ^2_β which is defined as the variance of an observation of unit weight.

Substituting the value of $\sigma^2_{f_\beta}$ from Eq. (14) in Eq. (15) the following simple equation is extracted:

$$P = \frac{1}{N} \qquad (16)$$

where $k = \sigma_\beta^2$

3.3. The standard deviation of angle from the perpendicular displacement

Standard deviation can also be calculated from the perpendicular displacement of the traverse (polygon or link). This linear perpendicular displacement comes from the errors in the observed angles in the traverse. The standard deviation of the angle could be expressed as a function of perpendicular displacement from the following relation:-

$$\sigma_\beta = \sqrt{\frac{[p_i f_u^2]}{R}} \qquad (17)$$

where:

f_u the perpendicular displacement
p_i the weight of lengths in the net, and
R the number of routes in the traverse.

The weight of the lengths could be known from the following relation (Lazaref, 1980):

$$P_i = \frac{C_i}{L_i^2} \qquad (18)$$

L is the length of the route, and
C is a value based on the number of point in the route (N_i).

$$C_i = \frac{12\rho^2}{N_i + 3} \qquad (19)$$

Combining Eqs. (17) and (18), the standard deviation could be expressed as follows:-

$$\sigma_\beta = \sqrt{\frac{\left[c\left(\frac{f_u}{L}\right)^2\right]}{R}} \qquad (20)$$

Section 5 below includes a numerical example to explain this technique.

4. ACCURACY ESTIMATION OF LENGTHS

It is not easy to calculate the standard deviation of the measured length in traverse, that is because there is no mathematical condition in traverses depending only on the sides. Besides, length measurements are functions of the type of the used instruments. It is possible to calculate the standard deviation of the measured distance by different method, such as; through calculating the difference between the measurements and the mean (residual), from the difference between two measurements, and from displacements. The aforementioned methods will be discussed in the following.

4.1. *Calculation of the standard deviation of a distance*

If distances are measured under the same condition and equal weights, accuracy of measurements could be calculated as in the case of angle measurements. But, if the condition and the weight of the measurements are changed, the standard deviation will be calculated from the following subsection.

4.2. *Calculation of the standard deviation of a measured distance from the difference between two measured values*

Myhkailofyh (1984) calculates the standard deviation for two measurements of a distance for a certain weight and is given as follows:

$$\sigma = \sqrt{\frac{[pd^2]}{2n}} \qquad (21)$$

where:

d the difference between two measurements for a distance, $d_i = s'_i - s''_i$
p_i the weight of difference d_i
n the number of the measured sides, and
[ ] summations such as $[pd^2] = p_1 d_1^2 + p_2 d_2^2 + \ldots + p_n d_n^2$

If the measured distances contain accidental errors, the following relation from Myhkailofyh (1984) could be used:

$$\sigma_{s_i} = \sigma\sqrt{s_i} \qquad (22)$$

and

$$P_i = \frac{K}{\sigma^2_{s_i}} = \frac{1}{s_i} \qquad (23)$$

when there are regular errors in the measurements the standard deviation could be modified as follows:

$$\sigma = \sqrt{\frac{[pd'^2]}{2(n-1)}} \qquad (24)$$

As d_i' is the difference between two consecutive distances which has no regular error.

$$d'_i = d_i - \lambda\, s_i \tag{25}$$

where

$$\lambda = [d] / [s] \tag{26}$$

It is obvious from the previous presentation that there is deficiency in equations (22) through (25). This is because they do not contain all the possible errors. Table (2) in Section 5 shows the way of calculating the standard error, by measuring traverse lengths twice.

4.3. Calculation of the standard deviation for a measured distance from linear displacements in traverses

In traverses the linear displacements are resulted from the errors in measuring lengths and initial errors in observations. By neglecting the initial errors in the observation, the linear displacements are considered as real errors and can be used to evaluate accuracy of the measured lengths (Myhkailofyh, 1984). The standard deviation for the weighted measurements may be calculated from the following formula

$$\sigma = \sqrt{\frac{[p f_l^2]}{R}} \tag{27}$$

where:

f_l the linear displacement in the traverse
p the weight
R the number of route in the traverse.

As the weight is expressed by Eq. (23) (the reciprocal of the length), then Eq. (27) could be written as:

$$\sigma = \sqrt{\frac{\left[\frac{1}{L} f_l^2\right]}{R}} \tag{28}$$

where L is the total length of the traverse (actual route). By taking into consideration the regular errors, the following equation may be used to calculate the SD:

$$\sigma = \sqrt{\frac{\left[\frac{1}{L} f_l'^2\right]}{R}} \tag{29}$$

where

$$f'_{li} = f_{li} - \lambda\, L_i\ ,\ \lambda = [f_l] / [L]$$

and λ is the factor of the regular errors. Fractional linear misclosure for a traverse may be also estimated from:

$$\sigma_R = \frac{\sigma\sqrt{L_s} + \lambda L_s}{L_s} \tag{30}$$

where L_s is the average length of the traverse net

$$L_s = \frac{1}{R}\sum_{i=1}^{R} L_i$$

The standard deviation is always relatively large because there is an error in the coordinate of the first point of the traverse, which has an effect on it. Table (3) in the following section shows the steps of calculations.

5. VERIFICATION OF THE SUGGESTED TECHNIQUE

To check over the precision and accuracy of the suggested method of measurement and evaluation technique, a numerical example is chosen for judgement. Measures are prescribed in Table (1) as linear perpendicular displacements for a link traverse, Figure (4). Solution steps are indicated in detailed calculations as application of Equations (19) and (20).

Table (2) and Figure (4) give an example for deriving the standard deviation by measuring twice every line of the traverse. The steps of calculating the standard deviation from the linear displacements in different routes for link traverse, are shown in Table (3).

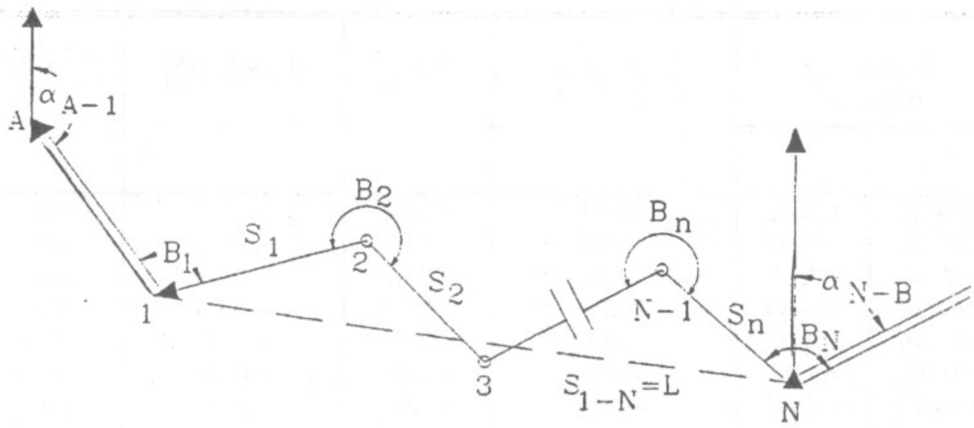

Figure 4.

Table 1. Results of the Numerical Example.

No.of Point	No. of measurements	f_u mts	L mts	$\frac{f_u}{L}10^{-4}$	$(\frac{f_u}{L})^2 10^{-8}$	C	$C\ (\frac{f_u}{L})^2 10^{-8}$
1	5	+0.059	834	0.71	0.5	1.5	0.80
2	5	+0.098	759	1.29	1.7	1.5	2.60
3	4	+0.254	674	3.77	14.2	1.7	24.10
4	3	+0.110	619	1.78	3.2	2.0	6.40
5	3	-0.095	692	1.37	1.9	2.0	3.80
6	4	-0.038	504	0.75	0.6	1.7	1.00
7	5	+0.081	825	0.98	1.0	1.5	1.50
8	5	+0.010	836	0.12	0.0	1.5	0.00
9	5	+0.210	730	2.88	8.3	1.5	12.40
10	4	-0.131	672	1.95	3.8	1.7	6.50
11	3	+0.195	620	3.15	9.9	2.0	19.80
12	3	+0.149	692	2.15	4.6	2.0	9.20
13	4	-0.002	504	0.04	0.0	1.7	0.00
14	5	+0.018	826	0.22	0.0	1.5	0.00
15	5	-0.070	301	2.33	5.4	1.5	8.10
							Σ = 96.2

$C = \frac{12}{n+3}$ and n is the number of sides in each route

$$\sigma_B = \sqrt{\frac{\left[C\left(\frac{f_u}{L}\right)^2\right]}{R}} = 52.2''$$

R is the number of routes.

Table 2. Solved example for a link traverse

Side	Horizontal distance		Difference $d_i = S'_i - S''_i$	$-\lambda S_i$	$d'_i = d_i - \lambda S_i$	$d'^2_i \times 10^{-4}$	$\frac{d'^2_i}{si} \times 10^{-8}$
	S'_i	S''_i					
1-2	124.36	124.32	+0.04	- 0.02	+ 0.02	4.0	322.0
2-3	218.42	218.37	+ 0.05	- 0.03	+ 0.02	4.0	183.0
3-4	87.18	87.21	-0.03	- 0.01	- 0.04	16.0	1839.0
4-5	164.20	164.14	+0.06	- 0.03	+ 0.03	9.0	548.0
5-6	215.46	251.38	+0.08	- 0.04	+ 0.04	16.0	636.0
6-7	69.00	69.02	-0.02	- 0.01	- 0.03	9.0	1304.0
7-8	130.49	130.47	+0.02	- 0.02	0.00	0.0	0.00
8-9	172.00	172.01	-0.01	- 0.03	- 0.04	16.0	930.0
[ ]	1217.11	1216.92	+ 0.19	-0.019	0.00	74.10	5762.0

$$\lambda = [d]/[S] = +1.56\times10^{-4}$$

$$\sigma = \sqrt{\frac{[d'^2/S]}{2(n-1)}} = \sqrt{\frac{5762.0\times10^{-8}}{14}} = 2.0 \text{ mm}$$

Table 3. The Use of Linear Displacement in Calculating SD.

No. of point	f_l mts	L mts	$-\lambda L$	$f'_l = f_l - \lambda L$	$f'^2_l \times 10^{-2}$	$\frac{1}{L} f'^2_l \times 10^{-4}$
1	-0.426	834	+0.236	-0.190	3.61	0.43
2	-0.252	759	+0.215	-0.037	0.14	0.02
3	+0.039	674	+0.191	+0.230	5.29	0.78
4	-0.115	619	+0.175	+0.060	0.36	0.06
5	+0.199	692	+0.196	+0.395	15.60	2.25
6	-0.200	504	+0.143	-0.057	0.32	0.06
7	-0.434	825	+0.234	-0.200	4.00	0.48
8	-0.376	836	+0.237	-0.139	1.93	0.23
9	-0.492	730	+0.207	-0.285	8.12	1.11
10	-0.094	672	+0.190	+0.096	0.92	0.14
11	-0.087	620	+0.176	+0.089	0.79	0.13
12	+0.152	694	+0.197	+0.349	12.18	1.80
13	-0.228	504	+0.143	-0.085	0.72	0.14
14	-0.489	826	+0.234	-0.255	6.50	0.79
15	-0.056	301	+0.085	+0.029	0.08	0.03
[ ]	-2.859	10090	+2.859	-0.001	-	8.45

$$\lambda = [f_l]/L = -2.859/10090 = -2.833 \times 10^{-4}$$

$$\sigma = \sqrt{\frac{\left[\frac{1}{L} f_l'^2\right]}{(R-1)}} = \sqrt{\frac{8.45\times10^{-4}}{14}} = 0.0078 \text{ m} = 7.8 \text{ mm}$$

6. CONCLUSION

In this paper, the accuracy of measurements are checked using different ways. Equations are derived which are useful to judge the accuracy of angular and linear measurements in the traverse. Equations (8), (9), (10) and (20) can be used to evaluate the accuracy of measurements in traverse even if they give different results. It is worth mentioning that the standard deviation (SD) obtained from Eqs. (8) and (9) is less than the one calculated from Eq. (20). This is because Eq. (20) reflects the effect of all sources of errors during angle measurements. The initial error is also taken into consideration. Eq. (10) gives an average value among the alternative values Eq. (20). Eq. (21), (24), (28) and (29) are used to evaluate the values of the distance measurements in traverse.

Sometimes, the suggested relation reveals unequal results, that is because they do not consider all circumstances and errors of the measurements. The standard deviations obtained from Eqs. (21) and (24) give values less than those obtained from Eqs.(28) and (29) for the same measurements.

In conclusion, the longitudinal and perpendicular displacements technique in traverses are highly recommended due to its simplicity, applicability, and accuracy.

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