DESIGN OPTIMIZATION OF FIBER REINFORCED LAMINATED COMPOSITE

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ABSTRACT

Composite materials are frequently considered for use in fusion reactors. A recent paper showed the importance of three-dimensional structural analysis for the design of the composite components. This paper discuses the importance of optimizing the composites for use in the fusion environments. Two failure criteria (maximum stress and Tsai-Wu) are used in the analysis. The optimized stresses are found to be significantly lower than the stresses found in the previous studies of non-optimized composite structures.

Keywords: Fiber reinforced laminated composite, optimized stresses, composite structure.

INTRODUCTION

Due to their excellent performance characteristics, advanced composite materials have gained wide use in many applications. The four most important requirements are light weight, high strength, high stiffness, and good fatigue resistance. Full realization of the potential of advanced composite materials to the fusion reactor structures requires advanced tools for analysis and an understanding of the response of failure characteristics of laminated composite structures.

The composite laminates provide some unique behaviors. Differing the ply arrangement on the thickness, orientation or stacking sequence would greatly alter laminate performance. Optimization provides us with a tool to actually design the material by tailoring the structural properties in an efficient manner. The optimum design may make a laminated structure meet some particular requirements with little waste of material capability. However, the optimization for laminates is more complicated than that for conventional isotropic materials.

In the past three decades, together with the creations of new composite materials and the developments of new fabrication techniques, various aspects of physical properties of composite materials have been investigated, and different approaches for the better understanding of the material behaviors have been proposed [1-4]. A better understanding of specimen testing and structure application is needed before composites can be successfully applied in design and manufacture. The influence of fiber and matrix constituent properties and bonding process on composite behavior is not always clear. Improved methods of determining the constituent and composite properties are needed so that reliable data can be generated to better predicted composite structures' long-term integrity under service conditions such as creep, impact, etc..

The determination of fiber orientation angles is the most important subject in optimum design of composite materials. Many investigations have been carried out on this point [5-7], but most of them yield optimum fiber angles for deterministic conditions where strengths and loads are constants. However, those results are considered to be invalid for probabilistic conditions where strengths and loads have some variations.

Cederbaum et al [8] proposed a method to evaluate the reliability of each ply in laminated plates subjected to in-plane random static loads, and Miki et al [9] developed the general method for optimum design of unidirectional fibrous composite under probabilistic conditions and found that the optimum fiber angle that yields the maximum reliability changes with the increase in the variation of applied loads.

For the optimization of complex structures, Sobieszczanski-Sobieski et al [10] suggested the multilevel composition method, in which an optimization problem is decomposed and the optimize process is performed by solving several subproblems with small scales. Another method proposed by Wang and Qian [11] is the optimization compound of different levels, in which the design variables are divided into two groups for separate purposes : one for minimizing the objective function and the other for relaxing constrains.

THEORY

Two failure criteria will be used in this analysis. These criteria are the maximum stress and Tsai-Wu failure criteria. Jenkins [12] extended the application of the maximum stress theory to predict the failure of a planar orthotopic material. In this theory, stresses acting on the orthotopic material are resolved along the material axis ($\sigma_1, \sigma_2, \tau_{12}$), where 1 refers to the fiber direction and 2 refers to the direction perpendicular to the fiber, and it is postulated that the failure will occur when one or all of these stresses attain the maximum values X, Y, and S obtained by uniaxial loading tests. This criterion stated that failure will not occur as long as the following conditions prevail:

$$-X < \sigma_1 < X, -Y < \sigma_2 < Y, \text{ or } |\tau_{12}| < S \tag{1}$$

where X, X′, Y, Y′ and S are the respective uniaxial tensile, compressive normal and shear strengths. There is no interaction among stresses or the modes of failure with this theory, and no stress- strain relationship is assumed.

For a unidirectionally reinforced material subjected to uniaxial tension σ at some angle θ to the fibers, and using the transformation equation, the maximum allowable stress is the smallest of the following:

$$\sigma = \frac{X}{\cos^2\theta}, \sigma = \frac{Y}{\sin^2\theta}, \text{or } \tau = \frac{S}{\sin\theta\cos\theta}$$

The main advantage of the maximum stress theory is the simplicity of concept and ease of use.

In an effort to more adequately predict experimental results, Tsai and Wu [3] proposed a lamina failure criterion, adding additional stress terms which do not appear in other theories such as the Hill analysis [14]. They assumed a failure surface in stress space of the form,

$$f(\sigma) = F_i \sigma_i + F_{ij} \sigma_i \sigma_j = 1 \quad (i;j=1,2,3,4,5,6) \tag{3}$$

The F_i and F_{ij} are second and fourth order lamina strength tensors. The linear term in σ_i takes into account internal stresses which can describe the difference between positive - and negative-stress induced failures. The quadratic terms $\sigma_i \sigma_j$ define an ellipsoid in the stress-space and it is similar to those in the Tsai-Hill formulation.. The F_{ij} ($i \neq j$) terms are new. Off-diagonal terms of the strength tensor provide independent interaction among the stress components. Equation 3 can be expanded as

$$f = \left(\frac{1}{X_t}+\frac{1}{X_c}\right)\sigma_1 + \left(\frac{1}{Y_t}+\frac{1}{Y_c}\right)\sigma_2 + \left(\frac{1}{Z_t}+\frac{1}{Z_c}\right)\sigma_3 - \frac{(\sigma_1)^2}{X_tX_c} - \frac{(\sigma_2)^2}{Y_tY_c} - \frac{(\sigma_3)^2}{Z_tZ_c} + \frac{(\sigma_{12})^2}{(S_{12})^2} + \frac{(\sigma_{23})^2}{(S_{23})^2} + \frac{(\sigma_{13})^2}{(S_{13})^2} + \frac{C_{12}\sigma_1\sigma_2}{\sqrt{X_tX_cY_tY_c}} + \frac{C_{23}\sigma_2\sigma_3}{\sqrt{Y_tY_cZ_tZ_c}} + \frac{C_{13}\sigma_1\sigma_3}{\sqrt{X_tX_cZ_tZ_c}} \tag{4}$$

where

f	is the Tsai-Wu failure criterion value,
$\sigma_1, \sigma_2, \sigma_3, \sigma_{12}, \sigma_{23}, \sigma_{13}$	are the computed layer component stresses,
X_t, X_c	are the tensile and compressive strength along the 1-axis,
Y_t, Y_c	are the tensile and compressive strength along the 2-axis,
Z_t, Z_c	are the tensile and compressive strength along the third axis,
S_{12}, S_{23}, S_{13}	are the shear strength components,
C_{12}, C_{23}, C_{13}	are the coupling coefficients for Tsai-Wu.

The compressive strengths X_c, Y_c, and Z_c are taken as positive numbers. These strength values are not sufficient to determine coefficients such as F_{12}. For its determination, biaxial tests are required. The latter have to be selected carefully to obtain accurate values for such interaction terms.

Since the failure of the material is insensitive to a change of sign of shear stress (in contrast to the change of sign of normal stress) all terms containing a shear stress to first order must vanish. Therefore, the only surviving unknown coefficients are F_{12}, F_{23}, and F_{13}. Tsai and Wu proposed using biaxial tests to determine these coefficients. In the absence of other data, Tsai and Hahn [15] suggested that

$$F_{12} = -0.5\sqrt{F_{11}\,F_{22}}\,.$$

While recognizing the difficulties in evaluating F_{12}, one must realize that relatively small changes in F_{12} can significantly affect the predicted strength.

The Tsai-Wu criterion is a significant improvement over previous criteria because of its generality, versatility and its good fits with test data. Nevertheless, its utilization and interpretation raise some intrinsic problems. Underlying these is the fact that a fiber composite consists of mechanically dissimilar phases : stiff elastic brittle fibers and a compliant yielding matrix. Consequently, the failure occurs in very different modes. Thus the fibers may rupture in tension or buckle in compression, or the matrix may fail due to loads transverse to the fibers. It is not evident that all of the distinct failure modes can be represented by a single smooth function such as Tsai-Wu.

MODEL AND FINITE ELEMENT ANALYSIS

The model shown in Figure (1) is a part of the first wall of a tokamak fusion reactor made from SiC/SiC composite [16]. The first wall coolant-channel wall thickness is 1 mm with an additional 2 mm-thick CVD layer of SiC facing the plasma. This layer provide a highly dense barrier to coolant leaks. The composite laminates provide some unique behaviors. Varying the ply arrangement in thickness, orientation or stacking sequence would greatly alter laminate performance. Optimization provides us with a tool to actually design the material by tailoring the structural properties in an efficient manner. The optimum design may make a laminated structure meet some particular requirements with little waste of material. However, the optimization for laminates is more complicated than that for conventional isotropic materials.

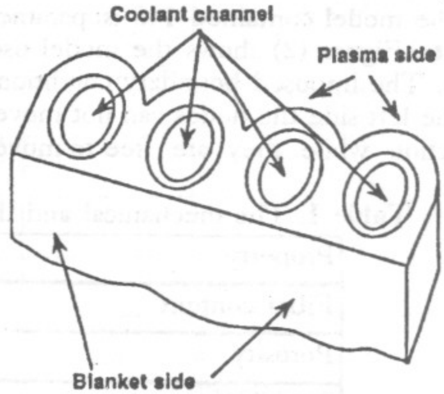

Figure 1. Schematic diagram for a part of the plasma facing of tokamak fusion reactor.

The failure analysis is particularly difficult, due to the many failure modes possible, and the many theories developed to account for these failure modes. Optimization of thick multilayer cylinders reveals that the radial layer sequence prediction is critical for maximizing burst pressure. This prediction can only be done by 3-D analysis, since 2-D analysis can not differentiate one sequence from another [17].

In order to obtain an optimum design with respect to the orientation of the fiber and the stacking sequence an automated design method is used. The laminate can be optimized with respect to the fiber orientation and the stacking sequence of each ply to minimize the peak stress. An optimization method, which is based on failure criterion is used to design the model. Three types of variables characterize the design problem, the objective function (the function to be minimized, in our case it is the failure criterion), the design variables (in our case they are the fiber orientation and the stacking sequence), and the state variables which represent the constraints of the model. The design variables are independent variables and subjected to upper and lower limits while the objective function and the state variables are dependent variables. State variables are also constrained by upper and lower limits. In this analysis the fiber direction 45° is chosen as basic fiber orientations. The mechanical and thermal properties of the material are shown in Table (1) [16].

A three dimensional finite element formulation is used in this analysis. Due to the symmetry of the geometry and the applied loads, the problem can be reduced to the calculation for half unit from the first wall. The model contained 145 isoparametric brick elements. Figure (2) shows the model used in this analysis. The imposed boundary conditions are:

- At the left side the nodes can not move in the x-direction while they are free to move in the y-direction.
- At the right side the nodes have coupled in the x-direction (all the nodes should move the same distance).

The first wall is subjected to severe mechanical and thermal loads. The pressure inside the tube is 10 MPa . Table (2) represent the thermal loads applied to the model.

Table 1. The mechanical and thermal properties of Laminated SiC/SiC composite [16].

Property	23 °C	1000 °C	1400 °C
Fiber content	40%	40%	40%
Porosity	10%	10%	41%
Tensile strength	200 MPa	200 MPa	150 MPa
Young's modulus, tensile	230 GPa	200 GPa	170 GPa
Poisson's ratio n_{12} n_{13}	 0.05 0.18	 - -	 - -
Compressive strength in plane through the thickness	 580 MPa 420 MPa	 480 MPa 380 MPa	 300 MPa 250 MPa
Interlaminar shear stress	40 MPa	35 MPa	25 MPa
Thermal expansion coefficient in plane through the thickness	 $3.0x10^{-6}$ /K $1.7x10^{-6}$ /K	 $3.0x10^{-6}$ /K $3.4x10^{-6}$ /K	 - -
Thermal conductivity in plane through the thickness	 19 W/K.m 9.5 W/K.m	 15.2 W/K.m 5.7 W/K.m	 - -
Specific heat	650 J/K.kg	1200 J/K.kg	-
Fracture toughness	30 $MPa.m_{1/2}$	30 $MPa.m^{1/2}$	30 MPa.m1/2

Table 2. The thermal loads applied to the first wall.

Volumetric heating	23.6 MW/m^3
Heat flux on the plasma side	0.55 MW/m^2
Heat flux on the blanket side	0.17 MW/m_2
Heat transfer coefficient inside the Coolant channel	2200 $W/K.m^2$
Blanket temperature	350 C°
Coolant temperature	650 C°

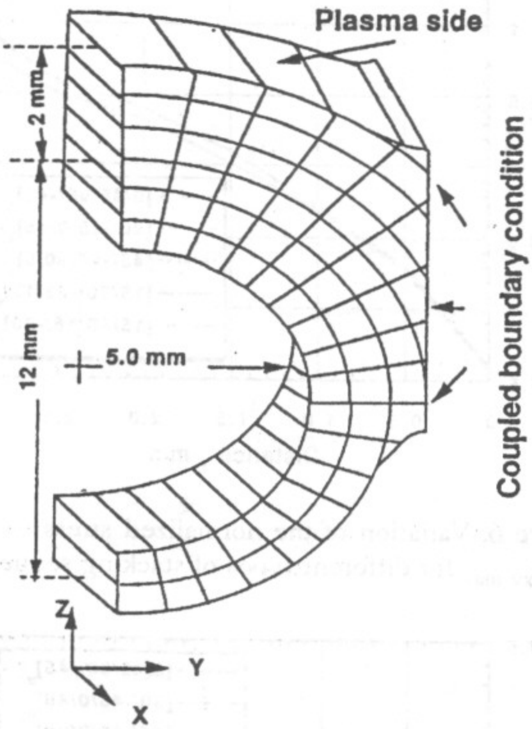

Figure 2. Finite element model for analysis.

RESULTS

Finite element analysis is used to find the internal forces and stresses in the composite structures. An algorithm optimization technique is used to find the optimum fiber orientation for our model in order to achieve a minimum peak stress in the structure, according to the maximum stress and Tsai-Wu failure criteria. The optimization technique is applied to the model starting with fiber orientation 45° as a base direction. The optimized fiber orientation is found to be $[15°/20°/-23°/13°]_s$ and for another sequence is $[15°/10°/18°/-30°]_s$.

Symmetric angle-ply laminates are built up by different fiber orientation and stacking sequence. A comparison between a traditional fiber orientation $[0°/45°/90°/-45°]_s$ with different stacking sequence and the optimized ones are presented. The optimization technique based on the nonlinear properties to achieve a minimum peak stresses. Two failure criteria are used in this analysis; the maximum stress criterion and Tsai-Wu criterion.

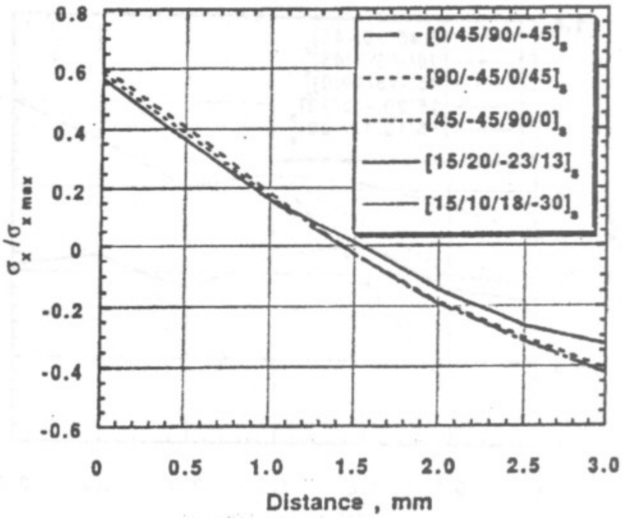

Figure 3. Variation of the normalized stress ($\sigma_x/\sigma_{x\,max}$ for different types of stacking sequence.

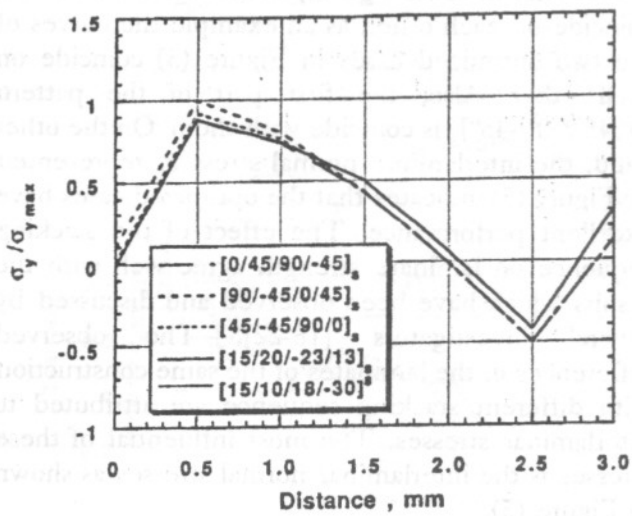

Figure 4. Variation of the normalized stress ($\sigma_y/\sigma_{y\,max}$ for different types of stacking sequence.

Five different sets of fiber orientations are used in this analysis. Three of them are traditional ones with different sequences, they are $[0°/45°/90°/-45°]_s$, $[90°/-45°/0°/45°]_s$ and $[45°/-45°/90°/0°]_s$. The other two are coming from the optimization technique which are $[15°/20°/-23°/13°]_s$ and $[15°/10°/18°/-30°]_s$. Figures (3) and (4) represent the normal stresses σ_x and σ_y and as shown the changes are very small for the different cases.

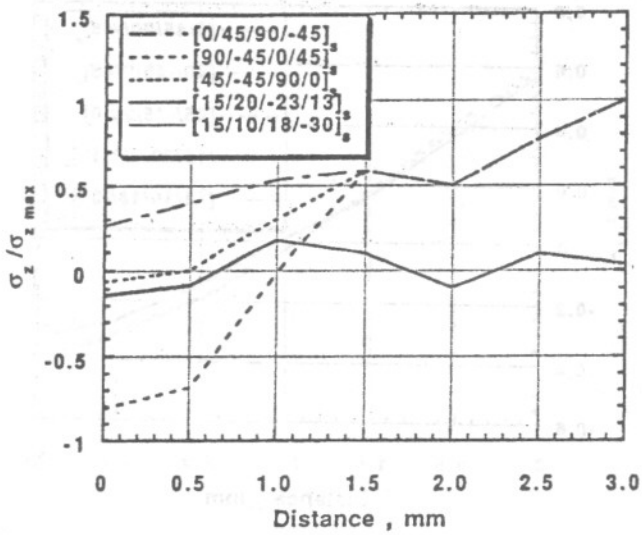

Figure 5. Variation of the normalized stress ($\sigma_z/\sigma_{z\ max}$ for different types of stacking sequence.

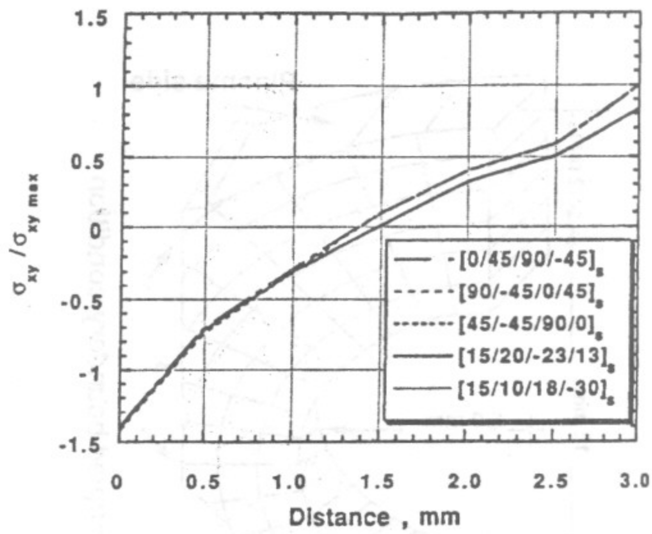

Figure 6. Variation of the normalized stress ($\sigma_{xy}/\sigma_{xy\ max}$ for different types of stacking sequence.

Note that in some figures, there are curves which coincide on each other, as an example the curves of the two optimized cases in Figure (3) coincide on each other. Also, the first part of the pattern $[0°/45°/90°/-45°]_s$ is coincide with them. On the other hand, the interlaminar normal stress σ_z represented by Figure (5) indicates that the optimized cases have excellent performance. The effect of the stacking sequence on laminate strength agree well with the results which have been observed and discussed by several investigators [18-22]. The observed differences in the laminates of the same construction with different stacking sequence are attributed to interlaminar stresses. The most influential of these stresses is the interlaminar normal stresses as shown in Figure (5).

Figure (6) represents the plane shear stress which indicate the effect of the applied loads on the bonding between the constituents of the composite material. As shown in this figure the influence of this stress appear more effective near the outer side of the model.

Figures (7), and (8) represent the shear stresses used to evaluate the interlaminar shear stress. In Figure (7) the most pronounced differences were observed in the layup $[45°/-45°/90°/0°]_s$. One of the explanations for this behavior is the existence of adjacent $\pm$ 45° layer creating a large mismatch between the material properties of the plies of the laminate.

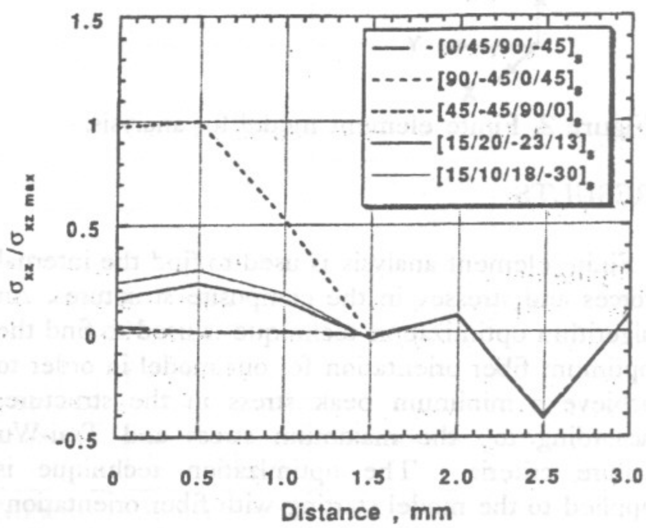

Figure 7. Variation of the normalized stress ($\sigma_{xz}/\sigma_{xz\ max}$ for different types of stacking sequence.

The interlaminar shear stresses, as illustrated in Figure (9), indicate that there is a noticed difference between the different stacking sequences of the same construction. The differences in Poisson's ratio between the various plies result in transverse stresses. These stresses are equilibrated by interlaminar shear stresses and couples produced by the interlaminar normal stresses.

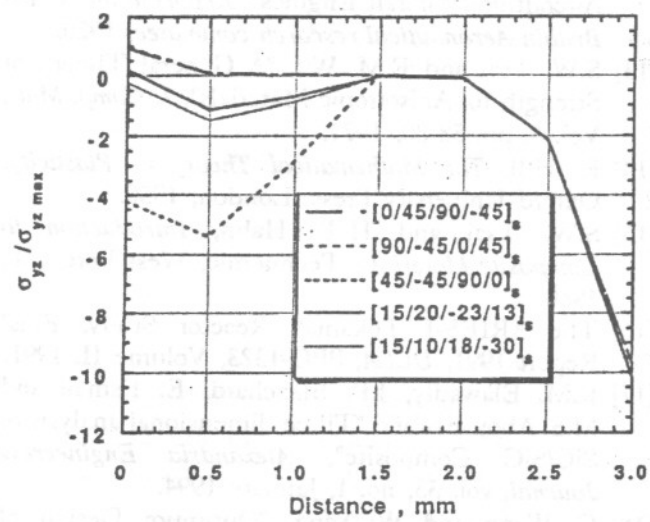

Figure 8. Variation of the normalized stress ($\sigma_{yz}/\sigma_{yz\,max}$ for different types of stacking sequence.

The varying of the maximum stress and Tsai-Wu failure criteria with different layups are illustrated in Figures (10), and (11). It is clear from these figures that the optimized layups have the best behavior under the same situation while one of the traditional layup, $[0°/45°/90°/-45°]_s$ failed toward the coolant side of the first wall.

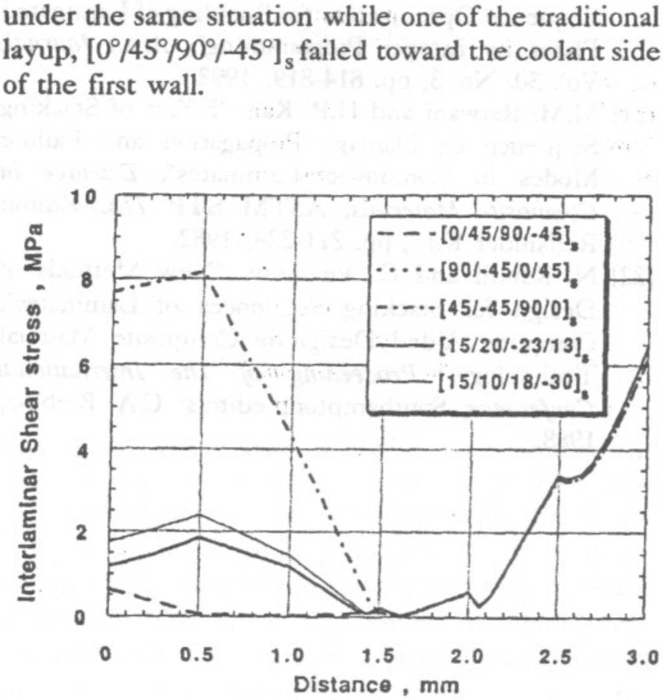

Figure 9. Variation of the interlaminar shear stress along the thickness of the first wall.

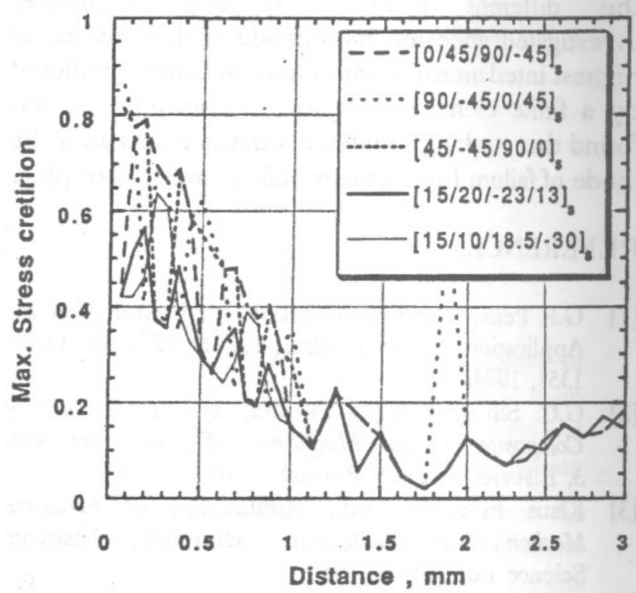

Figure 10. Variation of the maximum stress failure criterion through the thickness of the first wall.

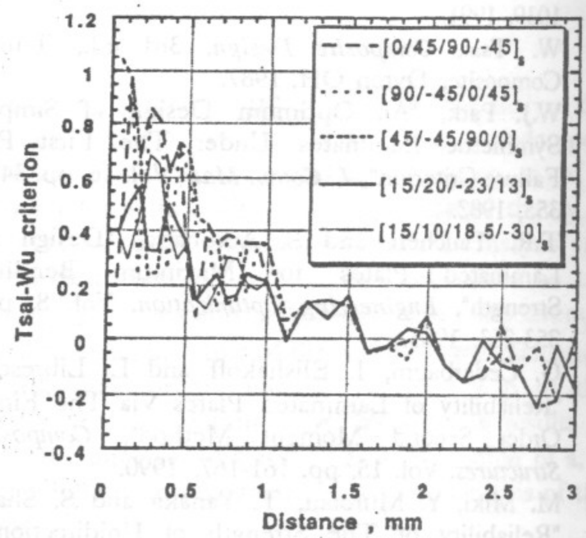

Figure 11. Variation of the Tsai-Wu failure criterion through the thickness of the first wall.

CONCLUSION

The main objective in the design of laminated fiber reinforced composite is the selection of suitable stacking sequences of layers, so as to fulfill some required properties. The results indicate that the strength of the composite laminates depends on the stacking sequence. The location of failure mode for the different laminate stacking sequences investigated appeared to coincide with positions of highest interlaminar shear or normal stress predicted by a finite element analysis. Furthermore, it was found that stacking sequence variations can alter the mode of failure from catastrophic to noncatastrophic.

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