A STUDY ON SEEPAGE FLOW FROM AN ARRAY OF TRIANGULAR CHANNELS

M.A. Mahmoud
Structural Engineering Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

The paper deals with a two dimensional flow problem of steady flow under gravity from an array of similar triangular channels. These channels are dug at equal spacings in a semi-pervious clay layer, of finite depth, underlain by a highly permeable layer of sand or gravel (aquifer), in which different piezometric head levels are considered. The boundary element method was used for evaluating the seepage discharge, distribution of velocity along the entrance and exit boundaries, location of the phreatic surface and velocity fields within the studied domain. The results indicated that, in planning a newly cultivated areas and in order to keep the location of the phreatic surface down between the channels, the irrigation channels should be dug, as possible, farthest away from each other. The results are also compared with the available experimental and theoretical results.

INTRODUCTION

Nowadays a big projects of new villages and new cultivated lands are going on in many places in Egypt. Irrigation channels of thousands kilometers in length were planned to be dug. In most of these agricultural areas, a surface layer of finite depth of relatively low permeability is underlain a highly pervious layer (aquifer). The water inside these aquifers is, usually, subjected to artezian pressure. The seepage loss from these channels, usually, occurs under the effect of gravity, and considered quantities of water may be lost if the surrounding soil is of high hydraulic conductivity. The seepage loss from these channels is considered to be a vital task in planning for a new developed agricultural areas, so a comparison between the cost of lining of these channels and the lost quantity of water should be investigated beforehand.

The problem of seepage from open channels through semi-pervious clay layer, underlain by a highly freely one was considered analytically by many authors, for instance, Bruch and Street [1] and [2], Hammad [3], El Nimr [4] and Hathoot [5]. Hammad [3] presented a solution of seepage loss from an array of parallel channels but he did not take into consideration the particular geometric shape of these channels. He also considered the free water surface is horizontal and slightly below the water surface in the channels, however El Nimr [4] presented a solution in which he considered the geometric channel shape. All of the above solutions used the conformal mapping technique for predicting the discharge and also shape of the free stream line. Bruch and Street [2] reported a number of numerical examples on 90° triangular array of channels. Hathoot [6] carried out experimental investigation covering a wide range of variables using a Hele-Show model and compared his experimental results with those obtained by Bruch and Street [2]. Hathoot [6] found that the theoretical discharge ratio results reported by Bruch and Street [2] were found to be higher than those recorded experimentally by a percent not exceeded 32 %.

The boundary element method was implied in dealing with many potential problems, for instance, Brebbia [7] and Abdrabbo and Mahmoud [8]. In the present study, the boundary element method has been used successfully for predicting the discharge, phreatic surface and velocity fields within the studied domain. The effect of the magnitude of the piezometric head within the aquifer on the flow within the semi-pervious clay layer was also considered. The present theoretical results have been compared with both the experimental results reported by Hathoot [6] and the theoretical results reported by Bruch and Street [2].

GOVERNING EQUATION

Figure (1) shows the geometry of the problem, whereas figure (2) demonstrates the idealization of the problem in which the surface (s) of the domain (D) is discretised into linear elements, each is represented by a node at its center and the potential u(x) is expressed as,

$$u(x) = \frac{p(x)}{\gamma} + x2(x) \qquad (1)$$

Where,

$p(x)$ hydrostatic pressure at point (x_i).
γ Fluid specific weight.
$x2(x)$ position head.

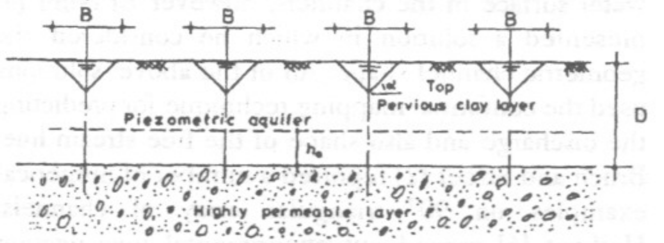

Figure 1. Physical section.

The boundary element formulation is given by Brebbia [7] as,

$$\alpha u_i + \int_s u\, q^* \, ds = \int_s q\, u^* \, ds \qquad (2)$$

where,

$q^* = \partial u / \partial n$
$\alpha = \omega / 2\pi$
ω integral angle of the surface at point (x) $\alpha = 1$ in the domain D.

Equation (2) was placed in a discretised form and written for each node (i), and thus n equations have been obtained and solved for the unknown boundary values of u and q.

Assuming AB a datum, the boundary conditions are as follows:

- The flux q normal to the lines of symmetry BC and AF is equal to zero.
- The flux q normal to the free water surface ED is taken as zero.
- The potential u along the submerged surface of the channel is taken as u=h.

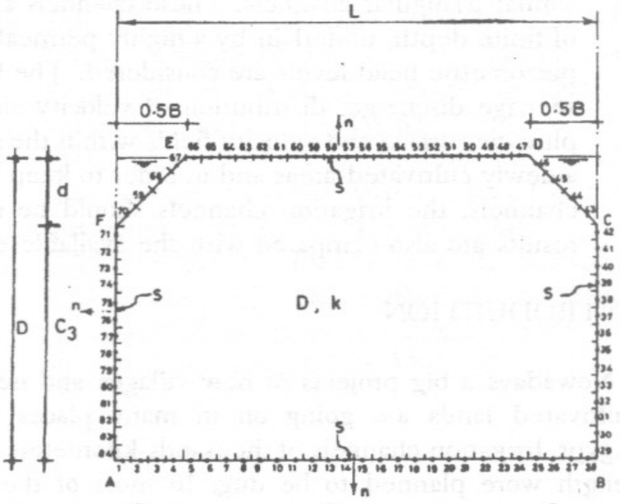

Figure 2. Discretization of the problem.

Introducing these boundary conditions, the solution gives the flux q along the submerged side slope of the channels and along the interface between the semi-pervious clay layer and the aquifer. A computer program was written to handle this problem.

DISCUSSION OF RESULTS

1. Seepage discharge

Figure (3) shows the variation of discharge ratio q/kB (where k is the coefficient of permeability) with both spacing ratio L/B and depth ratio D/B for two different values of piezometric head ratios h_o/D of 0.0 and 0.5 . It is obvious from figure (3) that (when the spacing ratio L/B is less than 2.5) as D/B increases, the discharge ratio decreases. However, beyond that limit it was found that as L/B increases and D/B increases, the discharge ratio q/kB increases. This is expected since any excess in the thickness of the semi-pervious clay layer is associated, eventually, with the increase in potential causing the flow and consequently resulting an increase in the quantity of flow. It can be concluded that when L/B is less than 2.5 (i.e the channels are adjacent), a reversed behaviour and inappreciable variation of seepage ratio are found. This behaviour is mainly due to the interaction of flow between the channels when they are located so close.

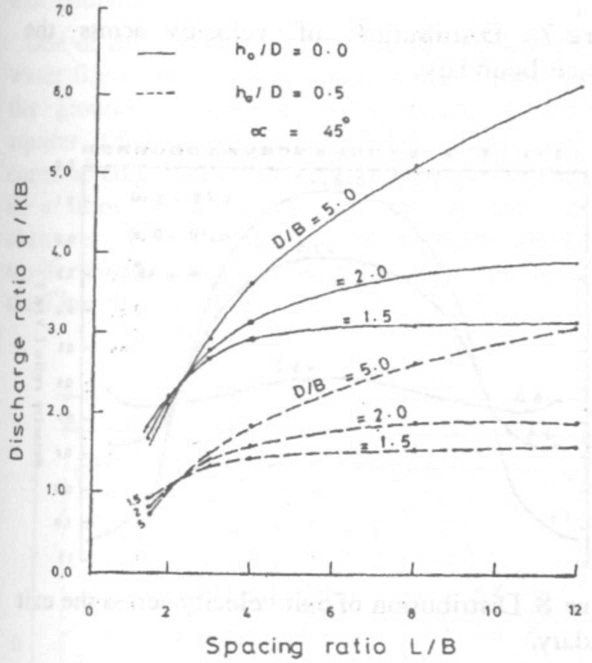

Figure 3. Discharge ratio versus spacing ratio.

Figure (4) demonstrates the variation of discharge ratio q/kB with spacing ratio L/B for different piezometric head ratios h_o/D of 0.0, 0.25, 0.5 and 0.75. It is clear from figure (4) that as the spacing ratio L/B increases and piezometric head ratio h_o/D decreases, the discharge ratio q/kB increases. It is worth to note that when the piezometric head is created within the aquifer so that $h_o/D>1.0$, a reversed flow is expected and the open channels will be gaining rather than loosing.

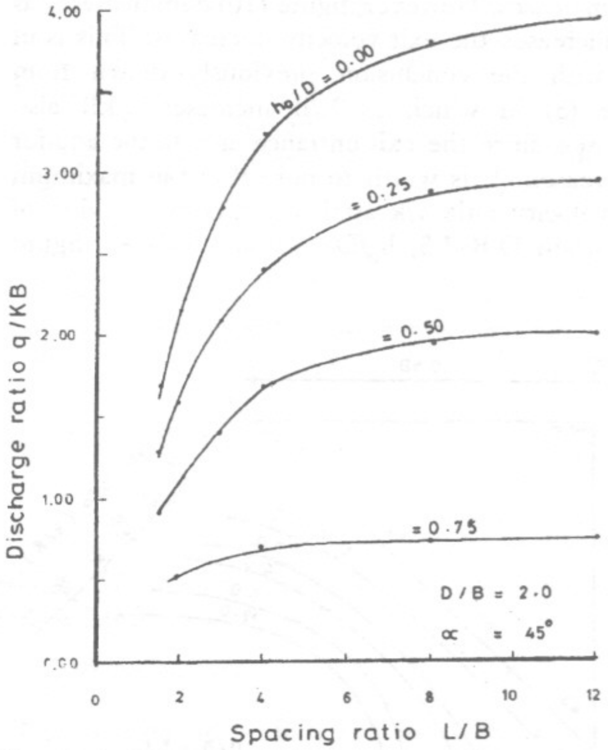

Figure 4. Discharge ratio versus spacing ratio.

2. Velocity field

Figures (5), (6) and (7) illustrate the distribution of entrance velocities along the open channel sides with the variations of L/B, D/B and h_o/D respectively. It can be seen that as both L/B , D/B increases and h_o/D decreases, the discharge ratio increases. The first glance to figure (5) indicates that the velocity ratio v/k attains a maximum value of 7 when D/B =5, L/B=12, $h_o/D=0.0$ and $\alpha=45°$. It is worth to note that the distribution of velocity along the side slope of the channel is not uniform. The lowest value is at the mid-point of side slope whereas the higher values are at the ends of side slope.

Figures (8-11) demonstrate the distribution of exit velocity along the interface boundary between the top semi-pervious clay layer and the aquifer. It can be seen from figure (8) that as the thickness of clay layer increases, the exit velocity tends to be uniform, however for smaller values of D/B, the seepage water enters the aquifer with relatively high velocity in the regions under the channels and with a relatively slower velocities in the regions between the channels. It is clear from figures (8-9) that as

D/B increases as well as h_o/D decreases, the velocity ratio increases. However, figure (10) demonstrates as L/B increases the exit velocity decreases. This is in line with the conclusion previously drawn from figure (3) in which as L/B increases q/kB also increases, since the exit entrance area to the aquifer is increased. It is worth to note that the maximum exit velocity ratio v/k attains a maximum value of 1.03 when D/B=1.5, h_o/D= 0.0 and L/B =8 ,figure (8).

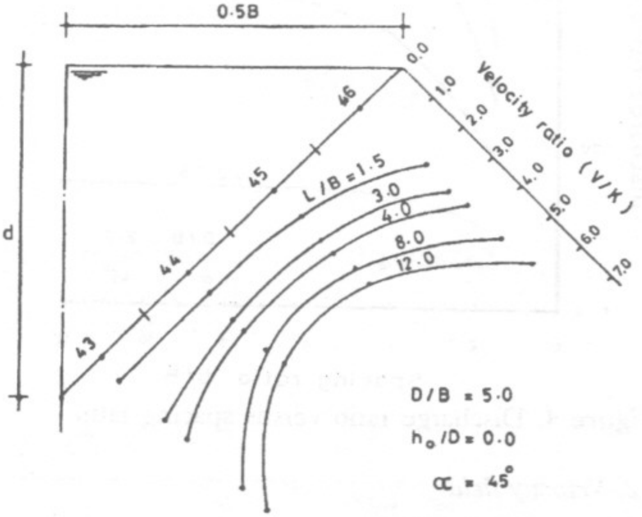

Figure 5. Distribution of velocity along the entrance boundary.

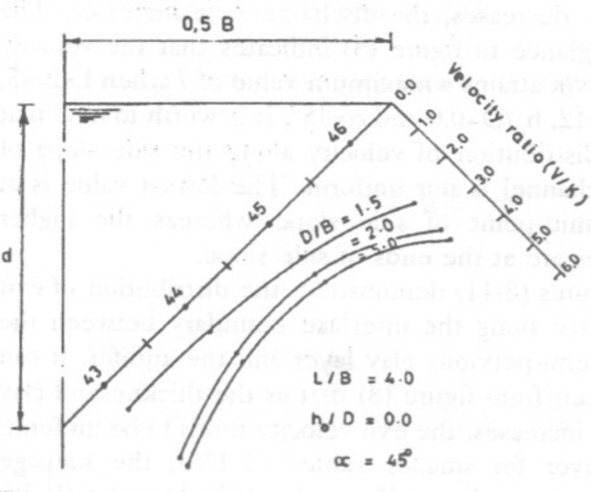

Figure 6. Distribution of velocity across the entrance boundary.

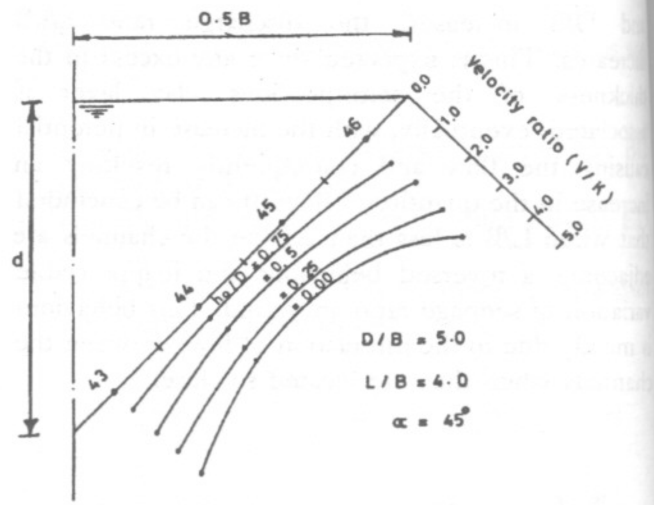

Figure 7. Distribution of velocity across the entrance boundary.

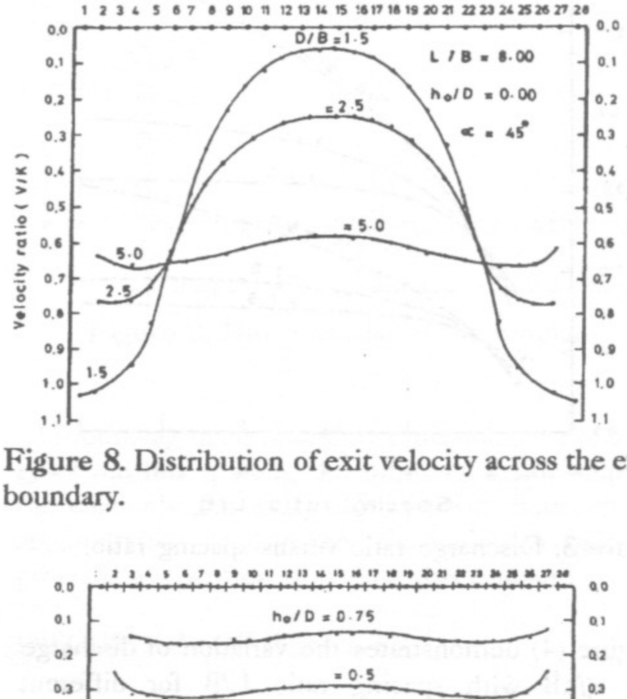

Figure 8. Distribution of exit velocity across the exit boundary.

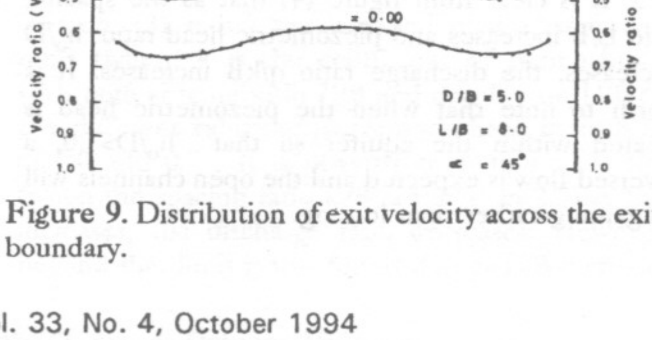

Figure 9. Distribution of exit velocity across the exit boundary.

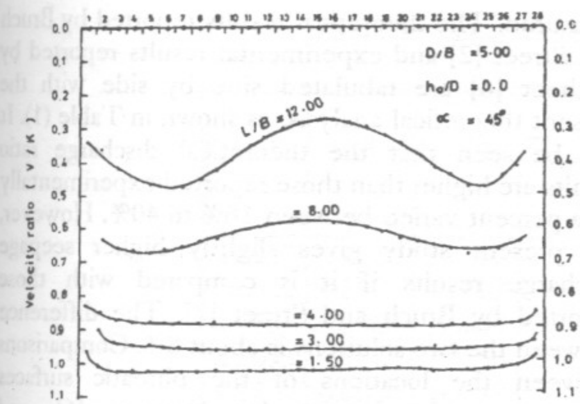

Figure 10. Distribution of exit velocity across the exit boundary.

One of the possible method to get rid of seepage water (i.e to keep the phreatic surface away below the ground surface) is to construct an artificial aquifer (drain). This artificial aquifer may be constructed by installing perforated pipes embedded in a filter, at a depth, underneath the bed of channels. The distribution of velocity along the aquifer surface may be used for a proper design of the filter of that drain.

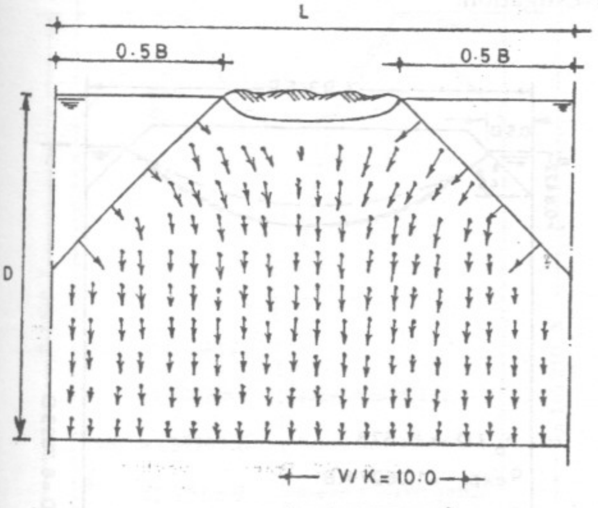

Figure 11. Velocity fields (L/B = 1.5, h_o/D = 0.0 & D/B = 2.0).

Figures (11) and (12) show the dynamic pattern of velocity fields within the studied domain for two cases. The first case for L/B=1.5 whereas the second case for L/B=12.0. The two patterns of flow demonstrate that as the distance between the adjacent channels increases, the interaction between them decreases and each channel works individually at L/B=12.0. It can be seen that when the channels are located so close, the flow goes vertically downwards towards the aquifer. However when the channels are constructed far away from each other the flows goes downward and away from the channel.

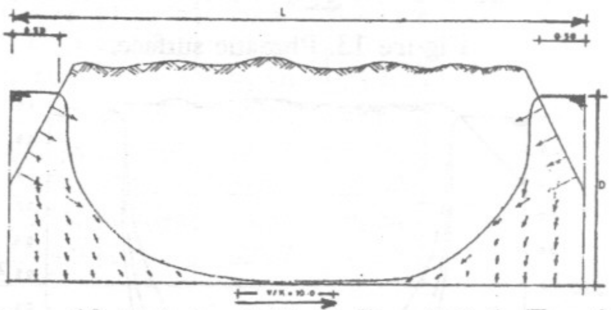

Figure 12. Velocity fields (L/B = 12.0, h_o/D = 0.0 D/B = 2.0 & $\infty = 45°$).

3. *Phreatic surface*

The location of the initial arbitrary free surface is initially assumed to be horizontal and coincides with the water level in the open channels. Iteration process was applied. After enough iterations, a steady state has been achieved and the location of the free water surface is determined precisely. Figures (13-15) show plots of the steady state location of the phreatic surface, each for typical configurations outlined in its figure. It is obvious from figures (13-15) that as the ratio L/B increases as well as D/B and h_o/D decreases, the entire position of the free surface moves downwards achieving maximum potential values at the channel sides and a minimum value at the mid-way between the channels. It is worth to say that the horizontal velocity components across the mid-plane between the channels are equal to zero since the inclination of the tangent to the horizontal surface is equal to zero. It can be seen from figure (13) that the free surface touches the plane of interface between the top semi-pervious clay layer and the aquifer when (L/B=12). Thus it can be concluded that each channel works individually at L/B=12, D/B=1.5 and h_o/D= 0.0. Practically in planning an array of channels, the spacing between them should be greater than 12B otherwise, eventually, the phreatic surface will raise resulting, a spoiling of the agricultural areas between them.

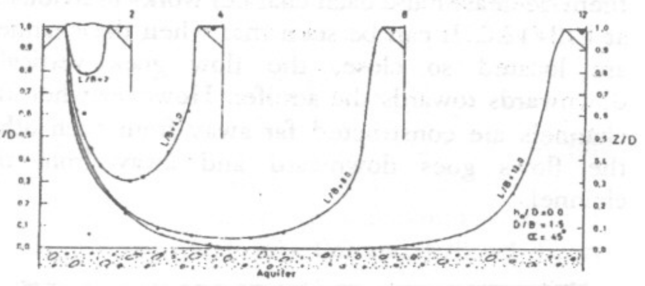

Figure 13. Phreatic surface.

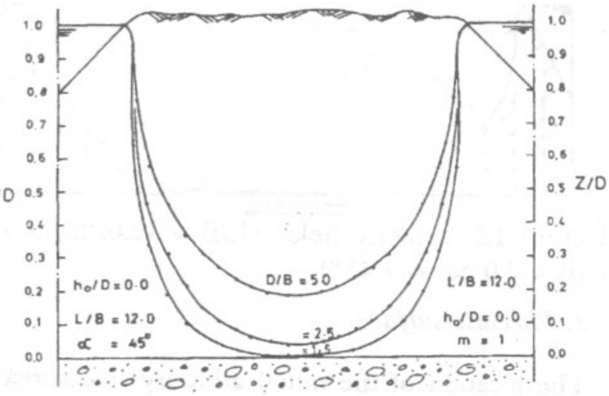

Figure 14. Phreatic surface.

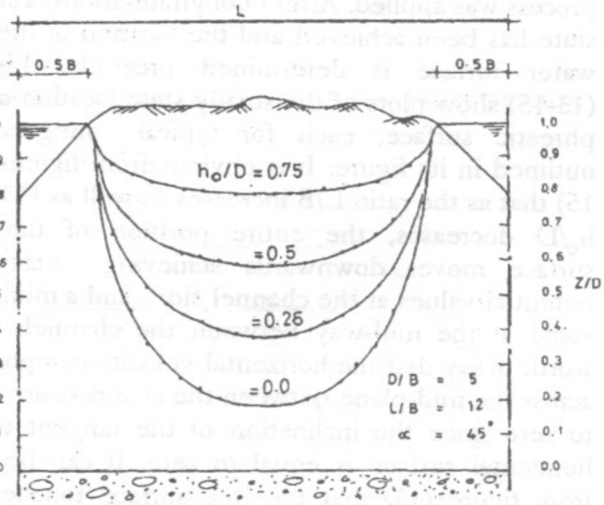

Figure 15. Phreatic surface.

COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL RESULTS

Both the present analysis and Bruch and Street solution [2] have neglected the effect of the capillary fringe and thus the computed discharge values are expected to be smaller than that experimentally measured. The theoretical results reported by Bruch and Street [2] and experimental results reported by Hathoot [6] are tabulated side by side with the present theoretical analysis, as shown in Table (1). It can be seen that the theoretical discharge ratio results are higher than those reported experimentally by a percent varied between 16% to 40%. However, the present study gives slightly higher seepage discharge results if it is compared with those reported by Bruch and Street [2]. The difference between the two solutions is about 8%. Comparisons between the locations of the phreatic surfaces recorded experimentally by Hathoot [6], and reported theoretically by Bruch and street [2] and that calculated numerically from the present investigation are demonstrated in figures (16) and (17). In figure (16) the theoretical and experimental results of the free surfaces are almost coincided but the free surface of the present investigation is pulled slightly upwards. In figure (17), Bruch and Street [2] phreatic surface is considerably shifted specially near the line of symmetry mid-way between the channels if it is compared with both that measured from experimental results presented by Hathoot [6] and numerically calculated from the present investigation.

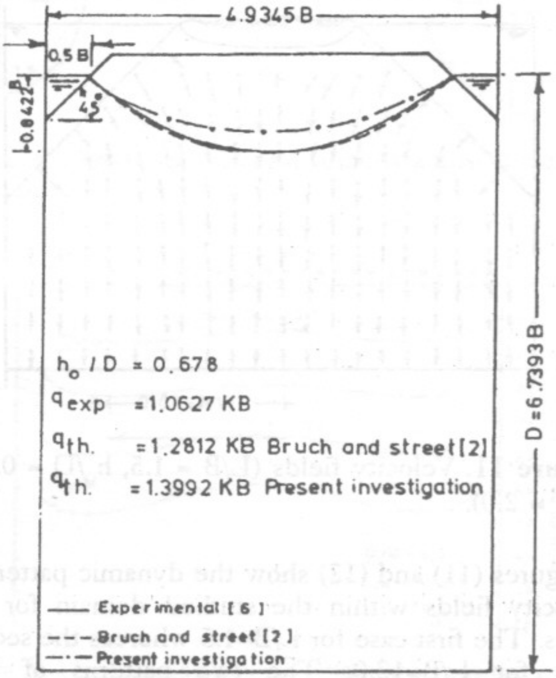

Figure 16. Phreatic surface.

Table 1. Theoretical discharge vs experimental discharge.

q/B	h_o/D	L/B	q/kB				
			Experimental	Theoretical		Difference (%)	
			(a)	(b)	(c)	(b)	(c)
1.2117	0.2622	1.6000	1.3652	1.61831	1.5890	18.54	16.39
1.5022	0.3078	1.6867	1.3186	1.5898	1.5948	20.57	20.95
2.0157	0.3068	1.6654	1.3231	1.6031	1.5992	21.16	20.87
2.5093	0.0775	1.9056	1.5965	1.9446	2.0003	21.80	25.40
2.9205	0.0008	1.9779	1.4582	2.0001	2.1087	37.16	44.61
3.5023	0.2102	2.0789	1.5191	1.9584	2.0497	28.92	34.92
3.9836	0.0414	2.0919	1.6254	1.9994	2.1671	23.01	33.30
4.4485	0.4891	2.9136	1.3624	1.7604	1.9024	29.21	39.64
5.7761	0.0005	1.9996	1.5121	2.0000	2.0833	32.27	37.78

(a) Hathoot [6]
(b) Bruch and Street (2)
(c) Present investigation.

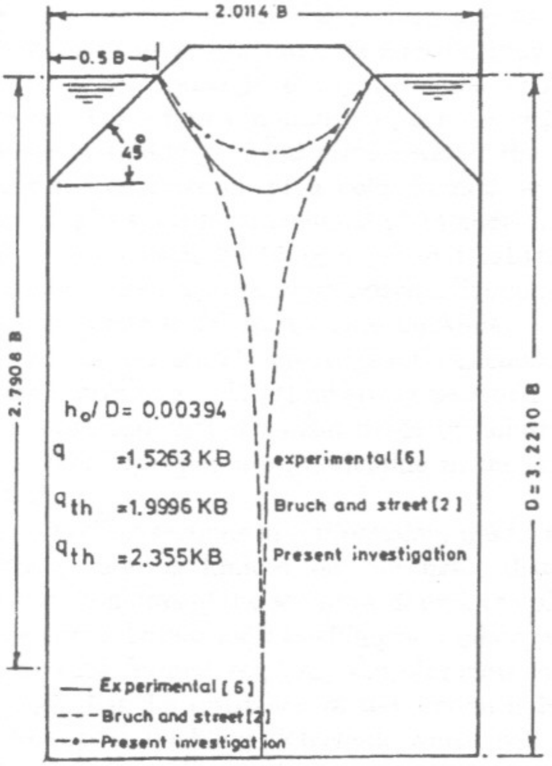

Figure 17. Phreatic surface.

CONCLUSIONS

The problem of flow from an array of triangular channels has been undertaken, the following are the main concluded points:

1. The versatility and simplicity of the boundary element method enables to solve the problem of seepage flow from an array of triangular channels.
2. In planning a newly cultivated area, the open channels should be dug, as possible, farthest away from each other with spacing not less than 12B.
3. One of the possible method to collect the seepage water is to construct an artificial aquifer (i.e drain) using perforated pipes, embedded in a filter underneath the open channels at a depth.
4. The seepage discharge ratio increases as spacing ratio L/B increases, depth ratio D/B increases and piezometric head ratio h_o/D decreases and when $h_o/D > 1.0$, the channels will be gaining rather than loosing.
5. Comparisons between present analysis, Bruch and Street solution [2] and experimental results reported by Hathoot [6] have been demonstrated. The discharge ratios obtained from the present analysis are found to be higher than those reported by Bruch and Street [2]. The difference

is about 8%. Theoretical seepage discharge ratios are found to be considerably greater than that reported experimentally. Differences are generally between 16% and 40%.

6. Phreatic surfaces predicted from the present analysis are nearly coincided with those reported experimentally by Hathoot [6]. However, the theoretical results reported by Brush and Street [2] are deviated considerably when h_o/D approaches zero.

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