DETERMINATION OF THE INITIAL DATA FOR TRIANGULATION NET WITHOUT BASE-LINE

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ABSTRACT

The present study gives a simplified solution to determine the initial data (base-line length and azimuth) for triangulation nets of low-order connected to a higher-order triangulation net. This may be achieved through the knowledge of the coordinates of two far-apart stations included in the lower-order triangulation net. The author gives a mathematical treatment for determining all the elements of the initial data in this case, and accordingly, the triangulation adjustment may be performed. A sample numerical example is also presented according to the suggested formulas.

1. INTRODUCTION

A triangulation system basically consists of a configuration of triangles all of whose angles have been directly measured. Sides whose lengths are actually measured are known as basis or baseline. The survey points or triangulation stations are located at the vertices of the triangles. By the use of the measured angles and bases, the lengths of all other sides in a connected system can be successively determined by trigonometry.

A cardinal principle in the extension of horizontal control is that of initially establishing master frameworks of reference, such as networks of high-order triangulation and subsequently subdividing such networks into secondary and tertiary systems of successively lower quality. This principle is sometimes termed "working from the whole to the part" [4].

In some practical cases to adjust a low-order triangulation network connected with a higher-order net, the initial data required may be missing and unknown due to various reasons.

In accordance, the present study aims to give a simplified procedure to obtain the initial data required for the adjustment of a secondary triangulation scheme. This procedure involves a mathematical treatment where the known coordinates of two stations of the network are used. These two stations do not represent the end-points of a base-line, but rather, are located in the triangulation net far apart from each other.

2. METHOD OF ANALYSIS

In a narrow triangulation system a chain of figures is employed, consisting of two triangles and a central-point figure (Figure (1)).

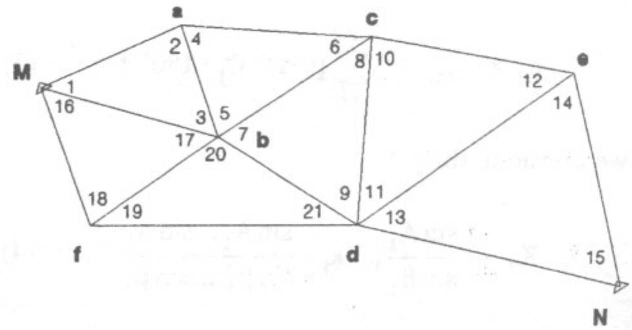

Figure 1.

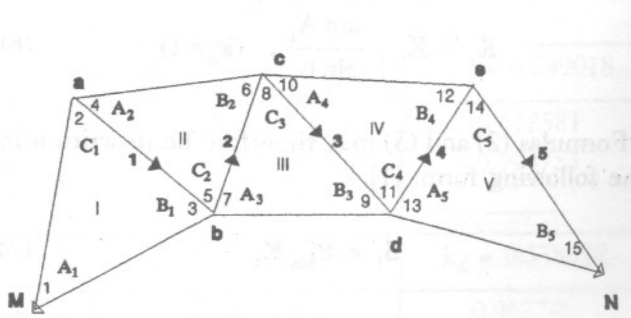

Figure 2.

From (Figure (1)) the chain of single triangles shown in (Figure (2)) is chosen; there is but one route by which distances can be computed through the chain. Also points M and N are fixed, and the coordinate-differences between them are ΔX_{MN}, ΔY_{MN}. All the angles of the triangles are denoted A_i, B_i and C_i.

From the length of the triangle sides Ma, 1,2,3,4,5 and through the route Ma-ab-bc-cd-de-eN in the chain (Figure (2)), the difference in coordinates between points M and N may be obtained from the following equation [3]:

$$\left.\begin{aligned}\Delta X_{MN} &= S_{Ma}\cos\alpha_{Ma} + S_1\cos\alpha_1 + \ldots + S_n\cos\alpha_n\\ \Delta Y_{MN} &= S_{Ma}\sin\alpha_{Ma} + S_1\sin\alpha_1 + \ldots + S_n\sin\alpha_n\end{aligned}\right\} \quad (1)$$

where: S_{Ma}, α_{Ma} are the length and azimuth of side Ma, and S_i, α_i are the length and azimuth of side "i".

To obtain the elements S_{Ma} and α_{Ma}, the following procedure is made [1]

$$S_i = S_{Ma}\,\frac{\sin A_1 \sin A_2 \ldots \sin A_i}{\sin\beta_1 \sin\beta_2 \ldots \sin\beta_i} \quad (2)$$

and

$$\alpha_i = \alpha_{Ma} + \sum_{j=1}^{i}\left[(-1)^j\, C_j + 180^\circ\right] \quad (3)$$

If we consider that:

$$K_1 = \frac{\sin A_1}{\sin\beta_1}, \quad k_i = \frac{\sin A_1 \ldots \sin A_i}{\sin\beta_1 \ldots \sin\beta_i} \quad (4)$$

$$Z_i = \sum_{j=1}^{i}(-1)^{j+1}\, C_j, \quad (5)$$

$$K_i = K_{i-1}\,\frac{\sin A_i}{\sin\beta_i}, \quad (k_o = 1) \quad (6)$$

Formulas (2) and (3) may therefore be re-written in the following form: [1]

$$S_i = S_{Ma}\, K_i \quad (7)$$

$$\alpha_i = (\alpha_{Ma} - Z_i) + 180^\circ . i \quad (8)$$

After simple conversion, formula (1) can be finally expressed in the form:

$$\left.\begin{aligned}\Delta X_{MN} &= \Delta X_{Ma}[1 - K_1\cos Z_1 + \ldots + (-1)^n K_n \cos Z_n] +\\ &\quad + \Delta Y_{Ma}[-K_1 \sin Z_1 + \ldots + (-1)^n K_n \sin Z_n],\\ \Delta Y_{MN} &= \Delta Y_{Ma}[1 - K_1\cos Z_1 + \ldots + (-1)^n K_n \cos Z_n] -\\ &\quad - \Delta X_{Ma}[-K_1 \sin Z_1 + \ldots + (-1)^n K_n \sin Z_n]\end{aligned}\right\} \quad (9)$$

To simplify the form of equation (9), consider that

$$\left.\begin{aligned}\epsilon_i &= (-1)^i\ K_i \,.\, \cos Z_i\\ \eta_i &= (-1)^i\ K_i \,.\, \sin Z_i\end{aligned}\right\} \quad (10)$$

also, for further simplification,

$$\left.\begin{aligned}E &= 1 + \sum_{i=1}^{n}\epsilon_i,\\ H &= \sum_{i=1}^{n}\eta_i\end{aligned}\right\} \quad (11)$$

Accordingly, formula (9) will have the form:

$$\left.\begin{aligned}\Delta X_{MN} &= E\ \Delta X_{Ma} + H\ \Delta Y_{Ma}\\ \Delta Y_{MN} &= -H\ \Delta X_{Ma} + E\ \Delta Y_{Ma}\end{aligned}\right\} \quad (12)$$

Through equation (12) the components of side Ma are determined [2]:

$$\left.\begin{aligned}\Delta X_{Ma} &= \frac{E\,\Delta X_{MN} - H\,\Delta Y_{MN}}{E^2 + H^2}\\ \Delta Y_{Ma} &= \frac{E\,\Delta Y_{MN} + H\,\Delta X_{MN}}{E^2 + H^2}\end{aligned}\right\} \quad (13)$$

Thus, the values of S_{Ma}, α_{Ma} for triangle I (Figure (2)) are given by:

$$S_{Ma} = \sqrt{\frac{\Delta \bar{X}^2_{MN} + \Delta \bar{Y}^2_{MN}}{E^2 + H^2}} = \sqrt{\frac{S^2_{MN}}{E^2 + H^2}} \quad (14)$$

$$\alpha_{Ma} = \arctan\frac{\Delta Y_{Ma}}{\Delta X_{Ma}} \quad (15)$$

3. DETERMINATION OF PROVISIONAL COORDINATES

The components of the sides of the chain are obtained from:

$$\left.\begin{aligned} \Delta X_i &= \epsilon_i \Delta X_{Ma} + \eta_i \Delta Y_{Ma} \\ \Delta Y_i &= \epsilon_i \Delta Y_{Ma} - \eta_i \Delta X_{Ma} \end{aligned}\right\} \qquad (16)$$

The provisional coordinates of the chain may be obtained by using the above components.

From the previous, the final adjusted coordinates may be obtained by applying a rigorous least squares method of adjustment.

4. A SAMPLE NUMERICAL EXAMPLE

Figure (2) illustrates a chain of adjoined triangles between two fixed stations M and N. All the angle of the triangles are observed [3]. The coordinates of the fixed stations M and N are: (20500.1 , 20500.2), (1521.9, 41792.8) respectively. The solution procedure and results are tabulated in tables (1) and (2).

ΔX_{MN} = -18978.20, ΔY_{MN} = +21292.60
From equation (13)
ΔX_{Ma} = + 5586.36, ΔY_{Ma} = + 14021.27
Also, from equations (14) and (15):
S_{Ma} = 15093.16 M, α_{Ma} = 68° 16' 34.1".

Table 1. Calculation of values Z and K..

No. of triangle	No. of angle	Observed angles	Corr.	Adjusted angles	sin A_i, sin β_i
I	1	22° 19′ 50′	-2″	22° 19′ 48″	0.379941
	2	120 22 45	-1	120 19 44	0.862675
	3	37 17 30	-2	37 17 28	———
		ω = +5″	-5″	Z_1 = 37° 17′ 28″	k_1 = 0.440422
II	4	54 59 20	-3	54 59 17	0.819037
	5	49 21 50	-3	49 21 47	0.758851
	6	75 39 00	-4	75 39 56	———
		ω = + 10″	-10	Z_2 =-38° 21′ 28″	k_2 = 0.475353
III	7	47 37 10	+2	47 37 12	0.738691
	8	39 45 25	+2	39 45 27	0.639577
	9	92 37 20	+1	92 37 21	———
		ω = - 5″	+5″	Z_3 = 54° 15′ 53″	k_3 = 0.549018
IV	10	37 47 35	0	37 46 35	0.612581
	11	84 07 40	0	84 07 40	0.994753
	12	58 05 45	0	58 05 45	———
		ω = 0.0″	0	Z_4 = 3° 49′ 52″	k_4 = 0.338092
V	13	47 18 10	+ 3	74 18 13	0.962709
	14	28 13 50	+ 4	28 13 54	0.473038
	15	77 27 50	+ 3	77 27 53	———
		ω = -10″	+ 10″	Z_5 = 73° 38′ 01″	k_5 = 0.688072

Table 2. Calculation of values ϵ, η, E, H, ΔX_i and ΔY_i.

No of sides	Z / cos Z / Sin Z	$(-1)^i K_i$	ϵ	η	$\epsilon_i \Delta X_{Ma} + \eta_i \Delta Y_{Ma}$ / ΔX_i	$\epsilon_i \Delta Y_{Ma} - \eta_i \Delta X_{Ma}$ / ΔY_i
M-a	_	1	1	0	+55866.36	+14021.27
1	37° 17′ 28″ 0.795568 0.605864	-0.440422	-0.350386	-0.266883	-1957.38 -3741.38 -5698.76	-4912.86 -1490.64 -3422.22
2	-38° 21′ 28″ 0.784137 -0.62057	+0.475353	+0.372742	-0.294999	+2082.77 -4136.14 -2053.87	+5226.32 -1647.92 +6874.24
3	54° 15′ 53″ 0.584041 0.811724	-0.549018	-0.320649	-0.44565	-1791.26 -6248.59 -8039.85	-4 495.91 -2 489.57 -2006.34
4	-3° 49′ 52″ 0.997766 -0.066816	+0.338092	+0.338092	-0.022590	+1884.48 -316.74 +1567.74	+4 729.89 -126.20 +4856.09
5	73° 38′ 01″ 0.281778 0.959479	-0.688072	-0.193884	-0.660190	-1083.10 -9256.72 -10339.82	-2718.50 -3688.06 +969.56
			E=0.845160	H=-1.690258		

Therefore, $E^2 + H^2 = 3.571268$

5. CONCLUSIONS

The paper involves the presentation of a straightforward simplified procedure to determine the necessary initial data, base-line length and azimuth which may by missing and unknown, required for the adjustment of a triangulation net.

The verification of the procedure is illustrated by a numerical example.

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