THE ACCURACY AND VALIDITY OF HAZEN-WILLIAMS AND SCOBEY PIPE FRICTION FORMULAS

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ABSTRACT

The authors investigate in this paper the accuracy and range of validity of both Hazen-Williams and Scobey's formulas, the criterion being the Darcy-Weisbach formula which is the most universally accepted. A computer program is designed and used in plotting graphs for the range of validity of both Hazen-Williams and Scobey's formulas. By means of these graphs and for a given pipe diameter, pipe roughness and permissible degree of accuracy the range of Reynolds' number may be evaluated. In addition the authors present examples to demonstrate how to use the graphs which are found simple and practical. Generally the conclusion is that the range of validity decreases as pipe roughness increases and for rough pipes the range of validity is not much affected as pipe roughness increases.

1. INTRODUCTION

There are many empirical formulas for the calculation of pipe flow, based upon tests of various types of pipes under different conditions. Of these are Darcy-Weisbach, Hazen-Williams and Scobey's formulas which are considered the most widely used. For their simplicity the Hazen-Williams and Scobey's formulas are preferred to the Darcy-Weisbach formula in many engineering applications, (Cuenca, 1989), (James, 1988) and (Finkel, 1982). However, the above two formulas are applicable only to problems involving flow of water at normal temperatures and at a relatively high degree of turbulence, as well as to ordinary commercial pipes (Morris and Wiggert, 1976). In addition they are non-homogeneous dimensionally and thus can be applicable only within the range for which they were derived. On the other hand Moody diagram, which is the basis of the Darcy-Weisbach formula, covers the practical range of Reynolds' number as well as the possible range of the relative roughness of pipes, Moody (1944). The objective of this paper is to determine the ranges of validity of both Hazen-Williams and Scobey's formulas, on the basis of the Darcy-Weisbach formula which is the most universally accepted, von Bernuth (1990).

THE DARCY-WEISBACH FORMULA

In this formula the friction head loss in a pipe is expressed as:

$$h_D = \frac{fLV^2}{2gD} \tag{1}$$

in which f is the coefficient of friction, L the pipe length, V the average velocity, g the acceleration due to gravity and D the pipe diameter. The coefficient of friction f, which generally depends upon Reynolds' number and the relative roughness of the pipe, may be obtained by using Moody diagram.

To facilitate solving friction loss hydraulic problems, Swamee and Jain (1976) presented the following friction coefficient formula:

$$f = \frac{1.325}{[\ln(\frac{\epsilon}{3.7D} + \frac{5.74}{R^{0.9}})]^2} \tag{2}$$

in which ϵ is the absolute roughness of pipe material, and R is the Reynolds' number. The preceding formula yields errors less than $\pm 1\%$ in the range $10^{-6} \leq \epsilon/D \leq 10^{-2}$ and $5x10^3 \leq R \leq 10^8$. A reference to

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Moody diagram (Streeter and Wylie, 1983) shows that the aforementioned range of Reynolds' number practically covers the turbulent zone, however a considerable area of the zone of complete turbulence where $\epsilon/D > 10^{-2}$ cannot be represented by Equation (2). The coefficient of friction for wholly rough turbulent flow (Vennard and Street, 1982) is given by

$$f = \frac{1}{[1.14 - 2.0 \log \frac{\epsilon}{D}]^2} \qquad (3)$$

It is worthy to note that laminar flow is not considered since flow of water in pipes is turbulent in engineering practice.

For convenience Eq. (1) is put in the following form:

$$S_D = \frac{h_D}{L} = \frac{fR^2\nu^2}{2gD^3} \qquad (4)$$

in which S_D is the friction head loss per unit pipe length (according to Darcy-Weisbach) and ν the kinematic viscosity of water. As both Hazen-Williams and Scobey's formulas apply to water at ordinary temperatures, ν is taken as 10^{-6} m^2/s (at 22°C). It is proved that at ordinary temperatures different from 22°C the ranges of validity of the above two formulas are not practically affected. Substituting $\nu = 10^{-6}$ m^2/s and g = 9.81 m/s^2 into Eq. 4 and simplifying;

$$S_D = 5.0968 \times 10^{-14} \frac{fR^2}{D^3} \qquad (5)$$

THE HAZEN-WILLIAMS FORMULA

The pipe friction head loss according to Hazen-Williams is given as (Cuenca, 1989)

$$h_H = 1.22 \times 10^{10} \frac{L(\frac{Q}{C})^{1.852}}{D^{4.87}} \qquad (6)$$

in which h_H is the friction head loss (m), L is the pipe length (m), Q the discharge (ℓ/s), C the Hazen-Williams coefficient and D the pipe diameter (mm).

Equation (6) may be put in the following form:

$$S_H = \frac{h_H}{L} = 10.7736 \frac{(\frac{Q}{C})^{1.852}}{D^{4.87}}$$

in which S_H is the friction head loss per unit pipe length (according to Hazen-Williams), Q the discharge (m^3/s) and D the pipe diameter (m). In terms of V rather than Q Eq. 7 may be put in the following form

$$S_H = 6.8876 \frac{(\frac{V}{C})^{1.852}}{D^{1.166}}$$

Introducing Reynolds' number we have:

$$S_H = 6.8876 \frac{(\frac{R}{C})^{1.852} \nu^{1.852}}{D^{3.018}}$$

substituting $\nu = 10^{-6}$ m^2/s and simplifying:

$$S_H = 5.3219 \times 10^{-11} \frac{(\frac{R}{C})^{1.852}}{D^{3.018}} \qquad (10)$$

THE SCOBEY'S FORMULA

The pipe friction head loss is given by Scobey as (Cuenca, 1989)

$$h_s = 2.587 \times 10^{-3} \frac{K_s\, L\, V^{1.9}}{D^{1.1}} \qquad (11)$$

in which K_S is the Scobey's coefficient and D is the pipe diameter (mm). Introducing Reynolds' number into Eq. (11) and rearranging:

$$S_s = \frac{h_s}{L} = 2.587 \times 10^{-3} \frac{K_s\, R^{1.9} \nu^{1.9}}{D^3} \qquad (12)$$

in which S_S is the friction head loss per unit pipe length according to Scobey. Substituting $\nu = 10^{-6}$ m^2/s and simplifying:

$$S_s = 1.0299x10^{-14} \frac{K_s R^{1.9}}{D^3} 4 \quad (13)$$

MATCHING REYNOLDS' NUMBER

To compare friction loss according to Hazen-Williams, S_H, or according to Scobey, S_S, with that according to Darcy-Weisbach, S_D, all the conditions should be the same. Though it is easy to consider the same values of Reynolds' number, R, and the pipe diameter, D, in Eqs. 5, 10 and 13, yet choosing corresponding values of ϵ, C and K_S is neither direct nor simple. Inspection of tables giving values of the above three parameters showed that there is often a lack of correspondence between them for the same pipe material (Morris and Wiggert, 1976). For this it is necessary to find a sort of direct correspondence between ϵ and each of C and K_S, regardless of the pipe material. For a given C or K_S and D the value of ϵ may be evaluated by equating S_D with S_H or S_S for the conditions at which they are all equal. In other words it is required to choose a matching Reynolds' number at which ϵ may be evaluated. As both Hazen-Williams and Scobey's formulas provide good results at $R>5x10^4$ (Morris and Wiggert, 1976) and as most practical flows occur around $R = 4x10^5$ (Vennard and Street, 1982), (Featherstone and Nalluri, 1982), (Streeter and Wylie, 1983), the latter value of Reynolds' number is taken as the matching Reynolds' number, R_o.

It is practically proved that R_o values which are moderately different from $4x10^5$ does not affect seriously the ranges of validity of Hazen-Williams and Scobey's formulas.

EVALUATION OF THE ABSOLUTE PIPE ROUGHNESS

a- Hazen-Williams Formula: For a certain pipe diameter and given C, the absolute roughness of pipe, ϵ, may be evaluated by equating the head loss per unit pipe length according to Hazen-Williams, S_H, to that according to Darcy-Weisbach, S_D, at the matching Reynolds' number, R_o. From Eqs. 5 and 10 we have:

$$5.0968x10^{-14} \frac{fR_o^2}{D^3} = \frac{5.3219x10^{-11}}{D^{3.018}} \left(\frac{R_o}{C}\right)^{1.852} \quad (14)$$

solving for f and simplifying:

$$f = \frac{1044.165}{D^{0.018} C^{1.852} R_o^{0.148}} \quad (15)$$

as all the parameters in the right hand side of the above equation are known in advance, f can be evaluated.

Substituting the value of f as estimated by Eq. 15 into Eq. 2 and solving for ϵ:

$$\epsilon = 3.7D\{\exp[-(\frac{1.325}{f})^{0.5}] - \frac{5.74}{R_o^{0.9}}\} \quad (16)$$

It is worthy to note that for rough flow in which $\epsilon/D > 10^{-2}$, the absolute roughness, ϵ, is to be evaluated from Eq. 3 instead of Eq. 2.

b- Scobey's Formula: Given the values of D, K_S and considering R_o, S_D equals S_S. From Eqs. 5 and 13 we have:

$$5.0968x10^{-14} \frac{fR_o^2}{D^3} = 1.0299x10^{-14} \frac{K_s R_o^{1.9}}{D^3} \quad (17)$$

solving for f:

$$f = \frac{0.2021 K_s}{R_o^{0.1}} \quad (18)$$

To determine the absolute roughness ϵ, the coefficient of friction f evaluated from Eq. 18 may be inserted into Eq. 16. For rough pipe flow where $\epsilon/D > 10^{-2}$ Eq. 3 may be used together with Eq. 18 to evaluate ϵ.

COMPUTER PROGRAMS

It is of practical importance to investigate the accuracy and range of validity of both Hazen-Williams and Scobey's formulas for the practical values of C, K_S, D and Reynolds' number, R. To achieve this two computer programs are designed, one for Hazen-Williams formula and the second for Scobey's formula. In both programs the diameter D ranges between 0.0254 m and 1.0 m and Reynolds' number between $5x10^3$ and 10^8, the latter being the range of validity of Swamee and Jain's formula.

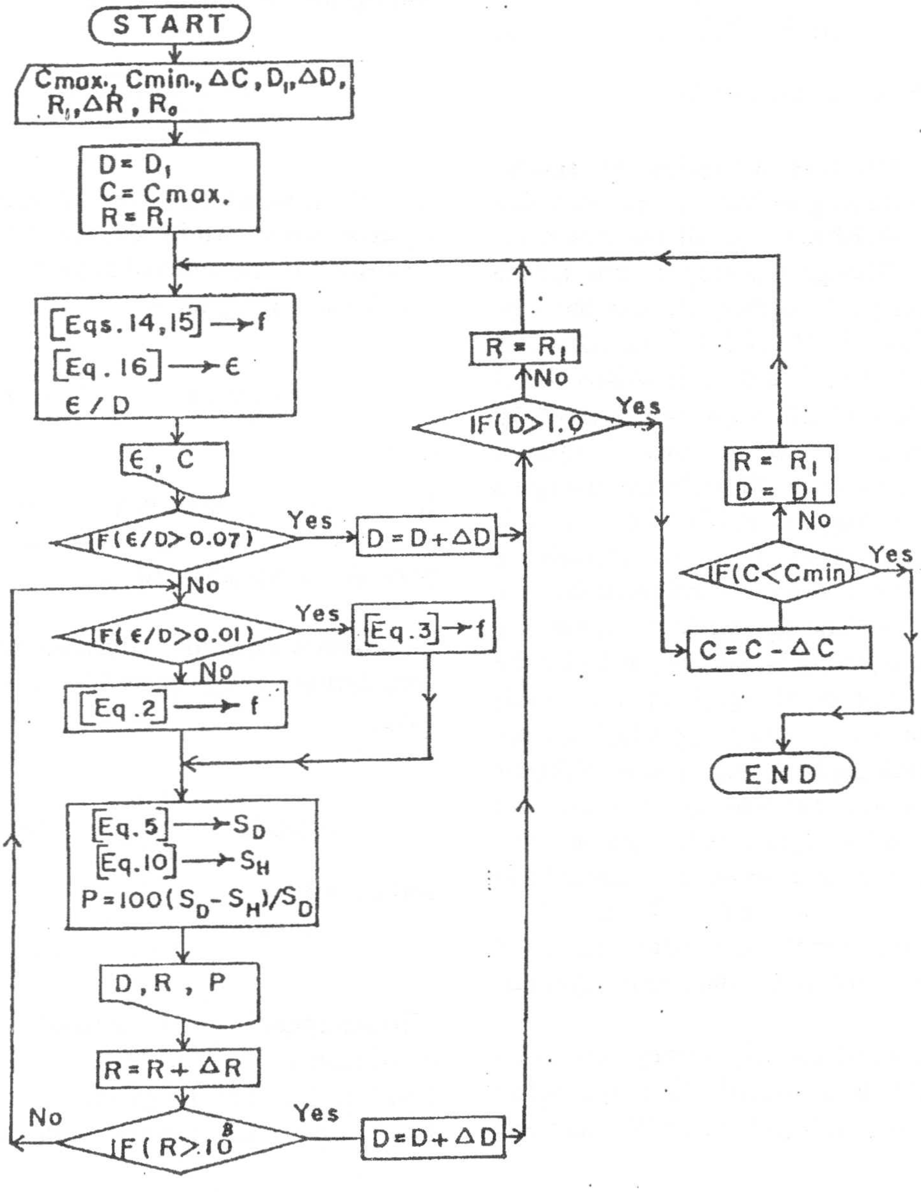

Figure 1. flow chart for the computer program for Hazen-Williams Formula.

In the Hazen-Williams program C ranges from 70 to 150, whereas in that of Scobey K_S ranges from 0.30 to 0.50. For each pipe diameter and C or K_S value the absolute roughness ϵ is first estimated, the friction head loss per unit length S_D, and S_H or S_S, then the percentage difference $100(S_D-S_H)/S_D$ or $100(S_D-S_S)/S_D$ for the whole range of Reynolds' number are evaluated. In Figure (1) is shown the flow chart of the computer program for the Hazen-Williams formula. The computer program corresponding to Scobey's formula is similar to that of Figure (1).

RANGES OF VALIDITY

a- Hazen-Williams Formula: According to the results of the computer program shown in Figure (1) graphs of Figs. 2 through 5 are plotted. In all figures curves corresponding to 0%, ±5%, ±10% and ±20% differences between S_D and S_H are plotted. The positive sign indicates that $S_D > S_H$ and vice versa.

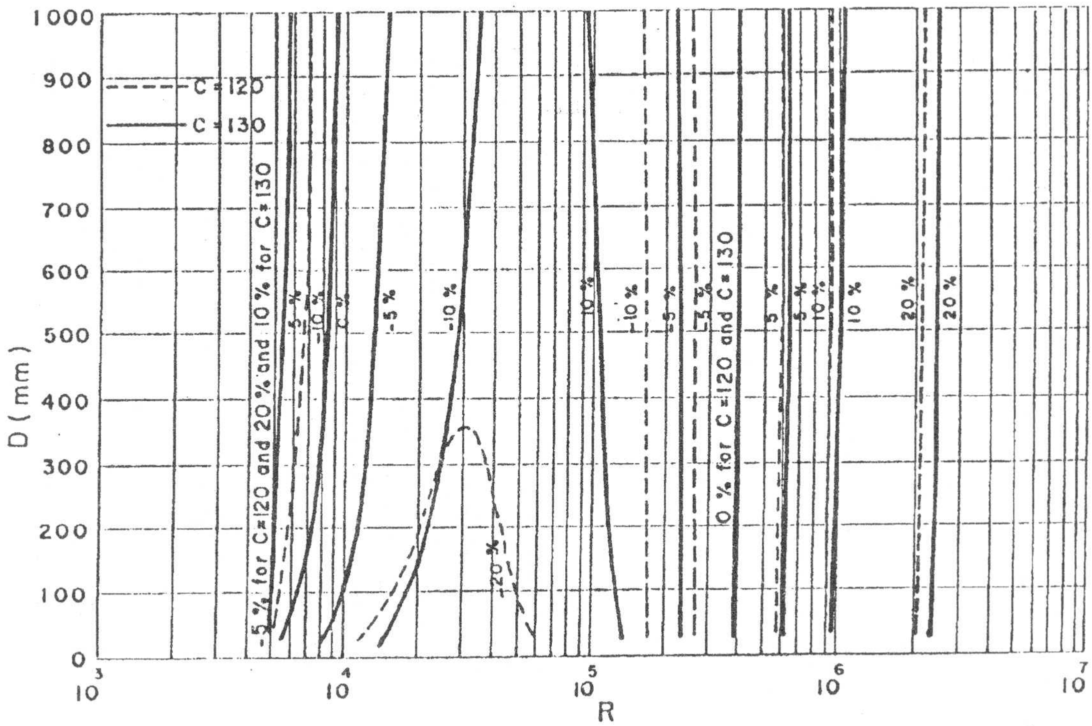

Figure 2. Ranges of validity of Hazen-Williams formula for C=140 and C=150.

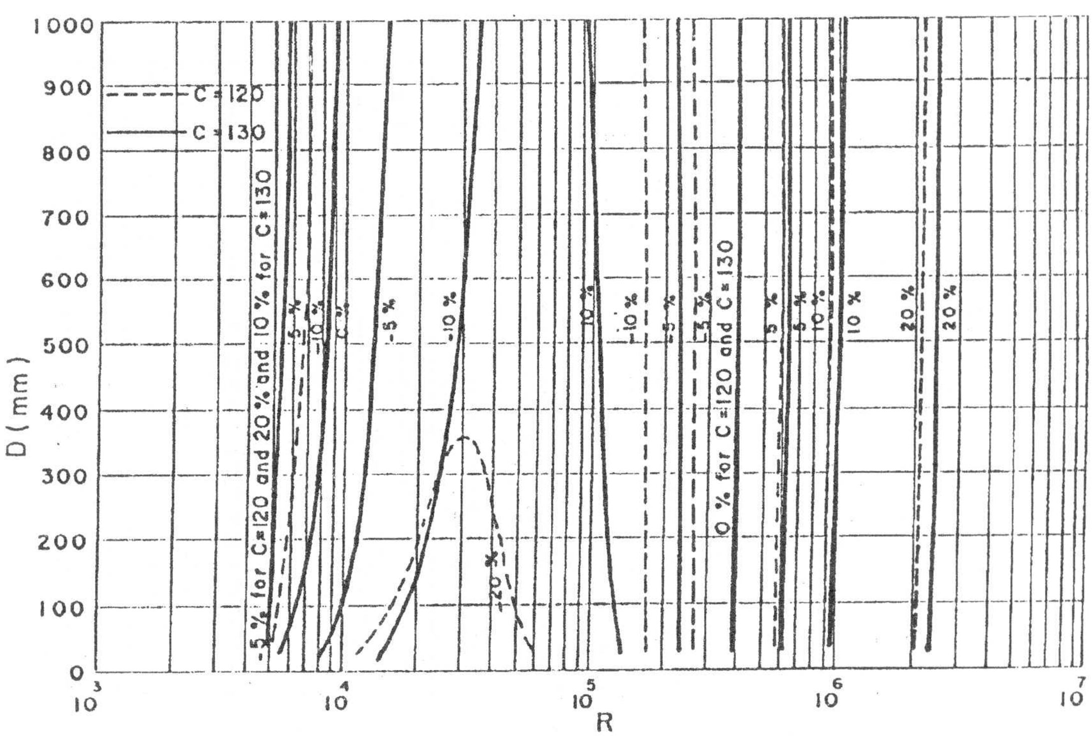

Figure 3. Ranges of validity of Hazen-Williams formula for C=120 and C=130.

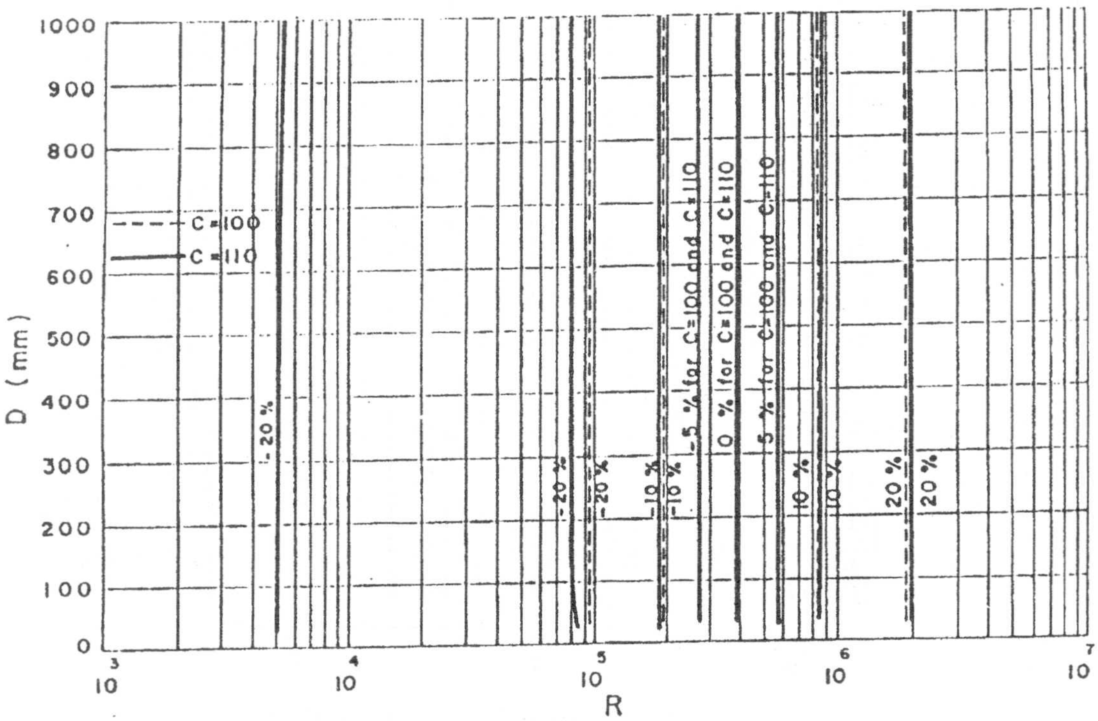

Figure 4. Ranges of validity of Hazen-Williams formula for C=100 and C=110.

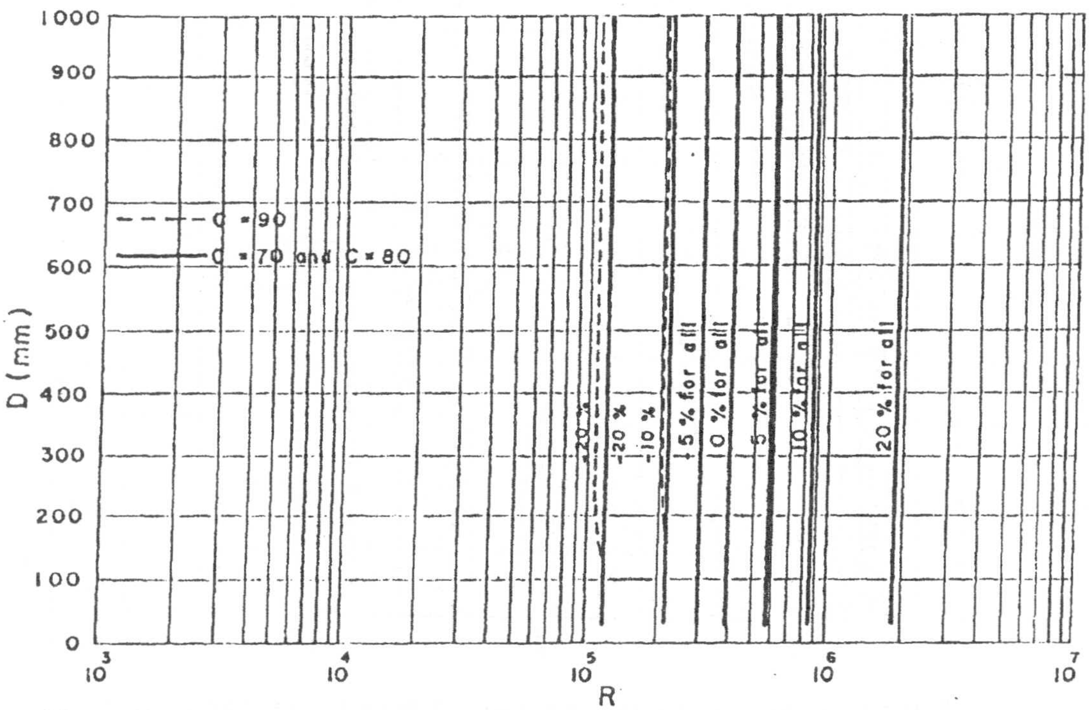

Figure 5. Ranges of validity of Hazen-Williams formula for C=70, C=80 and C=90.

As C decreases curves tend to flatten and for $C<90$ curves become nearly vertical lines which means that percentage differences are independent on the pipe diameter. Figures (2) through (5) are plotted to provide the ranges of validity of Hazen-Williams formula according to the permissible degree of accuracy. For example if the percentage error should not exceed ±10% for C=130 and D=400 mm, Figure (3) shows that flow should be within the ranges $5x10^3<R<2.6x10^4$ and $1.1x10^5<R<10^6$. If ±20% is allowable, the range becomes $5x10^3<R<2.45x10^6$. On the other hand if the permissible error reduces to ±5% the ranges are $5.2x10^3<R<1.28x10^4$ and $2.35x10^5<R<6.3x10^5$. In most figures the range of validity for ±5% permissible error is about $2.8x10^5<R<6x10^5$ and for ±10% the range is about $2x10^5<R<9x10^5$. According to Figures (2) through (5) it is clear that the ranges of validity of Hazen-Williams formula reduces as C decreases, i.e. as pipes become more rough. For D>1.0 m and/or for error percentages other than those considered, ranges of validity may be determined through extrapolation or interpolation, respectively.

b- Scobey's formula: Figures (6) through (10) are plotted according to a computer program similar to that shown in Figure (1) but designed to suit the Scobey's formula. It is evident that all curves are vertical straight lines, which should be expected since the coefficient of friction as given by Eq. 18 is independent on D. For example for D = 100 mm, K_S = 0.42 and ± 10% permissible error the range of validity of Scobey's formula is $6.1x10^3<R<1.4x10^6$ for ±20% the range is $5x10^3<R<5x10^6$ and for ±5% the ranges are $9x10^3<R<2.5x10^4$ and $1.8x10^5<R<7.6x10^5$. Similar to the case of Hazen-Williams formula, as K_S increases and pipes become more rough, the range of validity of Scobey's formula decreases.

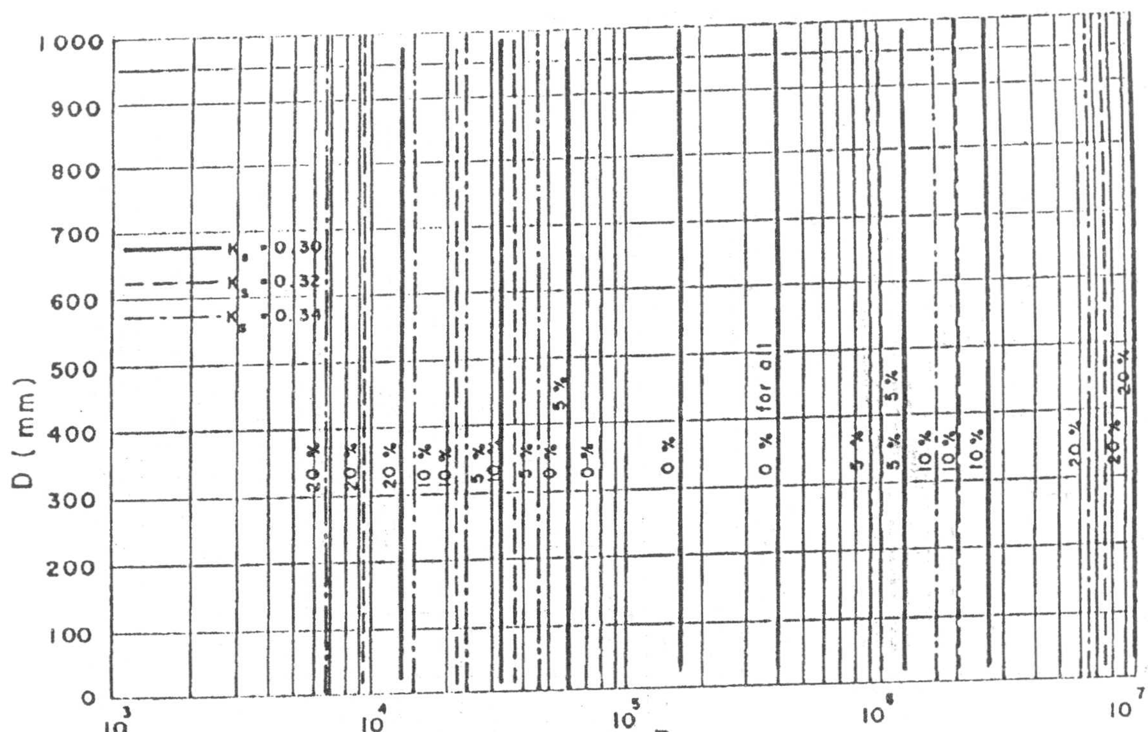

Figure 6. Ranges of validity of Scobey's formula for K_s=0.30, K_s =0.32 and K_s = 0.34.

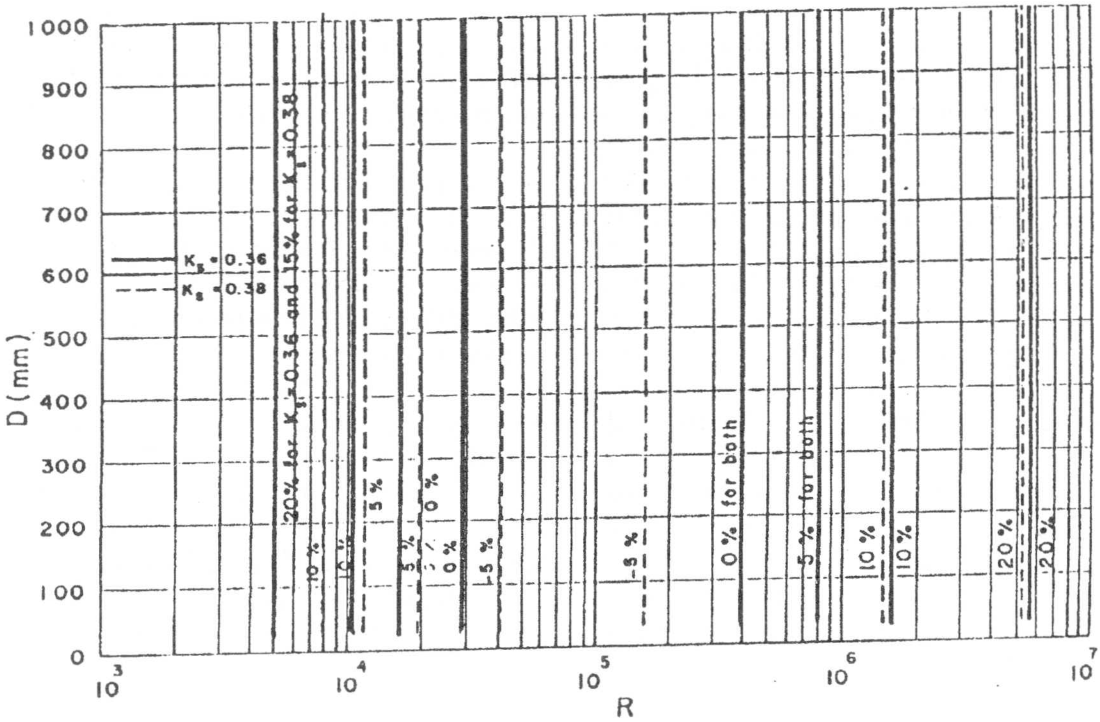

Figure 7. Ranges of validity of Scobey's formula for K_s=0.36, and K_s = 0.38.

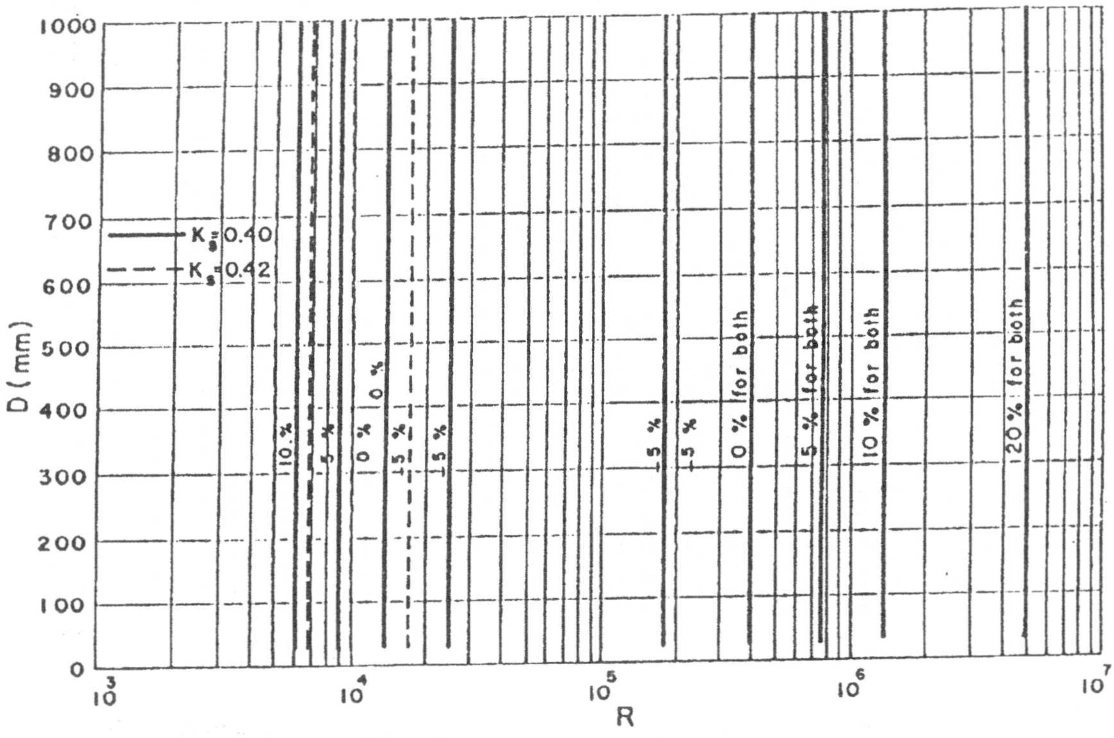

Figure 8. Ranges of validity of Scobey's formula for K_s=0.40, and K_s = 0.42.

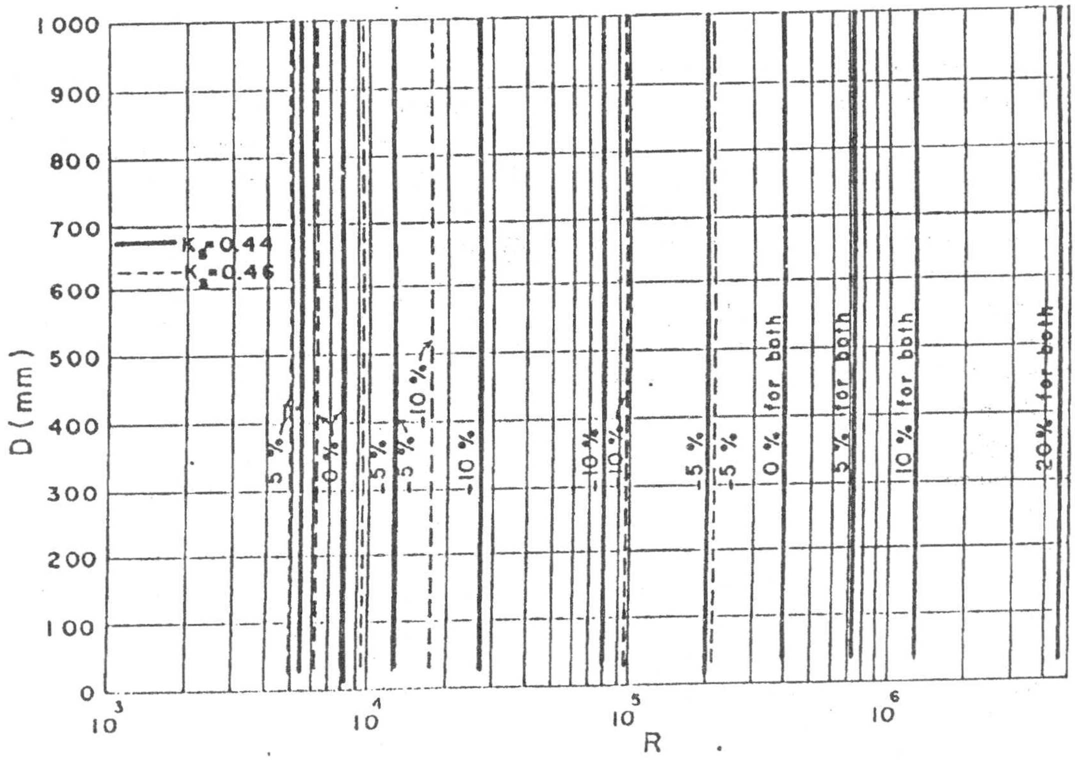

Figure 9. Ranges of validity of Scobey's formula for K_s=0.44, and K_s = 0.46.

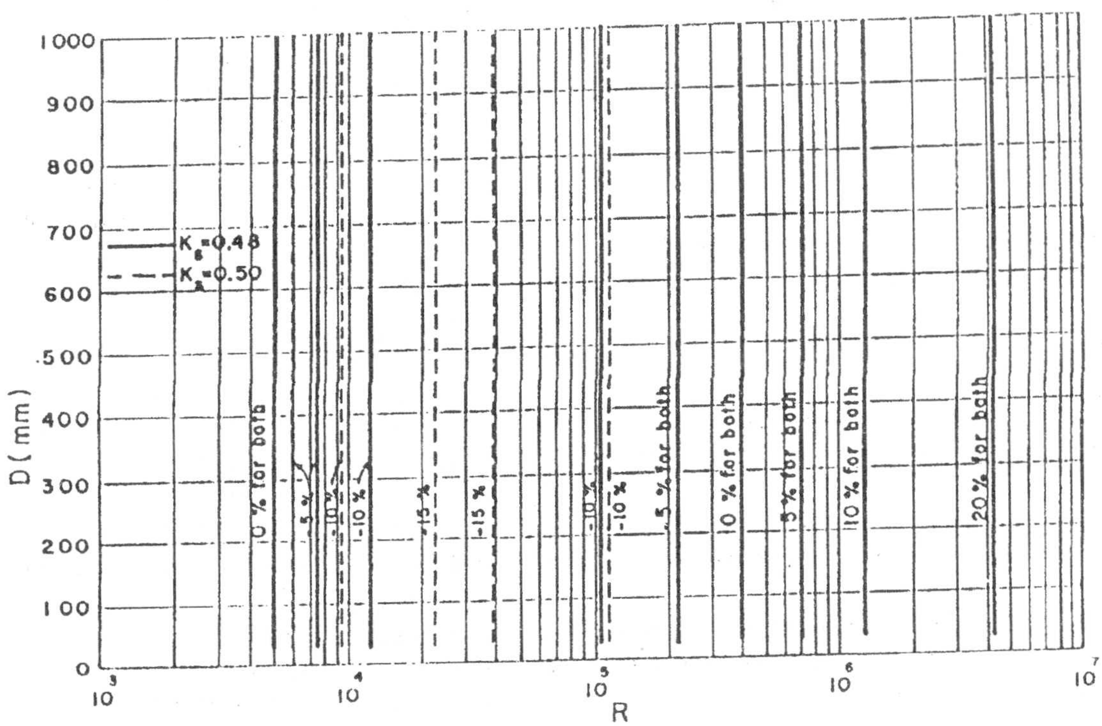

Figure 10. Ranges of validity of Scobey's formula for K_s=0.48, and K_s = 0.50.

CONCLUSIONS

The validity graphs presented in this paper are simple and of practical importance. For Hazen-Williams formula curves flatten as C increases and for $C<90$ curves are nearly vertical straight lines which means that percentage differences are independent on the pipe diameter. For relatively smooth pipes there are two ranges of validity of Reynolds' number; whereas for more rough pipes there is one range of validity. For a permissible error as low as $\pm 5\%$ the range of validity of Hazen-Williams formula is about $2.8\times10^5 < R < 6\times10^5$ in most cases.

In the case of Scobey's formula all curves are vertical lines since the percentage difference of friction losses according to Darcy-Weisbach and Scobey are independent on the pipe diameter. There are two ranges of validity for all values of K_S. In most cases, for $\pm 5\%$ permissible error the range of validity is nearly in the range $2.1\times10^5 < R < 7.4\times10^5$. For both Hazen-Williams and Scobey's formulas the range of validity decreases as the pipe roughness increases and for relatively rough pipes the range of validity is practically independent on C or K_S. Though for both Hazen-Williams and Scobey's formulas curves are plotted for water at 22°C, yet it is proved that for other normal temperatures the ranges of validity are not much affected.

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NOTATIONS

The following symbols are used in this paper:

C	=	Hazen-Williams coefficient;
D	=	pipe diameter;
f	=	coefficient of friction;
g	=	acceleration due to gravity;
h_D	=	friction head loss according to Darcy-Weisbach;
h_H	=	friction head loss according to Hazen-Williams;
h_S	=	friction head loss according to Scobey;
K_S	=	Scobey's coefficient;
L	=	pipe length;
Q	=	pipe discharge;
R	=	Reynolds' number;
R_o	=	matching Reynolds' number;
S_D	=	friction head loss per unit pipe length according to Darcy-Weisbach;
S_H	=	friction head loss per unit pipe length according to Hazen-Williams;
S_S	=	friction head loss per unit pipe length according to Scobey;
V	=	average velocity of pipe;
ν	=	kinematic viscosity; and
ϵ	=	absolute roughness of pipe material.