# OPTIMIZED PARAMETER VERSUS OPTIMAL CONTROLLERS FOR MARINE DIESEL ENGINES

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ABSTRACT

Based on the quadratic performance index and the reduced matrix Riccati equation, a proportional linear quadratic regulator is designed for optimal speed control of marine Diesel engines. The behavior of the automatic loop is discussed and compared with its behavior if adopting the recent concept of pseudo derivative feedback control with additionally optimized gain by the integral of the square of the error performance index and Parseval's theorem.

NOMENCLATURE

[A] = System matrix
[b] = Control vector
C = Propeller power constant(w/(rad/s)$^3$)
$\bar{C}_1$ = $\dot{m}_o / u_o$ (Kg/s/% rack stroke)
$\bar{C}_2$ = $\dot{m}_o / \omega_o$ (Kg)
$\bar{C}_3$ = $P_{eo} / \dot{m}_o$ (Joule/Kg)
$\bar{C}_4$ = $\eta_{tr} \cdot \eta_{prop} / I \omega_o$ (s/Kg.m$^2$)
$\bar{C}_5$ = $1 / I \omega_o$ (s/Kg.m$^2$)
$\bar{C}_6$ = $3 \eta_{tr} \cdot \eta_{prop} \cdot P_{eo} / \omega_o$ (Joule)
$C.V$ = Calorific value of fuel (Joule/Kg)
$D(s)$ = Polynomial denominator of $E(s)$
$e(t)$ or $E(s)$ = Error signal in time or Laplace domains.
$[F]^T$ = Desired optimal feedback row vector = $[\alpha_1 \ \alpha_2 \ \alpha_3]$
I = Mass polar moment of inertia of rotating parts including added mass of water (Kg. m$^2$)
$[\bar{I}]$ = Unity matrix.
i = $\sqrt{-1}$
J = Quadratic performance index.
$j_3$ = ISE index of second over third order polynomials of $E(s)$
k = Gain of proportional controller of PDF
$\bar{k}$ = Fuel pump constant (Kg/rad)
$k_D$ = Gain of derivative controller of PDF (s)
$k_I$ = Gain of integral controller of PDF (s$^{-1}$)
[L] = Output row vector
[M] = Observer's vector matrix = $[m_1 \ m_2 \ m_3]^T$
m = Order of $D(s)$
$\dot{m}_f$ = Rate of fuel injected into the cylinder (Kg/s)
$\dot{m}_j$ = Rate of fuel delivered by fuel pump (Kg/s)
$\dot{m}_o$ = Nominal value of $\dot{m}_f$ (Kg/s)
$N(s)$ = Polynomial numerator of $E(s)$
n = Rate of revolutions of the Diesel engine (r.p.m)
$n_o$ = Nominal rate of revolutions of the Diesel engine (r.p.m)
$n_{in}$ = Desired speed (r.p.m)
$[O]$ = Null matrix
$o$ = Suffix indicating the nominal value.
$[P]$ = Positive-definite real symmetric matrix extracted from the solution of Riccati equation.
$P_e$ = Brake power (w)
$P_{eo}$ = Nominal brake power. (w)
$P_f$ = Friction power lost in shaft bearings (w)
$P_l$ = Power delivered by the propeller. (w)
$P_{l_o}$ = Nominal value of $P_l$ (w)
$[Q]$ = Positive-definite (or positive-semi definite) real symmetric matrix.
$[r]$ = Positive-definite real symmetric matrix.

| | | | |
|---|---|---|---|
| $s$ | = | Laplacian operator | ($s^{-1}$) |
| $[...]^T$ | = | Transpose of a matrix | |
| $t$ | = | Time | (s) |
| $[U(t)]$ | = | Control vector | |
| $u$ | = | Fuel rack position | (%) |
| $u_o$ | = | Nominal value of u | (%) |
| $[W]$ | = | Reference vector | |
| $[X]$ | = | State variable vector of the plant $= [x_1\ x_2\ x_3]^T$ | |
| $[\hat{X}]$ | = | Reconstructed (estimated) state variable vector. | |
| $[Y]$ | = | Output vector. | |
| $[\hat{Y}]$ | = | Reconstructed output vector | |
| $[\tilde{Y}]$ | = | Output error vector. | |
| $y_o$ | = | Maximum stroke of fuel rack | (cm) |
| $z$ | = | Number of cylinders of the Diesel engine | |
| $\hat{\alpha}$ | = | Crank shaft angle | (rad) |
| $\alpha_1, \alpha_2, \alpha_3$ | = | Proportional state variable feedbacks $= [F]^T$ | |
| $\Delta...$ | = | Change in.. | |
| $[\xi(t)]$ | = | Desired state vector | |
| $\eta_{bth}$ | = | Brake thermal efficiency of Diesel engine | |
| $\eta_m$ | = | Mechanical efficiency of Diesel engine | |
| $\eta_{p.ow}$ | = | Propeller efficiency in open water. | |
| $\eta_{prop}$ | = | Propeller efficiency behind ship $= \eta_{r.r} \cdot \eta_{p.ow}$ | |
| $\eta_{r.r}$ | = | Relative rotative efficiency | |
| $\eta_{tr}$ | = | Transmission efficiency of propulsion shaft | |
| $\lambda$ | = | Lagrange multiplier, a positive constant indicating the weight of control cost w.r.t the minimized errors. | |
| $\tau_c$ | = | Delay time of fuel | (s) |
| $\tau_d$ | = | Transportation lag (dead time) of fuel | (s) |
| $\Phi$ | = | A dummy variable to write the state matrix equation | |
| $\omega$ | = | Angular speed of propulsion shaft | (rad/s) |
| $\omega_o$ | = | Nominal value of $\omega$ | (rad/s) |
| $\dot{\omega}$ | = | Angular acceleration of the propulsion sh | (rad |

## INTRODUCTION

The principles of appropriate selection conventional controllers have been established thro the few last decades. Two fundamental basis shoul taken into account; the dynamics, time delays transportation lags incorporated into the plant be the nature of the external disturbance affecting on plant whether being a standard mathematical functi stochastic, or having a periodic frequency.

The most simple and less expensive governor is proportional type. If swifter interference is requi (due to delays and lags in the plant or in case of se perturbations), a derivative action is introduced to the proportional action. Derivative action advantageous in its instantaneous interfere improving the closed loop performance, while influence vanishes in steady state. Therefore, pure derivative controllers could never be utilized. larger the time delays or the transportation lags of plant and/or the smaller the rotor's time constant propulsion engine, the higher should be the coeffic of the derivative action.

In order to build the D-property, it is preferabl adopt either minor partially relaxed feedback - ins of using minor delayed feedback or to measure dir both the speed and acceleration simultaneously in of speed governing. Such techniques reduce the t delays of governor; a matter which improves automatic control loop performance.

Should the static error be desired to be tot eliminated irrespective of the disturbance to which system is subjected, integral control action added either the proportional or the proportional deriva controller becomes a must.

In contrast to proportional controller, inte property controller has also the merit to interfere a instant when the error signal is momentarily z whereas the interference of both controllers evere in steady state. However, pure integral controll used with pure integral plant renders the loop unsta This is the reason why (PI) or (PID) controllers preferable rather than (I) controllers when inte control action is required. The sequential logic selecting the proper industrial controller to operate with periodic disturbances differs from the accustomed traditional order adopted for non-periodic perturbations where in the former case, the operating frequency has a significant role in the choice.

Owing to the friction and inertia forces inherent in all controllers, ideal controllers do not exist since time delays are inevitable.

Absolute and relative stability measures together with time and frequency domains behavior of the automatic loop, judge the adequacy of the regulating system in use.

Dynamic behavior of different property controllers as well as dead time elements in both time and frequency domains is pictorially demonstrated [1] in the appendix.

One of the most recent techniques in control engineering which came into appearance one half decade ago, is the adoption of pseudo derivative feedback (PDF), where only integral controller is used in the forward path and a proportional - derivative feedback is located in an inner feedback path [2]- instead of unreliable second order derivative placed in the main feedback path; the objective being the elimination of the dynamics of inputs. The mathematical form of a PDF control depends on the order of the plant. The aim of this research is to investigate the dynamic performance of the marine Diesel engine with PDF control whose proportional gain is optimized in accordance with the integral of the square of the error (ISE) performance index and to compare it with the engine's performance with an optimal controller designed on the basis of the quadratic performance criterion by Riccati reduced matrix equations.

## MODELLING OF THE MARINE DIESEL ENGINE

*Further* research advances took place through the last two decades concerning the mathematical simulation and dynamic analysis of naturally aspirated or supercharged marine Diesel engines with either conventional analog controllers or with microprocessor digital algorithmic controllers.

Among the earlier researchists in this territory are: Wozniak & Barret, Andersen, Thompson and Engja [3-6] who carried out academic or laboratory mathematical description of Diesel engines used for ship propulsion. Recent researches have focused on turbocharged Diesel engines particularly those performed by Woodward and Ford [7-8].

It has been shown that the laborious treatment of supercharger has almost worthless influence on results obtained [4,8].

Consequently, satisfactory results from the engineering point of view could be achieved by simplifying the model under study [9].

We consider now a navigating ship by direct slow speed Diesel propulsion where the sea is calm and the propeller blades are totally immersed in water. In other words, the propeller power depends only on the cube of the angular speed of its rotation. Dead time due to the discrete nature of firing of the cylinders beside delay time attributed to the time elapsed for fuel delivery from the fuel pump to the engine's cylinders are assumed too [7,9].

The dynamic analysis of the engine could be briefly derived as follows:

To analyze the fuel pump first:

Linearizing by Taylor's formula we have:

$$\dot{m}_j = \bar{k}\,\omega\,u$$

$$\Delta\,\dot{m}_j = \bar{C}_1\,.\,\Delta u + \bar{C}_2\,\Delta\omega \qquad (1)$$

and

$$\Delta\,\dot{m}_f = \frac{e^{-\tau_d\,.\,s}}{1+\tau_c\,.\,s}\,.\,\Delta\,\dot{m}_j \qquad (2)$$

where $\tau_c = \dfrac{\hat{\alpha}}{z\,.\,\omega}$ and $\tau_d = \dfrac{\pi}{z\,.\,\omega}$

Applying Pade′ approximations [9] it follows:

$$Ce^{-\tau_d.s} \approx \left[(1-\frac{\tau_d}{2r}.s)/(1+\frac{\tau_d}{2r}.s)\right]^r \approx \frac{1-0.5\,\tau_d.s+0.0833\,\tau_d^2 s^2}{1+0.5\,\tau_d.s+0.0833\,\tau_d^2 s^2}$$

$$\approx \frac{1-0.5\,\tau_d\,.\,s}{1+0.5\,\tau_d\,.\,s} \qquad (\text{for } r = 1) \qquad (3)$$

And to relate the fuel to the power:

$P_e = \bar{C}_3\,\dot{m}_f$ where $\bar{C}_3 = \eta_m\,.\,\eta_{bth}\,(c\,.\,v)$ Then:

$$\Delta P_e = \bar{C}_3 \, \Delta \dot{m}_f \tag{4}$$

For the dynamics of the propulsion shaft:

$$\frac{d}{dt}\left(\frac{1}{2} I \omega^2\right) = P_e - P_f - P_l \tag{5}$$

where $P_f = P_e - \eta_{tr} \; \eta_{prop} \; p_e$

Expanding equations (5)

$$I \omega \dot{\omega} = \eta_{tr} \; \eta_{prop} \; P_e - P_l$$

$$\dot{\omega} = \frac{\eta_{tr} \; \eta_{prop}}{I \omega} \, . \, P_e - \frac{1}{I \omega} P_l \tag{6}$$

Linearizing equation (6) yields:

$$\Delta\dot{\omega} = \frac{\partial \dot{\omega}}{\partial P_e} \Big|_o \, . \, \Delta P_e + \frac{\partial \dot{\omega}}{\partial P_l} \Big|_o \, . \, \Delta P_l + \frac{\partial \dot{\omega}}{\partial \omega} \Big|_o . \Delta \omega$$

but

$$\frac{\partial \dot{\omega}}{\partial \omega} \Big|_o = -\frac{\eta_{tr} \eta_{prop} P_e}{I \omega^2} \Big|_o + \frac{P_l}{I \omega^2} \Big|_o = \frac{-\eta_{tr} . \eta_{prop} P_{eo} + P_{lo}}{I \omega^2} = 0$$

and

$$\frac{\partial \dot{\omega}}{\partial P_e} \Big|_o = \frac{\eta_{tr} \cdot \eta_{prop}}{I \omega_o},$$

$$\frac{\partial \dot{\omega}}{\partial P_l} \Big|_o = - \frac{1}{I \omega_o}$$

or $\Delta \dot{\omega} = \bar{C}_4 \, . \, \Delta P_e - \bar{C}_5 . \Delta P_l$ (7)

The propeller performance could be approximately but satisfactorily written as:

$$P_l = C \omega^3$$

which yields after linearization

$$\Delta P_l = \frac{\partial P_l}{\partial \omega} \Big|_o . \, \Delta \omega \tag{8}$$

where

$$\bar{C}_6 = \frac{\partial P_l}{\partial \omega_o} \Big|_o = \frac{3 \, \eta_{tr} \, \eta_{prop} \cdot P_{lo}}{\omega_o}$$

*Data of the Plant*

Chosen numerical data for the marine Diesel eng are as follows: B&W two stroke , 6 cylinders

$P_e$ = 10.24512 Mw , $\dot{m}_f$ = 0.584 Kg/s

$I$ = 83750 Kg.m$^2$ , rpm = 99.8

$u_o$ = 73.8 % , $\eta_{tr}$ = 98%

$\eta_{prop}$ = 60% which result in the follow numerical constants

| | |
|---|---|
| $\bar{C}_1 = 0.8$ | (Kg/s)/% rack stroke |
| $\bar{C}_2 = 0.056$ | Kg |
| $\bar{C}_3 = 17.534 * 10^6$ | J/Kg |
| $\bar{C}_4 = 0.672 * 10^{-6}$ | s/Kg. m$^2$ |
| $\bar{C}_5 = 1.14 * 10^{-6}$ | s/Kg. m$^2$ |
| $\bar{C}_6 = 1.72924 * 10^6$ | J |
| $\tau_c = 0.1$ | s |
| $\tau_d = 0.05$ | s |

Equations from (1) to (8) inclusive represent simulation of marine Diesel engine whose bl diagram is indicated in Figure (1).

*State Space Description of the Plant*

Referring to Figure (1-b), the three simultaneous order differential equations of the plant could derived as:

$$\dot{X}_1 = - 1.9713 \, X_1 + 11.78886 \, \Phi$$

$$\dot{X}_2 = - 40 \, X_2 + 40 \, X_3$$

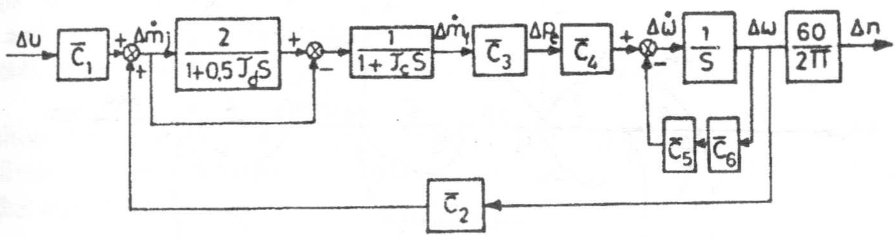
$$\dot{X}_3 = 0.56 \, X_1 - 10 \, X_3 + 8 \, \Delta u$$

where

$$\Phi = X_2 - 0.025 \, \dot{X}_2$$

and

$$\Delta n = 9.55 \, X_1$$



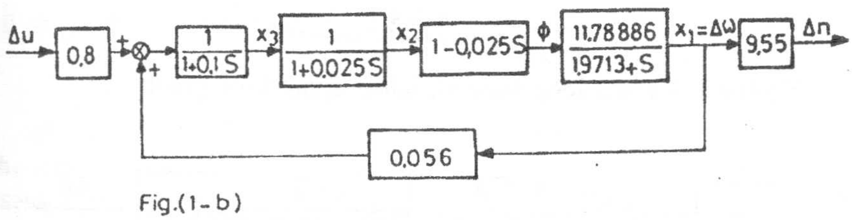




**Figure 1**. Modelling of the engine with selected coefficients.

Equations (9) yield the following matrix equations:

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\end{bmatrix} = \begin{bmatrix}-1.9713 & 23.57772 & -11.8886\\ 0 & -40 & 40\\ 0.56 & 0 & -10\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 8\end{bmatrix}\Delta u$$

$$\Delta n = \begin{bmatrix}9.55 & 0 & 0\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\end{bmatrix} \qquad (10)$$

or in terms of brief notations:

$$[\dot{X}] = [A]\,.\,[X] + [b]\,.\,\Delta u,$$

$$\Delta n = [L]\,[X] \qquad (11)$$

*Optimal Regulator's Design*

The quadratic performance index for optimal controller design minimizing a generalized state error function $[\xi(t)] - [X(t)]$ and imposing constraints on the control vector -in order to compromise the control cost too- is given by [10,11]:

$$J = \int_0^{\tau} [\underline{\xi}(t) - \underline{X}(t)]^T Q [\xi(t) - X(t)]\,dt + \lambda \int_0^{\tau} \underline{U}^T(t)\,\underline{r}\,\underline{U}(t)\,dt,,$$

$$0 \le t \le \tau$$

If the desired states $\xi(t)$ are chosen as the origin, the Lagrange multiplier $\lambda$ is included in $\underline{r}$ matrix and $\tau = \infty$, the quadratic index becomes:

$$J = \int_0^{\infty} \left([X]^T[Q][X] + [U]^T[r][U]\right)dt \qquad (12)$$

This concept when being applied to the system matrix equations (10,11), yields the reduced matrix Riccati equation for proportional linear quadratic regulator's design namely:

$$[A]^T[P] + [P][A] - [P][b][r]^{-1}[b]^T[P] + [Q] = [0]$$

$$\text{Where: } -[F]^T = -[r]^{-1}[b]^T[P] \qquad (13)$$

where $[Q]$ is a positive - definite (or positive semi-definite) real symmetric matrix and $[r]$ is a positive-definite real symmetric matrix.

Since the system has only single input variable, the matrix $[r]$ should be here a scalar quantity.

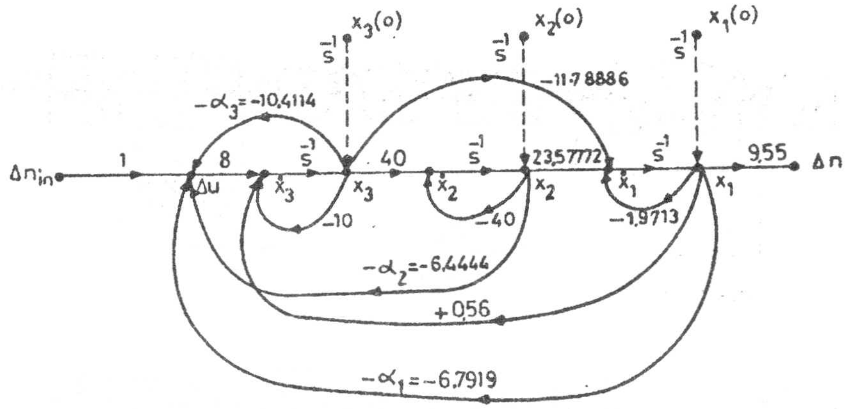



**Figure 2**. Closed loop state variable signal flow graph.

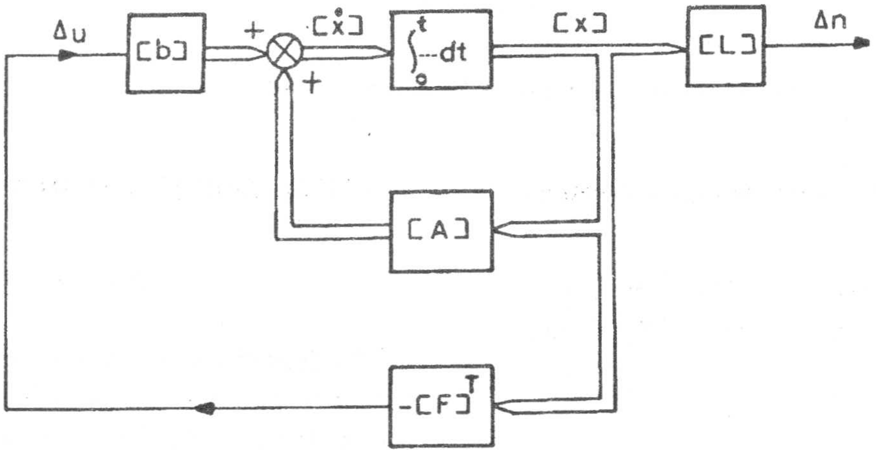



**Figure 3-a**. Optimal regulator's design by proportional linear state feedback.

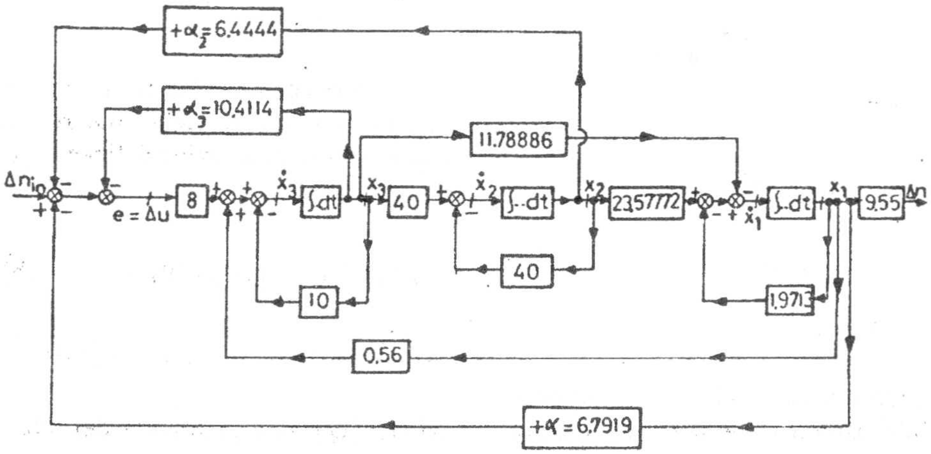



**Figure 3-b**. Decomposed closed loop block diagram of Figure (3-a).

The matrix $[P]$ which should be positive-definite real symmetric matrix is deduced from the solution of equation (13) and is used to find the optimal proportional linear state feedback to the regulator [10,11].

Testing of positive-definiteness of matrices could be performed by Silvester's theorem [11].

It is apparent that the system is controllable since the test matrix:

$$\left[\underline{A}^{n-1}\underline{b}\ \underline{A}_{n-2}\underline{b}\ \underline{A}_{n-3}\underline{b}\ \dots\ \underline{b}\right] = \left[\underline{A}^2\underline{b}\ \underline{A}\,\underline{b}\ \underline{b}\right]$$

has a rank = 3.

The concept of controllability involves the dependence of the state variables of the system on the inputs. It is essential to certify before designing a controller.

Now let: $-[F]^T = \left[-\alpha_1 \ -\alpha_2 \ -\alpha_3\right]$

Then the state variable signal flow and block diagrams of the optimal closed loop with proportional linear state feedback are indicated in Figures (2) and (3 a&b) respectively.

Referring to Figure (2), the S.F.G. has two paths, nine loops, eight (two non-touching loops) and two (three non-touching loops). In accordance with Mason's rule, the optimal closed loop transfer function in terms of $\alpha_1$, $\alpha_2$ and $\alpha_3$ is:

$$\frac{\Delta n(s)}{\Delta n_{in}(s)} = \frac{\text{Numerator}}{\text{Denominator}} \qquad (14)$$

where

$Numerator = -900.668904s + 36026.75616$, and

$$Denmoniator = s^3 + (51.9713 + 8\alpha^3)s^2 + (505.1667616 - 94.31088\alpha_1 + 320\alpha_2 + 335.770\alpha_3)s + (524.449536 + 3772.4352\alpha_1 + 630.816(\alpha_2 + \alpha_3))$$

Selecting $r = 1$ and preforming an iterative procedure for the values of $[Q] = m[I]$, $m = 5, 100, 5$ in order to obtain satisfactory static error and closed loop performance we get the following computer solution of equation (13):

$r = 1$ ,

$$[Q] = \begin{bmatrix} 90 & 0 & 0 \\ 0 & 90 & 0 \\ 0 & 0 & 90 \end{bmatrix},$$

$$[P] = \begin{bmatrix} 11.3684 & 5.3524 & 0.849 \\ 5.3542 & 3.7619 & 0.8055 \\ 0.849 & 0.8055 & 1.3014 \end{bmatrix},$$

$$-[F]^T = [-6.7919 \ \ -6.4444 \ \ -10.4114]$$

and equation (14) is transformed to:

$$\frac{\Delta n(s)}{\Delta n_{in}(s)} = \frac{-900.668904s + 36026.75616}{s^3 + 135.2625s^2 + 5422.664638s + 36779.3605} \qquad (15)$$

The final value theorem shows that $\Delta n(\infty) = 0.9795318$ with about 2% static error. The poles of equation (15) are located at

$-8.4538$, $-63.4044 \pm i\,18.1808$

Both the unit step and frequency responses of equation (15) are displayed in Figures (4) and (5) respectively.

To obtain the transfer function of the Diesel engine without regulator either $\alpha_1$, $\alpha_2$ and $\alpha_3$ are set equal to zero in equation (14) or using the retransformation from state space to conventional transfer function relationship (from equations 10 and 11) namely:

$$\frac{\Delta n(s)}{\Delta n_{in}(s)} = [L]\,[s[I] - [A]]^{-1}\,[b] = \frac{-900.668904s + 36026.75616}{s^3 + 51.9713s^2 + 505.1667616s + 524.449536} \qquad (16)$$

The unit step response of only the plant represented by equation (16) is shown in Figure (6).

### *Determination of State Estimator*

If the state variables of the system are not accessible by direct measurement and the system is observable, a state reconstructor could be designed to generate and build indirectly the states of the system by measuring the outputs and the inputs [12].

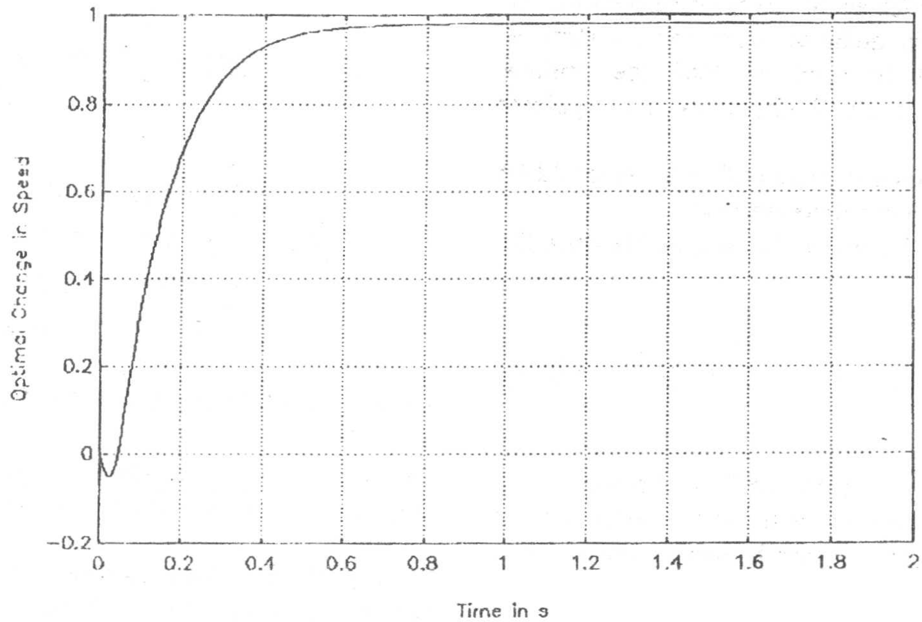



**Figure 4**. Riccati solution.

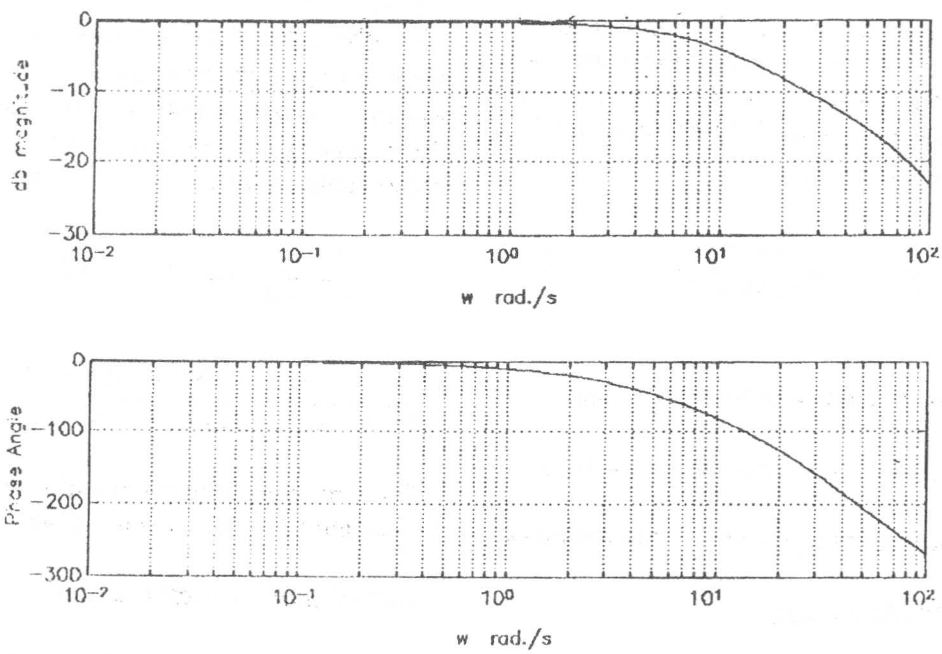



**Figure 5**. Optimal frequency response.

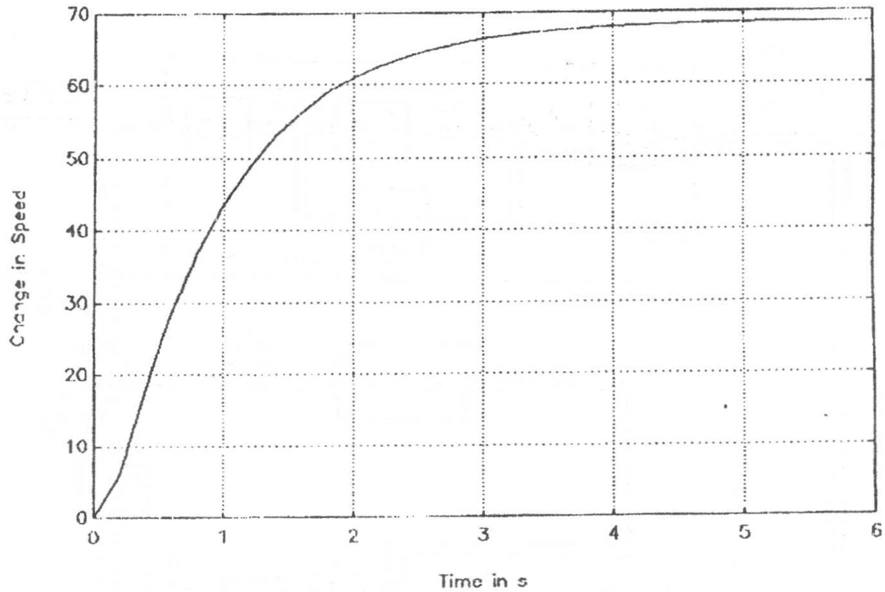



**Figure 6.** Speed of plant.

Observability is concerned with the problem of determining the states by observing the outputs. The mathematical condition for observability of the system is satisfied since:

$\left[\underline{L}^T\ \underline{A}^T\underline{L}^T\ \ldots..\ (\underline{A}^T)^{n-1}\ \underline{L}^T\right]$ has a rank = 3.

The duality principle of Kalman [10,11] for the analogy between controllability and observability can be applied too for this test, namely:

The pair $\underline{A}\underline{b}$ is controllable implies that the pair $\underline{A}^T\underline{b}^T$ is observable. Likewise, the pair $\underline{A}\,\underline{L}$ is observable implies that the pair $\underline{A}^T\underline{L}^T$ is controllable.

In the regulation of the Diesel engine the state $X_3$ is inaccessible while the state $X_2$ is difficult to measure despite having physical interpretation. Therefore a state observer is indispensable for the system and is shown in Figure (7), [12].

Reduced order or Luenberger observer will not be used due to the limited number of states. The additional complexity of the whole system after constructing the state estimator which raises the order of the control loop implies the significance of the stability problem.

Exact and estimated state matrix equations for Figure (7) are:

$$[\dot{X}] = [A]\,[X] + [b]\,[U]$$

$$[Y] = [L]\,[X]$$

$$[\dot{\hat{X}}] = [A]\,[\hat{X}] + [b]\,[U] - [M]\,[\tilde{Y}]$$

$$[\hat{Y}] = [L]\,[\hat{X}]$$

$$[\tilde{Y}] = [Y] - [\hat{Y}] \quad (17)$$

or

$$[\dot{\hat{X}}] = \left[\,[A] + [M]\,[L]\right]\,[\hat{X}] + [b]\,[U] - [M]\,[Y] \quad (18)$$

To determine the observer's matrix $\left[m_1\ m_2\ m_3\right]^T$, location of the roots of the new resulting characteristic equation should be selected by pole placement concept. For the rapid decay of the state errors $[\tilde{x}] = [x] - [\hat{x}]$ which may result due to noise or perturbations, the poles of equation (18) are chosen far enough from the imaginary axis at $s = -2, -3$ and $-4$.

It follows that the characteristic polynomial of equation (18) is:

$$(s+2)\,(s+3)\,(s+4) = \det\left[\,s\,[I] - \{\,[A] + [M]\,[L]\,\}\,\right] \text{or}$$

$$\det\begin{bmatrix} (s + 1.9713 - 9.55\,m_1 & -23.57772 & 11.78889 \\ -9.55\,m_2 & (s+40) & -40 \\ -0.56 - 9.55\,m_3 & 0 & (s+10) \end{bmatrix} =$$

$$s^3 + 9\,s^2 + 26\,s + 24 \quad (19)$$

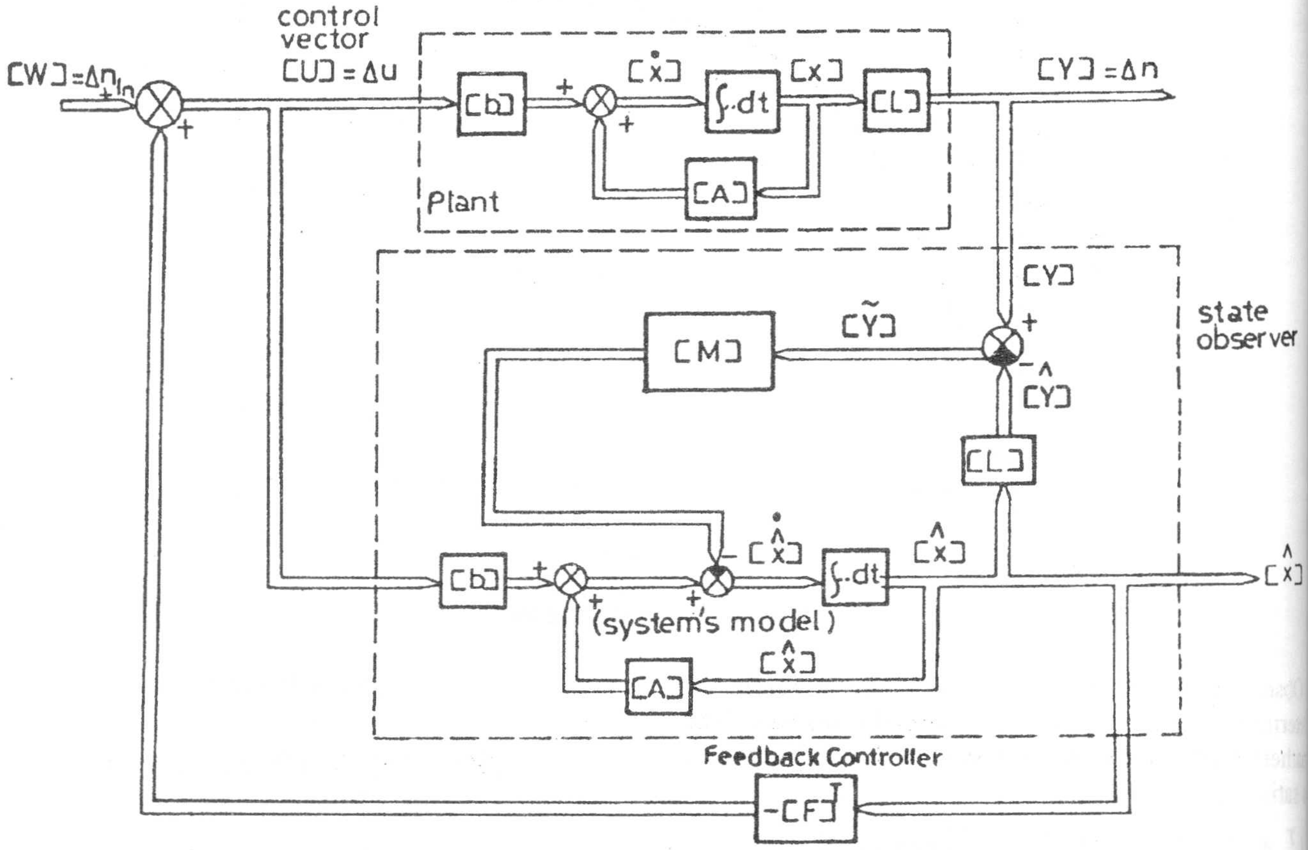



Figure 7. Block diagram of state reconstructor.

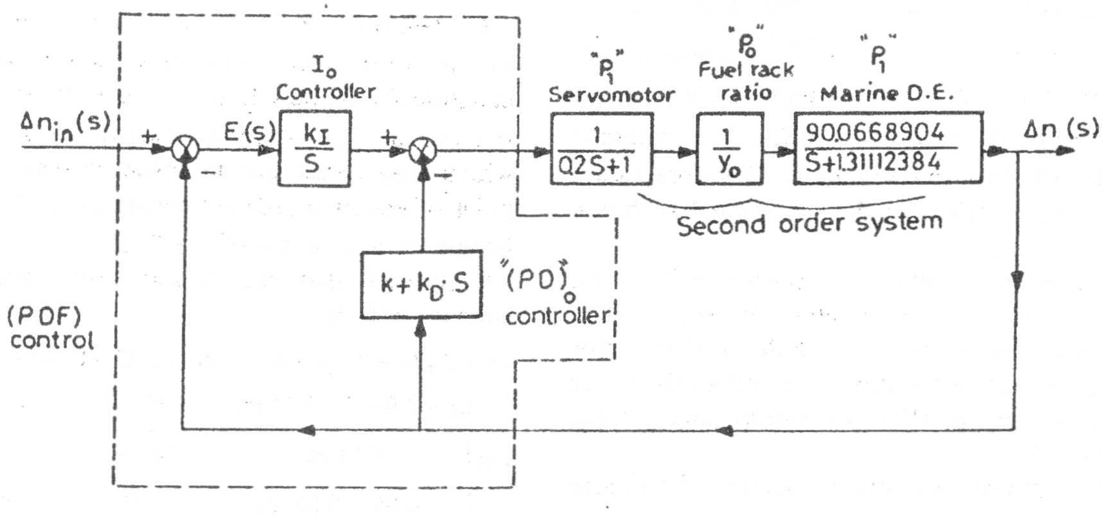



Figure 8. Regulation of marine D.E. with (PDF) control.

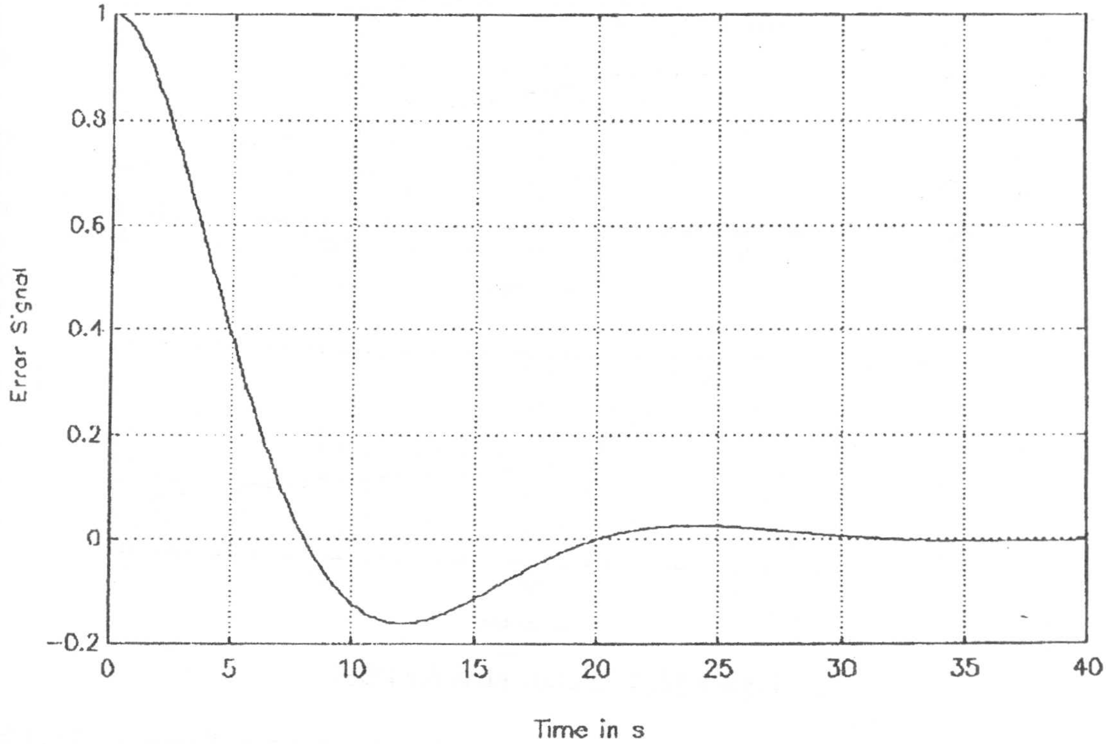



**Figure 9**. Display of error signal with PDF.

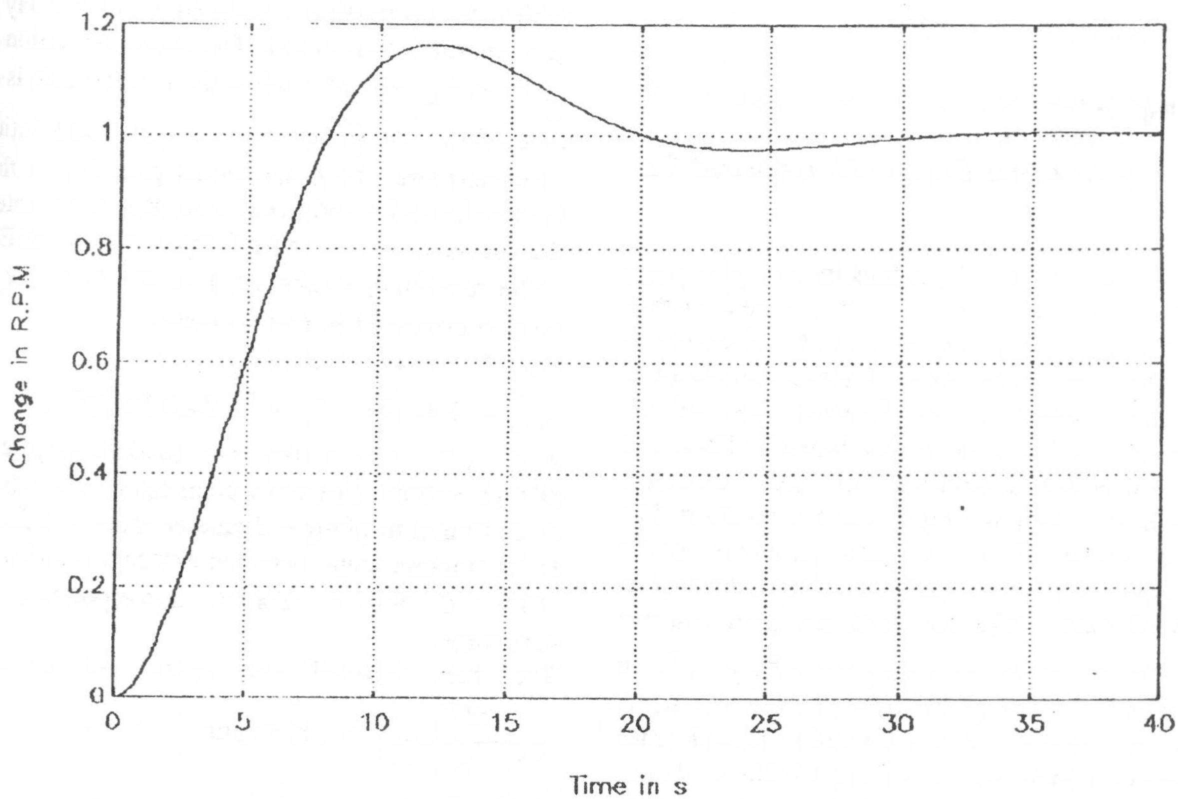



**Figure 10**. Speed dynamics with PDF.

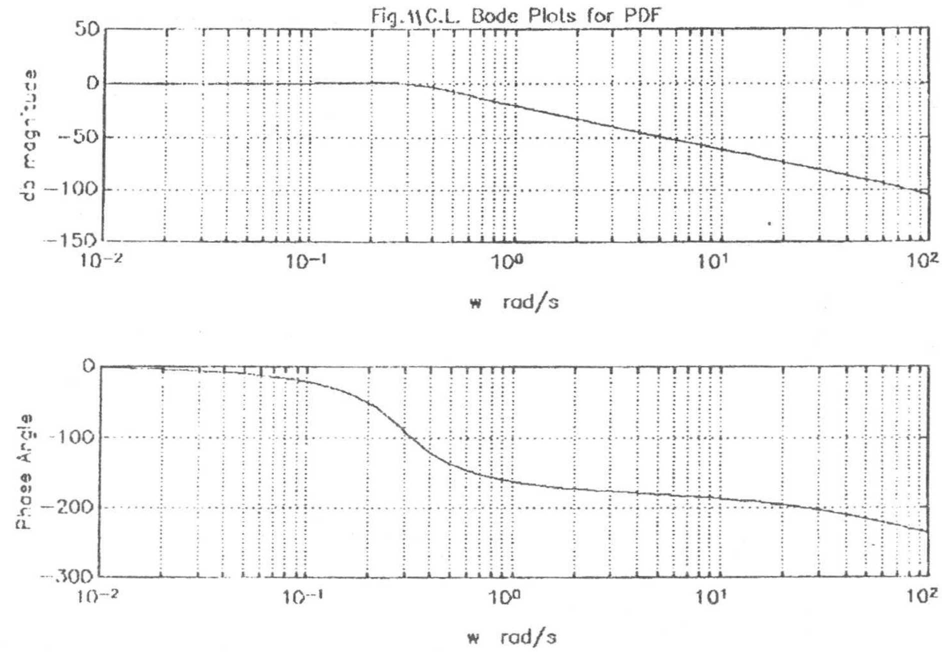



**Figure 11**. C.L Bode plots for PDF.

The values of $[A]$ and $[L]$ are given in equations (10,11).
solution of equation (19) yields:

$$[m_1 \;\; m_2 \;\; m_3]^T = [4.499612565 \;\; -7.409050595 \;\; 0.0099738623]^T$$

*Regulation of the Diesel Engine with Optimized Gain PDF Control*

One of the most recent algorithms in control strategy is pseudo derivative feedback (PDF), a new control structure that captures the advantages of derivative (D) action without the attendant difficulties caused by a differentiator located in the forward path of the controller [2,13]. This concept developed by Phelon of Cornell university eliminates all the numerator dynamics in the command input transfer function. For a second order plant -as this problem under investigation- only the integral controller is located in the forward path while the feedback path transfer function has the form: $(1 + K_{D1} \, . \, s + K_{D2} \, . \, s^2)$ in order that the control signal depends on the output, its derivative and its integral. Since reliable second order derivatives of signals are almost impossible to obtain, the problem is over-ridden by adopting only proportional- derivative action in a feedback path of a minor loop as shown in figure (8). Ideal integral proportional derivative control actions are concer while the proportional with first order delay (0 servomotor is introduced. The derivative action gai taken as $K_D = 0.85 \; s$ while the integral gain is cho as $0.085 \; s^{-1}$ to match with the previous solution.

Furthermore, the proportional gain $(K)$ of the P is intended to be optimized according to the integra the square of the error performance index (ISE).

The maximum stroke $(y_o)$ of the fuel rack sho first be estimated in this sequence:

For $C.V = 41870$ KJ/Kg,

$\eta_{bth} = 0.41898$ , $P_e = 10.24512 * 10^6$ w,

$z = 6$ , $n_o = 99.8$ rpm (two stroke), specific gra of fuel = 0.88, Stroke to diameter ratio of plunge = 5, Pinion to plunger diameter ratio = 2.25. Helix rotation angle between extreme positions (40%-110 %) = $2\pi/3$ corresponding to periphery.

The fuel injected per cycle per cylinder

$$\frac{60 \, P_e}{z \, n_o \;\; \eta_{bth} \;\; C.V} = 58.52 \text{ gm}$$

Then:

$$y_o = \frac{2.25\,\pi}{3} \sqrt[3]{\frac{58.52 * 4}{5\,\pi * 0.88}} = 6 \text{ cm}$$

Now all numerical data for the block diagram shown in Figure (8) are determined with the exception of K which will be optimized.

It is to be noted that both dead and delay times of the fuel are neglected in the transfer function of the Diesel engine for being minute.

*Parseval's Theorem and Application*

According to the ISE performance index the frequency-time correlation derived from Fourier integrals [11], it could be written:

$$\int_0^{\infty} e^2(t)\,dt = \int_0^{\infty} e(t) \left[ \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} E(s)\, e^{st} ds \right] dt \quad (20)$$

provided that;

$$\int_0^{\infty} |\, e(t)\,|\, dt < \infty$$

Interchanging the order of integration in equation (20) and applying the definition of Laplace transforms then:

$$\int_0^{\infty} e^2(t)\,dt = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} E(s) \left[ \int_{t=0}^{t=\infty} e(t)^{+st} dt \right] ds =$$

$$\frac{1}{2\pi i} \int_{-i\infty}^{i\infty} E(s)\, E(-s)\, ds \quad (21)$$

Jury [14] computed and published tables for the solution of equation (21) provided that $E(s)$ can be written in the form:

$$E(s) = N(s) / D(s) \text{ where:}$$

$$N(s) = b_o + b_1 s + \dots + b_{n-1} s^{n-1},$$

$$D(s) = a_o + a_1 s + \dots + a_n s^n$$

where $D(s)$ has zeros only in the left half of the complex plane.

The result for $n = 3$ for continuous systems (the case of the D.E., Figure (8)) is [11,14]:

$$J_3 = \frac{b_2^2 a_o a_1 + a_o a_3 (b_1^2 - 2 b_o b_2) + b_o^2 a_2 a_3}{2 a_o a_3 (-a_o a_3 + a_1 a_2)} \quad (22)$$

*Assessment of the PDF Control with the Marine Diesel Engine*

The error function $E(s)$ can be deduced from the closed loop block diagram displayed in figure (8) with unit step input as:

$$E(s) = \frac{0.2 s^2 + 13.87158943 s + (1.31112384 + 15.0111484 K)}{0.2 s^3 + 13.87158943 s^2 + (1.31112384 + 15.0111484 K) s + 1.260936466} \quad (23)$$

applying equation (22) to equation (23) yields:

$$J_3 = \frac{625.1497453 K^2 + 108.4482094 K + 53.229179}{105.0251572 K + 9.04605116} \quad (24)$$

Differentiating equation (24) w.r.t (K) and equating to zero gives:

$$14.24411173 k^2 + 2.453754261 k - 1 = 0$$

or $k = 0.192477189$

Substituting the optimized value of $k$ and reducing the block diagram represented in Figure (8) we get:

$$\frac{\Delta n(s)}{\Delta n_{in}(s)} = \frac{3.15234}{0.5 s^3 + 34.6789 s^2 + 10.501 s + 3.15234},$$

$$E(s) = \frac{0.2 s^2 + 13.87158943 s + 4.200427488}{0.2 s^3 + 13.87158943 s^2 + 4.200427488 s + 1.260936466} \quad (25)$$

The static error of the control system is clearly zero. Artificial intelligence symbolic as well as numerical and graphics manipulation packages [15,16] were used. Demonstration of the error signal, speed dynamics and closed loop Bode plots for the control system composed of the marine Diesel engine with optimized gain PDF control are shown in Figures (9), (10) and (11) respectively.

## DISCUSSION

Analysis of results plotted in Figures (4,5,6,9,10,11) reveals that the time response of the plant without controller possesses the behavior of self regulating plant but with extremely excessive impractical values. Regarding the Riccati solution which comprises optimization of errors as well as control energy, it represents much more idealized response in comparison with the PDF control with optimized gain. In what concerns the speed of response, delay and rise times values picked are: 6 versus 0.12,0.15 versus 4s and 0.2 versus 5s, for Riccatti solution and PDF control respectively.

Moreover, settling times and static errors are 0.8 versus 27s and 2% versus 0% sequentially. Maximum overshoot and the corresponding time which exist only with the PDF control are 15% and 12s respectively.

On the other hand, values of bandwidth are 7 and 0.3 rad/s with no resonant frequency- for optimal and optimized gain controls respectively. Similarly, the change of phase angle w.r.t the variation of the operating frequency $\omega$ keeps almost unchangeful over a wide range in case of optimal regulator's design.

Summing up, optimal regulator's design is undoubtedly advantageous to the PDF control with optimized gain. Nevertheless dynamic deviations between the two techniques may not overweigh the additional instrumentation and design of state reconstructor with the associated augmentation of the complexity of the control system and the encountered arising stability problems.

## CONCLUSION

The automatic speed control loop of marine Diesel engines was investigated in time and frequency domains with both proportional linear quadratic optimal regulator- compromising both the error function and the control energy and pseudo derivative feedback control with optimized proportional gain according to ISE performance criterion. The reduced matrix Riccati equation as well as Fourier integrals and Parseval's theorem were applied. Building of state estimator was carried out for the measurement of inaccessible states.

It can be concluded that merits gained with the optimal regulator over the optimized PDF control do not overweigh the increased instrumentation and complication of the system due to raising its order.

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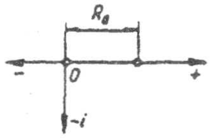
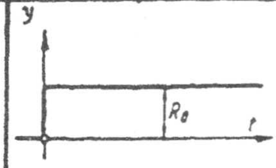
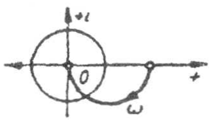
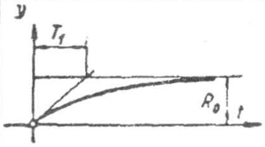
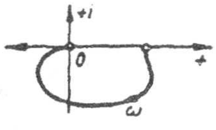
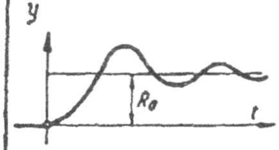
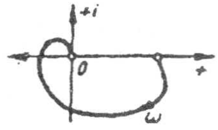
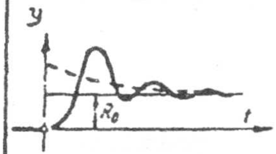
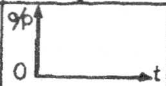
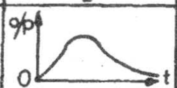
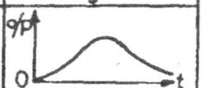
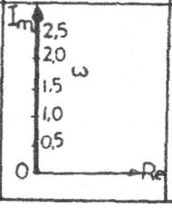
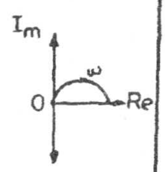
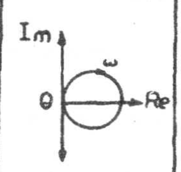
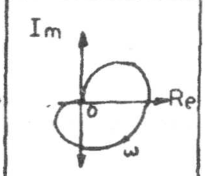
## Appendix

Tables of Transfer-Function Plots of Principle Elements and Controllers.

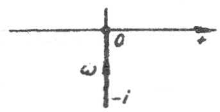
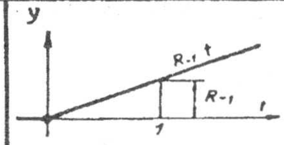
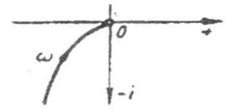
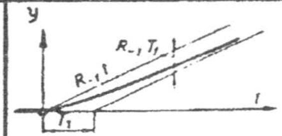
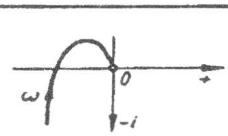
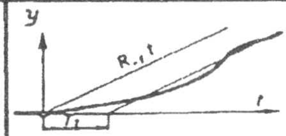
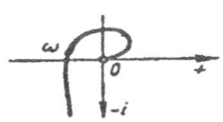
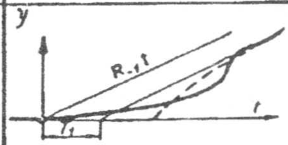
| Differential Equation | Frequency Response | Polar Plot | Unit-Step Response |
|---|---|---|---|
| $y = R_0 x$ | $Y(i\omega) = R_0$ |  |  |
| $T_1 y' + Yy = R_0 x$ | $Y(i\omega) = \frac{R_0}{T_1 i\omega + Y1}$ |  |  |
| $T_2 y'' + T_1 y' + Yy = R_0 x$ | $Y(i\omega) = \frac{R_0}{T_2(i\omega)^2 + T_1 i\omega + 1}$ |  |  |
| $T_3 y''' + T_2 y'' + T_1 y' + Yy = R_0 x$ | $Y(i\omega) = \frac{R_0}{T_3(i\omega)^3 + T_2(i\omega)^2 + T_1(i\omega) + 1}$ |  |  |

(A)- Proportional Elements or Controllers ( P-Property )

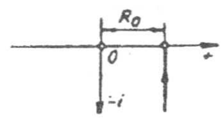
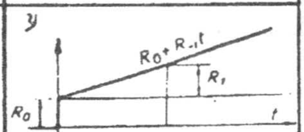
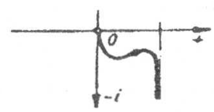
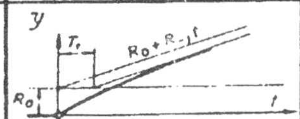
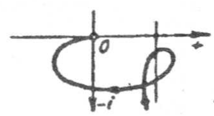
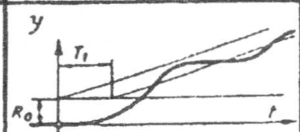
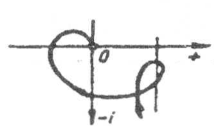
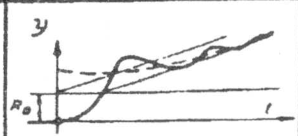
| "O" Element | Delay order | | | |
|---|---|---|---|---|
| | no delay 0 | 1 | 2 | 3 |
| Transient Response |  |  |  |  |
| Frequency Response |  |  |  |  |

(B)

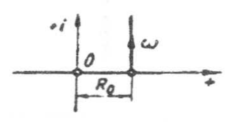
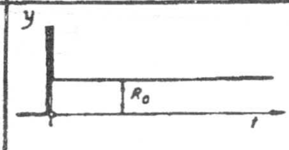
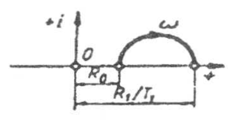
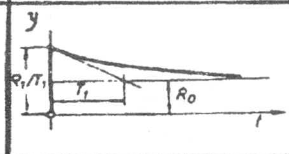
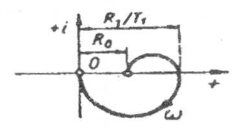
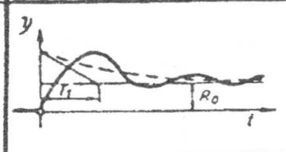
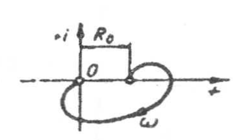
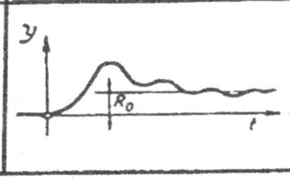
O- Property

| Differential Equation | Frequency Response | Polar Plot | Unit-Step Response |
|---|---|---|---|
| $y = R_{-1}\int x\,dt$ | $Y(i\omega) = R_{-1}(i\omega)^{-1}$ |  |  |
| $T_1 y' + y = R_{-1}\int x\,dt$ | $Y(i\omega) = \frac{R_{-1}(i\omega)^{-1}}{T_1 i\omega + 1}$ |  |  |
| $T_2 y'' + T_1 y' + y = R_{-1}\int x\,dt$ | $Y(i\omega) = \frac{R_{-1}(i\omega)^{-1}}{T_2(i\omega)^2 + T_1 i\omega + 1}$ |  |  |
| $T_3 y''' + T_2 y'' + T_1 y' + y = R_{-1}\int x\,dt$ | $Y(i\omega) = \frac{R_{-1}(i\omega)^{-1}}{T_3(i\omega)^3 + T_2(i\omega)^2 + T_1 i\omega + 1}$ |  |  |

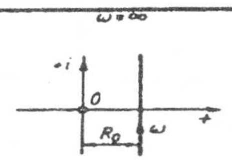
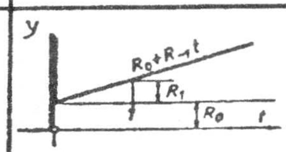
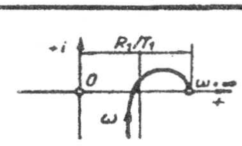
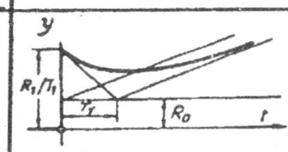
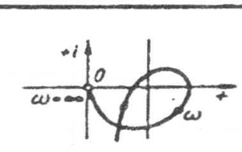
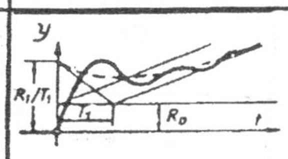
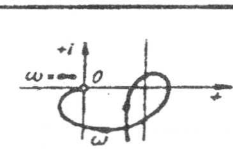
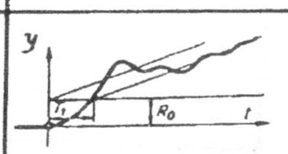
(C) Integral Elements or Controllers ( I-Property )

| Differential Equation | Frequency Response | Polar Plot | Unit-Step Response |
|---|---|---|---|
| $y = R_0 x + R_{-1}\int x\,dt$ | $Y(i\omega) = R_{-1}(i\omega)^{-1} + R_0$ |  |  |
| $T_1 y' + y = R_0 x + R_{-1}\int x\,dt$ | $Y(i\omega) = \frac{R_{-1}(i\omega)^{-1} + R_0}{T_1 i\omega + 1}$ |  |  |
| $T_2 y'' + T_1 y' + y = R_0 x + R_{-1}\int x\,dt$ | $Y(i\omega) = \frac{R_{-1}(i\omega)^{-1} + R_0}{T_2(i\omega)^2 + T_1 i\omega + 1}$ |  |  |
| $T_3 y''' + T_2 y'' + T_1 y' + y = R_0 x + R_{-1}\int x\,dt$ | $Y(i\omega) = \frac{R_{-1}(i\omega)^{-1} + R_0}{T_3(i\omega)^3 + T_2(i\omega)^2 + T_1 i\omega + 1}$ |  |  |

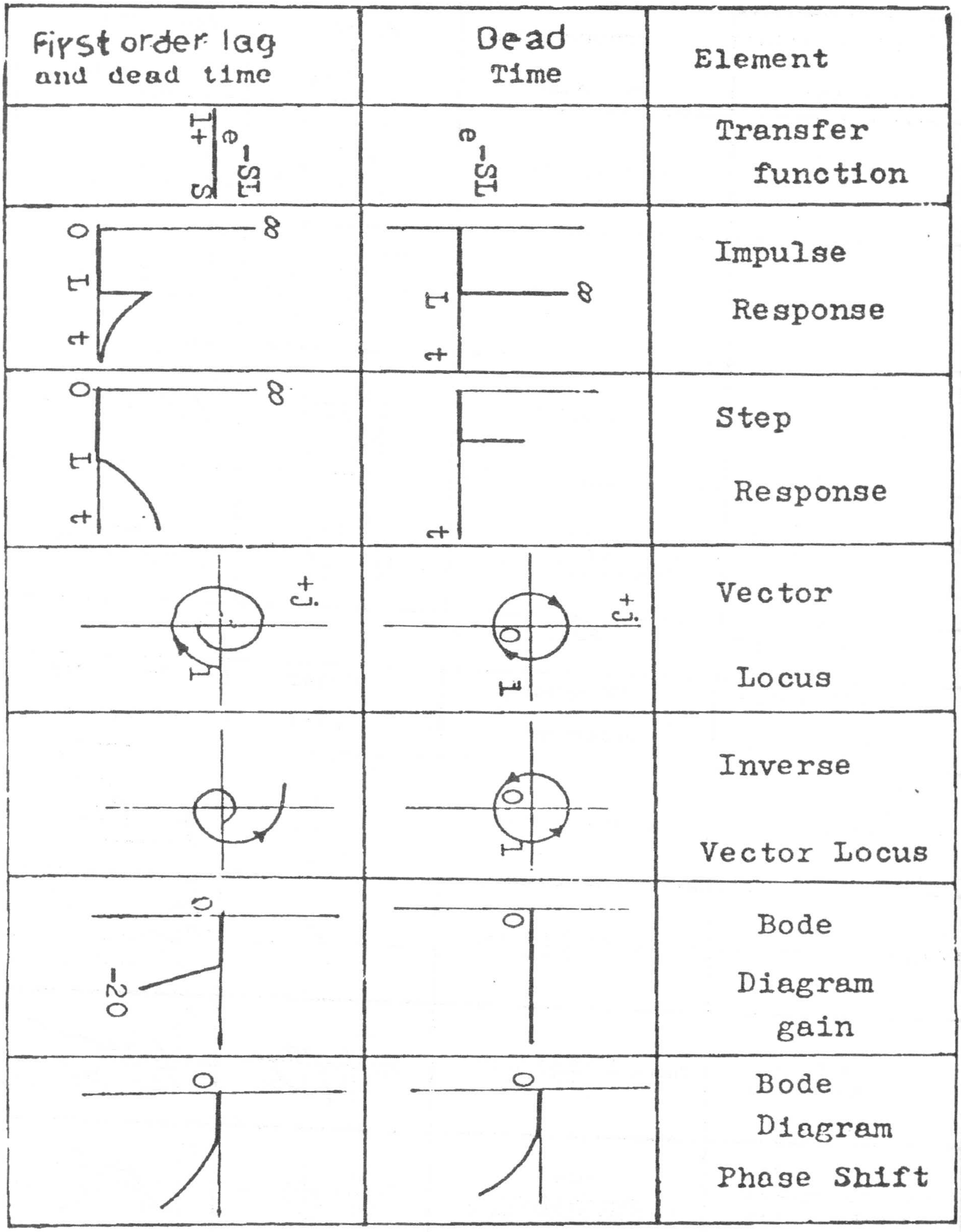
(D) PI -Property

| Differential Equation | Frequency Response | Polar Plot | Unit-Step Response |
|---|---|---|---|
| $y = R_1x' + R_0x$ | $Y(i\omega) = R_1 i\omega + R_0$ |  |  |
| $T_1y' + y = R_1x' + R_0x$ | $Y(i\omega) = \frac{R_1 i\omega + R_0}{T_1 i\omega + 1}$ |  |  |
| $T_2y'' + T_1y' + y = R_1x' + R_0x$ | $Y(i\omega) = \frac{R_1 i\omega + R_0}{T_2(i\omega)^2 + T_1 i\omega + 1}$ |  |  |
| $Ty''' + T_2y'' + T_1y' + y = R_1x' + R_0x$ | $Y(i\omega) = \frac{R_1 i\omega + R_0}{T(i\omega)^3 + T_2(i\omega)^2 + T_1 i\omega + 1}$ |  |  |

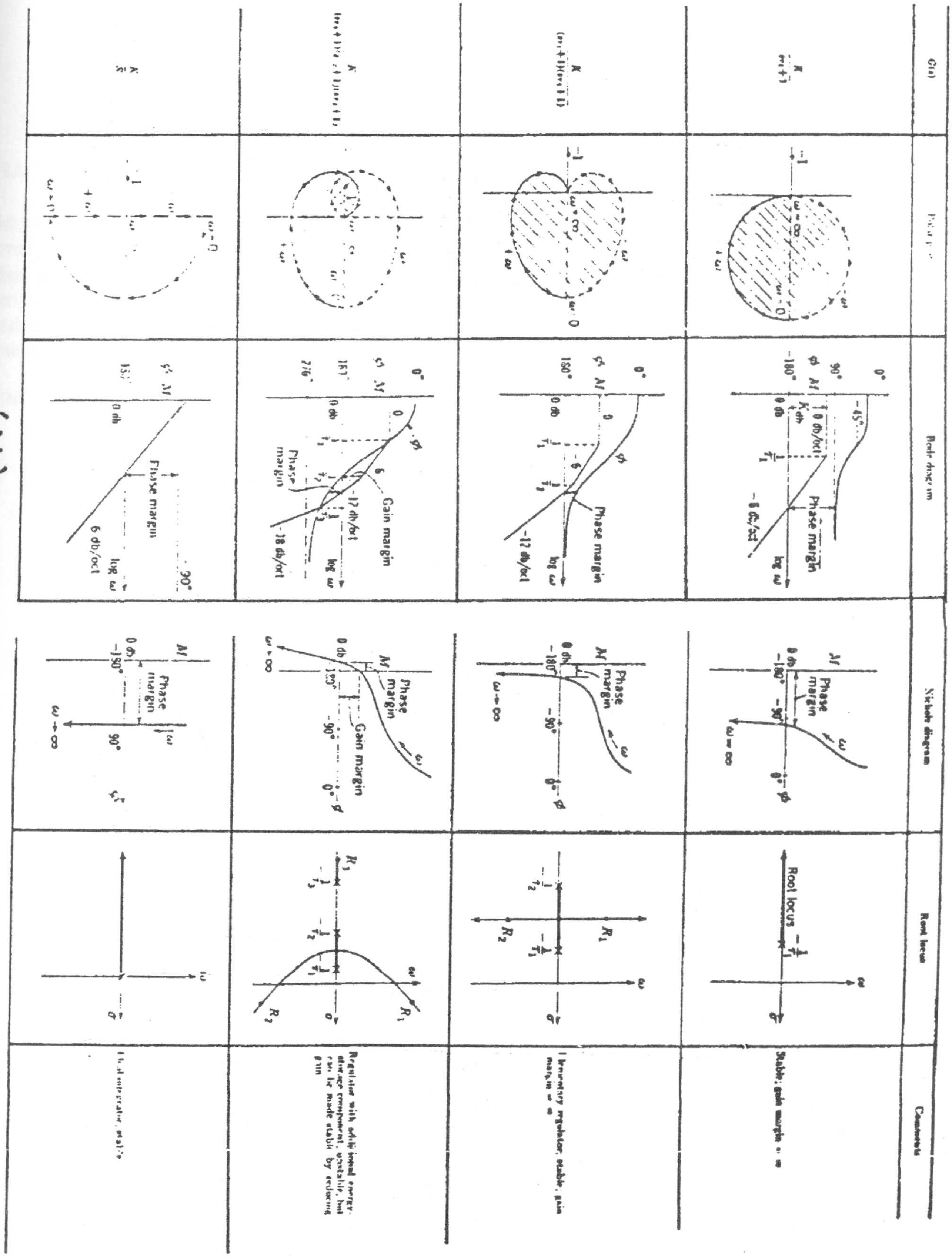
(E) PD-Property

| Differential Equation | Frequency Response | Polar Plot | Unit-Step Response |
|---|---|---|---|
| $y = R_1x' + R_0x + R_{-1}\int x\,dt$ | $Y(i\omega) = R_1 i\omega + R_0 + R_{-1}(i\omega)^{-1}$ |  |  |
| $T_1y' + y = R_1x' + R_0x + R_{-1}\int x\,dt$ | $Y(i\omega) = \frac{R_1 i\omega + R_0 + R_{-1}(i\omega)^{-1}}{T_1 i\omega + 1}$ |  |  |
| $T_2y'' + T_1y' + y = R_1x' + R_0x + R_{-1}\int x\,dt$ | $Y(i\omega) = \frac{R_1 i\omega + R_0 + R_{-1}(i\omega)^{-1}}{T_2(i\omega)^2 + T_1 i\omega + 1}$ |  |  |
| $T_3y''' + T_2y'' + T_1y' + y = R_1x' + R_0x + R_{-1}\int x\,dt$ | $Y(i\omega) = \frac{R_1 i\omega + R_0 + R_{-1}(i\omega)^{-1}}{T_3(i\omega)^3 + T_2(i\omega)^2 + T_1 i\omega + 1}$ |  |  |

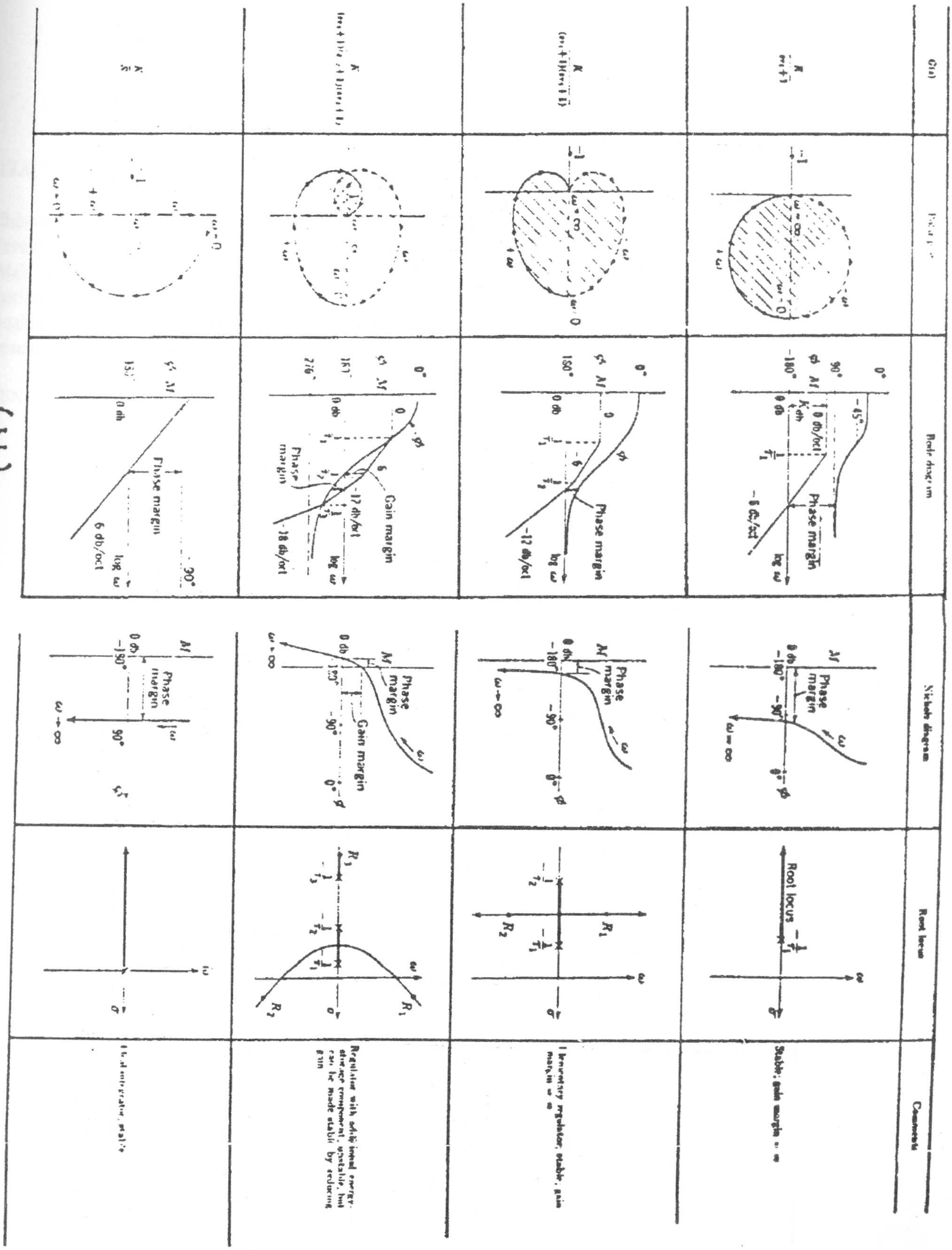
(F) PID-Property


First order lag and dead time
Dead Time
Element
Transfer function
Impulse Response
Step Response
Vector Locus
Inverse Vector Locus
Bode Diagram gain
Bode Diagram Phase Shift
+j
-20

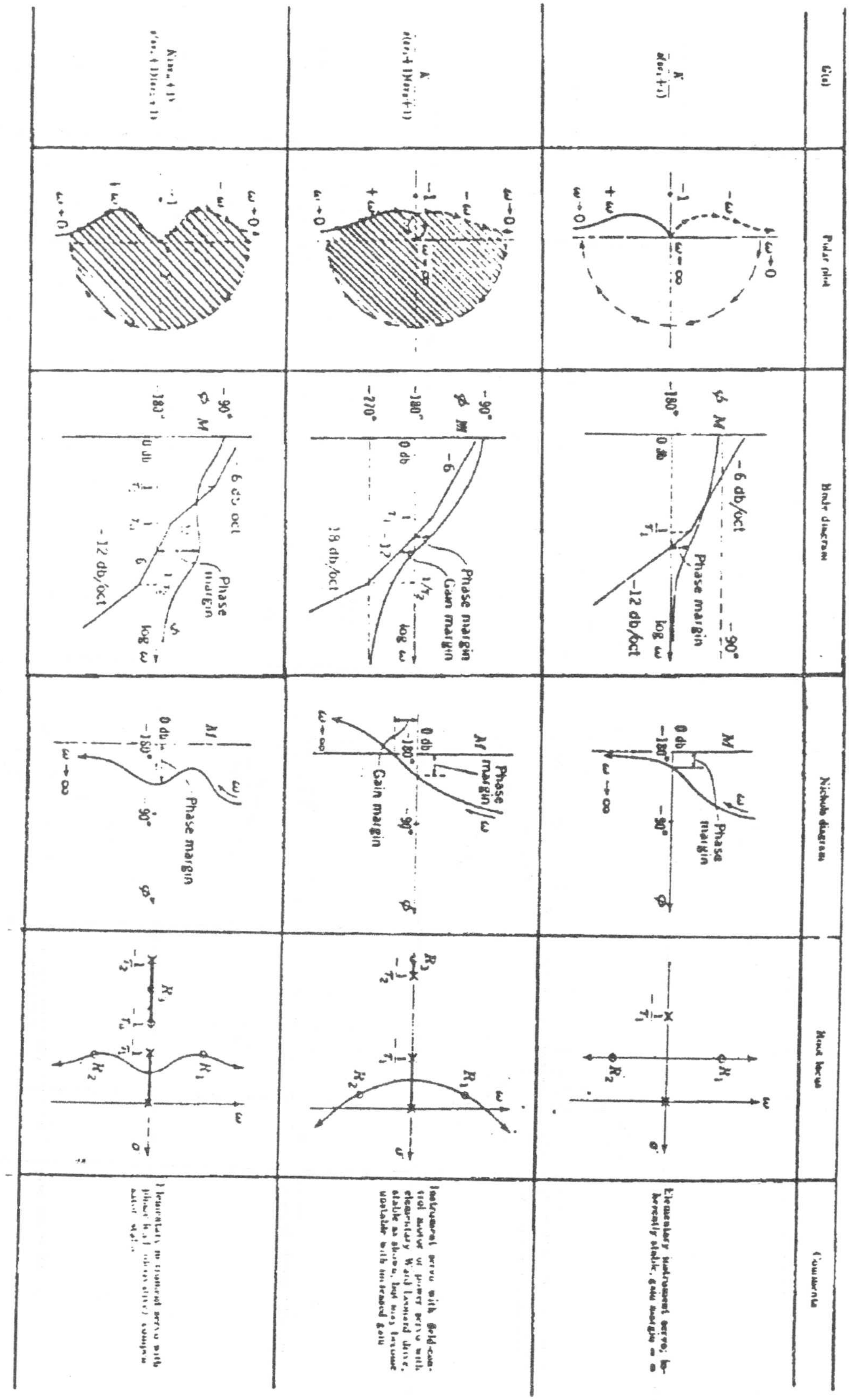
(G)


G(s)
Polar plot
Bode diagram
Nichols diagram
Root locus
Comments
Phase margin
Gain margin
-6 db/oct
-12 db/oct
-18 db/oct
log ω
Stable; gain margin = ∞
Elementary regulator; stable; gain margin = ∞
Regulator with additional energy-storage component; unstable, but can be made stable by reducing gain
Ideal integrator; stable

(H)


G(s)
Polar plot
Bode diagram
Nichols diagram
Root locus
Comments
Phase margin
Gain margin
Root locus
Stable; gain margin = ∞
Elementary regulator; stable, gain margin = ∞
Regulator with additional energy-storage component, unstable, but can be made stable by reducing gain
Ideal integrator, stable

(H)


G(s)
Polar plot
Bode diagram
Nichols diagram
Root locus
Comments
Phase margin
Gain margin
-6 db/oct
-12 db/oct
log ω
Elementary instrument servo; inherently stable, gain margin = ∞
Instrument servo with field-control motor or power servo with elementary Ward Leonard drive, stable as shown, but may become unstable with increased gain

(1)