RELATION BETWEEN THE ERROR IN THE HEIGHT DIFFERENCE AND THE COEFFICIENT OF REFRACTION IN TRIGONOMETRIC LEVELLING

Sayed El-Naghi
Transportation Engineering Department, Faculty of Engineering
Alexandria University, Alexandria, Egypt.

ABSTRACT

The problem of the proper refraction correction to be used in reduction of trigonometric levelling is discussed. A method for the determination of the atmospheric refraction in trigonometric levelling is presented. The study includes the analysis and application of an approximate formula to compute the probable error in the height difference between two stations. The proposed theoretical method yields numerical results which seems to be satisfactory and saves time and effort.

1. INTRODUCTION

Trigonometrical levelling is the process of determining the differences of elevations of stations from observed angles and known distances, which are assumed to be either horizontal or geodetic lengths of mean sea-level (M.S.L.)

The accuracy of direct observations is always influenced by the irregularities in the coefficient of refraction (K), while in the reciprocal method the equality of the refraction effects is merely assumed, and for this reason simultaneous observations are desirable, if not always practicable; and in consequence, the work is often carried out on different days at the time minimum refraction effect.

The results in the trigonometric levelling can be obtained with a good accuracy if we solve the refraction problem. The aim of this paper is to give, in a straight-forward and simplified procedure, values of the refraction difference (ΔK) to be used in calculating the probable error in the height difference (Δh) between two stations when using trigonometric levelling.

2. NOTATION

In developing reduction formulae, it will be supposed that it is required to determine the elevation of a station B from that of a station A, assumed known. The notation adopted is as follows:

h_1 = elevation of A above M.S.L.
h_2 = elevation of B above M.S.L.
S = geodetic or M.S.L. distance between A and B.
R = radius of the earth at the mid-latitude of A and B.
θ = angle subtended at the centre of the earth by S.
α = observed vertical angle from A to B, corrected in reciprocal observations for the difference in height of the instrument and the signal above the ground.
β = " from B to A.
K = coefficient of refraction

3. METHODS OF TRIGONOMETRICAL LEVELLING

Two general methods are employed to obtain the difference of elevation of two points of known distance apart. In the first, the difference of elevation is computed from the vertical angle measured at one of the stations only, and a knowledge of the value of the refraction coefficient is required. In the second, it is determined from vertical angles observed from each station to the other. The object of such reciprocal observations is to remove the effect of uncertainty regarding the value of the coefficient of refraction. The angle of refraction is taken as being the same for both.

3.1. First method

Let A (Figure (1)) be the station occupied, and let α

be the uncorrected angle of elevation observed to B. The following formulae are applicable if the corresponding angle β at B is deduced as follows [1]:

$$\beta = \alpha + \theta - 2K\theta \tag{1}$$

α being positive for an elevation and negative for a depression: a negative value of β indicates an angle of elevation B. The value of β so deduced includes all errors of observation and in the assumed value for K. The required difference of elevation is [2]:

$$h_2 - h_1 = S \tan\left[\alpha + \left(\frac{1}{2} - K\right)\theta\right]\left(1 + \frac{h_1 + h_2}{2R} + \frac{S^2}{12R^2}\right) \tag{2}$$

in which α is given a negative sign if depression. The $\frac{S^2}{12R^2}$ term may safely be omitted, and $\frac{h_1 + h_2}{2R}$ may be treated as negligible, in rough determination. For the first method of solution we have $\beta' = (\alpha' + \Theta)$, α' being given the negative sign for a depression, and

$$h_2 - h_1 = (R + h_1)\left[\frac{\cos\alpha'}{\cos(\alpha' + \theta)} - 1\right] \tag{3}$$

3.2 *Second method*

Figure (1) illustrates the case in which B, the point of unknown elevation, is higher than A. The angles α and β are the observed angles corrected for difference in height of eye and object, and may be treated as the angles which would be observed at and to A and B, the respective ground points.

The required difference of elevation $(h_2 - h_1)$ is obtained by formula [2]:

$$h_2 - h_1 = S \tan\frac{\alpha + \beta}{2}\left(1 + \frac{h_1 + h_2}{2R} + \frac{S^2}{12R^2}\right) \tag{4}$$

In applying this formula, a first approximation is derived from:

$$h_2 - h_1 = S \tan\frac{\alpha + \beta}{2}$$

and the value of h_2 so obtained is used for a second approximation by the formula.

An alternative method of solution is based on obtaining the true vertical angles H_1 AB and H_2 BA by eliminating the refraction angles,

$$H_1 AB = (\alpha - K\theta), \text{ and } H_2 BA = (\beta + K\theta),$$

but

$$H_2 BA - H_1 AB = \theta,$$

$$\therefore K\theta = \frac{\theta - \beta + \alpha}{2} \tag{5}$$

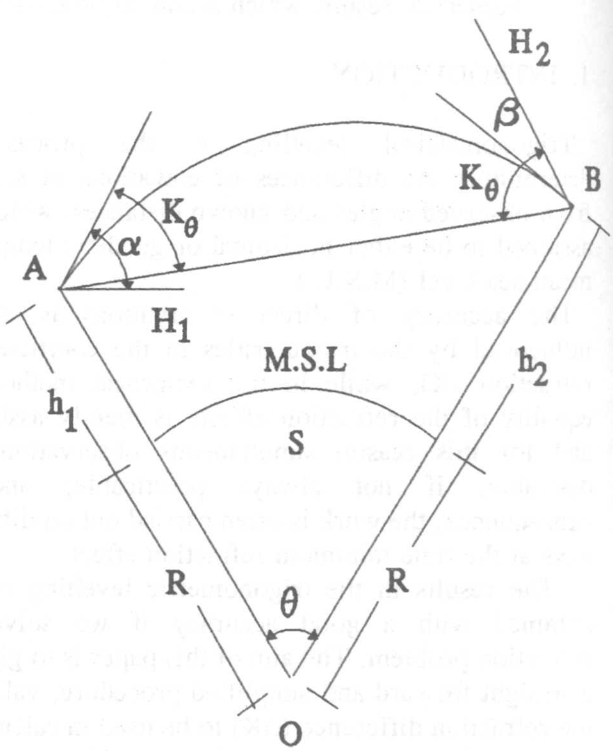

Figure 1.

4. ERRORS OF TRIGONOMETRICAL LEVELLING

The probable error computed from the differences of elevation given by repeated vertical angle observations is not likely to be a trustworthy index of the precision of a result. Even in simultaneous reciprocal observation, constant errors are introduced in the assumptions made regarding the effect of refraction. The quality of a system of trigonometric levelling is best ascertained by connecting it at intervals to lines of spirit levelling or by reference to its own errors of closure.

If expression (4) is differentiated with respect to α and the last two terms in the bracket are neglected, we have [3]:

$$\Delta h = \frac{1}{2} S \sec^2 (\alpha + \beta) . \Delta\alpha \qquad (6)$$

in which Δh may be considered to be the probable error in (h_2-h_1) resulting from an error $\Delta\alpha$ in the measurement of the angle α. Hence, if the angular errors are assumed to be independent of the length of the line, the probable error in the computed difference of elevation will be directly proportional to the length of the line.

As regards errors in refraction, it is obvious from equation (2), that, provided they obey the ordinary laws of accidental error, these will behave exactly similar to accidental angular errors when the line is observed in one direction only. If reciprocal observations are taken, and the coefficients of refraction at the two stations are not equal, and taking into consideration that the effect of earth curvature and refraction equals $\left[\frac{S^2}{2R}(1-2K)\right]$, it is easy to show that equation (4) will contain the extra term

$$\frac{S^2}{2R}(K_2-K_1)\left[1+\frac{(h_2+h_1)}{2R}+\frac{S^2}{12R^2}\right]$$

in which the first term will be much larger than either of the other two, and K_2 and K_1 represent the coefficients of refraction at B and A respectively. Hence, if $(K_2-K_1) = \Delta K$ is treated as an error in the refraction, we have

$$\Delta h = \frac{S^2}{2R}\Delta K \text{ approximately} \qquad (7)$$

and, in this case, the error (Δh) varies as the square of S and ΔK.

5. APPLICATIONS AND RESULTS

In actual practice by using equation (6), the angle $\frac{1}{2}(\alpha+\beta)$ is generally small so that $\sec^2 \frac{1}{2}(\alpha + \beta)$ may be put equal to unity.

On the basis of the present study the author furnished a simple method for the determination of the relation between the error in the height difference (Δh) and the refraction difference (ΔK) in trigonometric levelling.

From the foregoing discussions a comparison between the two equation (6) and (7), shows that the use of the former equation requires field observations and is rather tedious; where as the application of equation (7) is more simplified for the determination of the relation between Δh and ΔK.

By using equation (7), different values for the variables (ΔK) and S are assumed, and accordingly, the error in the height difference (Δh) may be obtained.

Results of the calculations are shown in Figure (2) and recorded in Table (1), which shows the values of (Δh) according to the following:

a. R = 6371 KM.
b. The value of S ranges from 3 KM. to 10 KM
c. The value of $\Delta K = K_2 - K_1$ is assumed from 0.01 to 0.20.

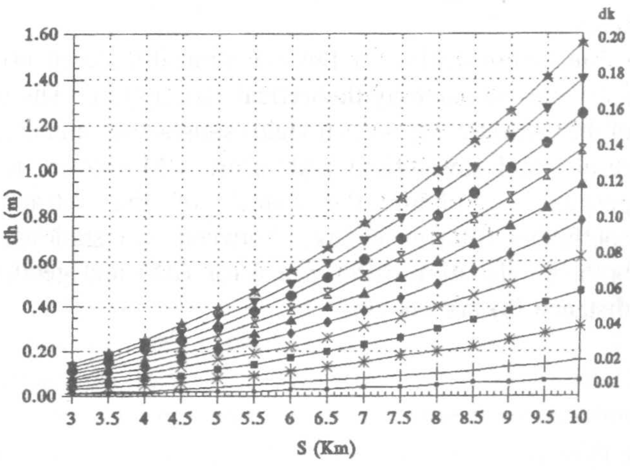

Figure 2. Relation between the Error in the Height Difference (dh) versus Distance (S)

Table 1.

ΔK / S, KM	0.01	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20
Values Δh, M											
3.0	0.01	0.01	0.03	0.04	0.06	0.07	0.08	0.10	0.11	0.13	0.14
3.5	0.01	0.02	0.04	0.06	0.08	0.10	0.12	0.13	0.15	0.17	0.19
4.0	0.01	0.02	0.05	0.08	0.10	0.12	0.15	0.18	0.21	0.23	0.25
4.5	0.02	0.03	0.06	0.09	0.13	0.16	0.19	0.22	0.25	0.28	0.32
5.0	0.02	0.04	0.08	0.12	0.16	0.20	0.23	0.27	0.31	0.35	0.39
5.5	0.02	0.05	0.09	0.14	0.19	0.24	0.28	0.33	0.38	0.42	0.47
6.0	0.03	0.06	0.11	0.17	0.22	0.28	0.34	0.39	0.45	0.50	0.56
6.5	0.03	0.07	0.13	0.20	0.26	0.33	0.40	0.46	0.53	0.59	0.66
7.0	0.04	0.08	0.15	0.23	0.31	0.38	0.46	0.54	0.61	0.69	0.77
7.5	0.04	0.09	0.18	0.26	0.35	0.44	0.53	0.62	0.70	0.79	0.88
8.0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.89	0.90	1.00
8.5	0.06	0.11	0.22	0.34	0.45	0.56	0.68	0.79	0.90	0.01	1.13
9.0	0.06	0.13	0.25	0.38	0.50	0.63	0.76	0.88	1.01	1.14	1.26
9.5	0.07	0.14	0.28	0.42	0.56	0.70	0.84	0.98	1.12	1.27	1.41
10.0	0.07	0.16	0.31	0.47	0.62	0.78	0.94	1.09	1.25	1.40	1.56

6. CONCLUSIONS

The following conclusions are drawn from the present study:

The determination of the elevation difference error (Δh) may be made by theoretical assumption. The use of the theoretical approach yields satisfactory numerical results and saves time and effort. Moreover, it is possible to obtain the values of the refraction coefficient difference (Δk), between two stations, if both the elevation difference error (Δh) and geodetic distance (S) are known.

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[2] Lazaref. G.E., Camyshken E.M. "Higher Geodesy" Moscow, Nedra, 1980, (in Russian).

[3] Mikhailofh, K., "Higher Geodesy" Moscow, 1984 (in Russian).