THE STUDY OF COMBINED EFFECT OF THE SYSTEMATIC AND RANDOM ERRORS OF PHOTOGRAPHS ON THE BLOCK PHOTOTRIANGULATION CONSTRUCTION ACCURACY

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ABSTRACT

Systematic errors occur according to some definite pattern which may or may not be known . When the law of occurrence of the systematic errors is known, it can sometimes be modeled by a mathematical expression and the measurements can be corrected accordingly. Lens distortion , atmospheric refraction and other factors are good examples of systematic errors. Random errors are caused by the inherent incapability of instruments and human observers to make exact measurements and by uncontrollable variations in the operating conditions during the measurements. Random errors are generally very small in magnitude , but they can be of any sign. To minimize the effect of random errors, measurements should be made under as wide a range of operating conditions as possible and with the greatest care.
The aim of this research paper is to study (using simulation) the effect of systematic and random errors on all elements of the block phototriangulation network. The studied parameters in this research are:

- Effect of errors by photographic interior orientation elements.
- Effect of the distortion of the aerophotocamera objectives.
- Effect of the atmospheric refraction.
- Combined effect of the systematic errors.
- Effect of random errors of the photographs.
- Combined effect of the systematic and random errors.

This work distinguishes from all the familiar works which show the effect of these errors on separate models or on the image of the ground point position.

1- INTRODUCTION

A rigid theory of construction and adjustment of block phototriangulation achieved very high level, highly precise stereocomparators with automatic recording of results of image measurements were created , technology of different methods of aerotriangulation and methods of systematic error effects were worked out in detail.

Mathematical model of any aerotriangulation method is based on a supposition that the picture itself represents the central projection.

In reality deviations from this projection in the form of systematic errors occur in the photo, the deviations being caused by photo material warping, atmospheric refraction, lens distortion and other factors.

Therefore the problem of accuracy increase in aerotriangulation requires the improvement of methods used for determination and taking into account systematic and random errors.

2- DESCRIPTION

Model photographs were made by the author to study the effects of various kinds of errors on the accuracy of construction of block photogrammetric network.

The block contains (15) photographs. Each stereo-pair has (10) points. The scale of the photographs is 1 : 5000 , the camera focal length 100 mm , forward over lap 60% and the side lap is 40%.

The given block was constructed and adjusted by using a strict method for construction and adjustment of photogrammetric block triangulation which was worked out by the author [6].

This strict method is introduced to investigate two variants for construction and adjustment of the analytical block phototriangulation. The basis of coplanarity for all corresponding (control) points and collinearity for tie points are used. The first variant is the construction of a free general model followed by the exterior orientation. The second variant is the construction of the system of geodetic coordinates , where the control points are supplemented by collinearity equation.

The represented variants of block phototriangulation is realized in the computer program as follows:

1. Provisional treatment for measurement results must be realized by the familiar method and includes itself all processes of interior orientation of photographs considering the deformation and rejection of blunder measurements by using tie points.
2. Provisional construction of block phototriangulation is accomplished with the aim to receive the preliminary approximation for block adjustment. It is established in coplanarity Equation and is accomplished by the method of under-orientation of each next photograph relatively to the previous photographs. Then, rejection of blunder measurements is checked by the values of residual parallaxes.
3. Compatible determination of exterior orientation elements of photographs is fulfilled on the basis of Equations coplanarity and collinearity. Observation equation of coplanarity is composed for each pair of consequent points in the limits of the whole block. Then each application of this equation is inserted in common system of equations with its weight. Observation equation of collinearity can be written for the control points. As a result of compatible solution of these two groups of equations under the condition $V^T P V$ = min. by the method of consequent approximations , receive the probable values of the exterior orientation elements of the block phototriangulation in a geodectic coordinate system.
4. Block points coordinates can be determined by using intersection method for solving collinearity equation. The given block was treated as the above described program.

Table (1) gives systematic errors (δ) and mean square errors (m) in determination of ground coordinate points in the block network , and Table (2) gives the same errors in determination of the exterior orientation elements of photographs after adjustment of block phototriangulation by this method.

Table 1.

Errors	$X_{(mt)}$	$Y_{(mt)}$	$Z_{(mt)}$
m	0.06	0.08	0.14
σ	-0.01	-0.03	0.02

Table 2.

Errors	X_L (mt)	Y_L (mt)	Z_L (mt)	ω ' "	ϕ ' "	K ' "
m	0.06	0.09	0.16	00 01	00 01	00 0[illegible]
δ	-0.01	-0.03	0.04	00 00	00 00	00 0[illegible]

2-1 The errors caused by the distortion of th objective were calculated by using the followi formula [2] which was realized in the compute program.

$$\Delta x = x(k_1 r^2 + k_2 r^4 + ..) + (P_1 (r^2 + 2x^2)$$

$$+ 2P_2 xy) (1 + P_3 r^2 + ..)$$

$$\Delta y = y(k_1 r^2 + k_2 r^4 + ..) + (P_2 (r^2 + 2y^2)$$

$$+ 2P_1 xy) (1 + P_3 r^2 + ..)$$

where:

$\Delta x, \Delta y$ correction for combined effects of radi and decentering distortion.

x, y photographic coordinates referred t principal point.

k_1, k_2, k_3 coefficients of radial distortion.

$P_1 = J_1 \sin \phi_o$

$P_2 = J_1 \cos \phi_o$

$P_3 = J_2 / J_3$

J_1, J_2, J_3 - coefficients of decentering distortion.

$\phi_o = \arcsin (x - x_o)/r$

$= \arccos (y - y_o) / r$

$r = [(x - x_o)^2 + (y - y_o)^2]^{0.5}$

In the analytical process of calibration the customar set of elements of interior orientation (f, x_o, y_o) i broadened to include the coefficients of radial an decentering distortion $k_1, k_2, k_3, \ldots , P_1, P_2, P_3, \ldots$ [2].

2-2 The errors caused by the atmospheric refraction were calculated by using the following formula [2] which was realized in the computer program.

$$\Delta x = x \frac{\Delta r}{r}$$

$$\Delta y = y \frac{\Delta r}{r} \qquad (2)$$

where:

$$\Delta r = k \left(r + \frac{r^3}{f^2}\right)$$

$$k = \left[\frac{2410\,H}{H^2 - 6H + 250} - \frac{2410h}{h^2 - 6h + 250}\left(\frac{h}{H}\right)\right] * 10^{-6}$$

$\Delta x, \Delta y$ - correction for effects of atmospheric refraction
x,y - photographic coordinates referred to principal point
k - (ARDC) model of the atmosphere is in microradians
H - flying height is in kilometers above M.S.L
h - elevation of the ground point is in kilometers above M.S.L
r - radial distance of the measured photographic point
f - focal length

3- NUMERICAL APPLICATION (SIMULATED)

3-1- *The effect of errors by photographic interior orientation elements 3*

The interior orientation of a photograph is one of the main processes in the analytical photogrammetry, its main elements being the focal length of the photo camera and principal point coordinates of the photograph [1,3,5].

To study the effect of errors of the interior orientation elements , a block network was constructed by distorted model photographs, in which assumed simulated errors were added to the coordinates of the principal point in every photograph and the focal length, as follows:

Case a- $\Delta x_o = \Delta y_o = \Delta f = 10$ mmc*.

Case b- $\Delta x_o = \Delta y_o = \Delta f = 50$ mmc*.

*1 mmc = 0.001 mm

In this case Tables 3, 4 give systematic errors (δ) and mean square errors (m) in determination of ground coordinate points and exterior orientation elements of photographs respectively.

Table 3.

Case	Errors	X_{mt}	Y_{mt}	Z_{mt}
a	m	0.067	0.086	0.148
	δ	-0.010	-0.030	0.017
b	m	0.087	0.102	0.167
	δ	-0.017	-0.046	0.024

Table 4.

Case	Errors	X_L (mt)	Y_L (mt)	Z_L (mt)	ω ' "	ϕ ' "	K ' "
a	m	0.152	0.176	0.233	00 01	00 02	00 03
	δ	0.102	0.099	0.134	00 01	00 02	00 03
b	m	0.515	0.564	0.627	00 03	00 06	00 07
	δ	0.430	0.445	0.497	00 02	00 03	00 03

3-2- *The effect of distortion by the aerophoto-camera objective*

To study the effect of distortion by the objective a block network of model photographs was constructed where the errors caused by the distortion of objective were intentionally input into the photo-coordinate points.

These errors were calculated from Formula (1). Distortion has caused large perversion in the coordinates of the block network coordinate points and exterior orientation elements of the photographs, as shown in Tables 5, 6

Table 5.

Errors	X_{mt}	Y_{mt}	Z_{mt}
m δ	0.126 0.015	0.163 -0.053	0.944 -0.501

Table 6.

Errors	X_L (mt)	Y_L (mt)	Z_L (mt)	ω ′ ″	φ ′ ″	K ′ ″
m δ	1.502 0.519	0.512 0.239	1.925 1.205	03 27 00 37	01 02 00 35	00 09 00 06

3-3- *The effect of the atmospheric refraction*

To study the effect of atmospheric refraction a block network of model photographs was constructed, where the errors caused by the atmospheric refraction were input into the network photo-coordinate points.

These errors are calculated from Formula (2). The analysis show that the atmospheric refraction had caused slight distortion of coordinates of the block network coordinate points and exterior orientation elements of the photographs, as shown in Tables 7, 8

Table 7.

Errors	X_{mt}	Y_{mt}	Z_{mt}
m δ	0.064 -0.012	0.083 -0.031	0.158 0.026

Table 8.

Errors	X_L (mt)	Y_L (mt)	Z_L (mt)	ω ′ ″	φ ′ ″	K ′ ″
m δ	0.062 -0.011	0.091 -0.032	0.201 0.077	00 00 00 00	00 03 00 00	00 01 00 01

3-4- *The effect of random errors of the photographs*

Random errors at the construction of hototriangulation networks appear due to the presence of random errors in measurements of coordinates and parallax of aerophoto points, random material deformation and other random errors [4].

To study the effect of random errors in photographic measurements, there was constructed a block network of model photographs, various random errors were input into the coordinates of the photographic network points with mean square errors as follows:

Case a- $m_x = m_y = 2$ mmc.

Case b- $m_x = m_y = 10$ mmc.

Consequently it was stated that the coordinates of the network points have systematic and random errors, as shown in Tables 9 , 10.

Here the systematic amount of the errors is much less than the random.

Table 9.

Case	Errors	X_{mt}	Y_{mt}	Z_{mt}
a	m δ	0.078 -0.015	0.108 -0.033	0.176 0.017
b	m δ	0.112 -0.043	0.354 -0.177	0.397 0.064

Table 10.

Case	Errors	X_L (mt)	Y_L (mt)	Z_L (mt)	ω ′ ″	φ ′ ″	K ′ ″
a	m δ	0.086 -0.017	0.115 -0.047	0.173 0.043	00 04 00 01	00 04 -00 01	00 0[illegible] 00 0[illegible]
b	m δ	0.305 -0.046	0.646 -0.452	0.249 0.085	00 48 00 03	00 32 -00 03	01 0[illegible] 00 3[illegible]

3-5- *Combined effect of the systematic errors on the accuracy of block phototriangulation*

Various systematic errors , modelling
- The distortion by the objective
- Atmospheric refraction
- Errors of the interior orientation elements :

$$\Delta f = \Delta x_o = \Delta y_o = 50 \text{ mmc.}$$

These errors were input into the coordinates of the photographic points.
The results of the experiments are shown in Tables 11, 12

Table 11.

Errors	X_{mt}	Y_{mt}	Z_{mt}
m	0.127	0.163	0.952
δ	-0.012	-0.013	-0.506

Table 12.

Errors	X_L (mt)	Y_L (mt)	Z_L (mt)	ω ' "	φ ' "	K ' "
m	1.532	0.801	1.036	00 59	03 26	00 10
δ	0.503	0.702	-0.812	00 38	00 31	00 05

3-6- *Combined effect of the systematic and random errors on the accuracy of block phototriangulation*

a- Various random errors with mean square errors :

$$m_x = m_y = 10 \text{ mmc.}$$

were input into coordinates of the photographic points.

b- Various systematic errors , modelling ,

- Distortion by the objective
- Atmospheric refraction
- Errors of the interior orientation elements

$$f = 20 \text{ mmc.}$$

$$\Delta x_o = \Delta y_o = 10 \text{ mmc.}$$

These errors were input into the coordinates of the photographic points.
The results of the experiments work are shown in Tables 13, 14.

Table 13.

Errors	X_{mt}	Y_{mt}	Z_{mt}
m	0.233	0.331	1.143
δ	0.023	0.116	0.561

Table 14.

Errors	X_L (mt)	Y_L (mt)	Z_L (mt)	ω ' "	φ ' "	K ' "
m	1.721	0.994	1.576	01 34	03 53	01 05
δ	0.144	0.759	-1.222	00 25	00 24	00 38

4- CONCLUSIONS

a- Comparison of the above results shows that random errors of photo points with mean square errors $m_x = m_y = 2$ mmc. do not affect the accuracy of definition of ground points coordinates and values of the exterior orientation elements practically, while the errors more than 5 mmc. do notably start to affect the final results of the network adjustment. Therefore phototriangulation network requires fine measuring instruments.

b- The analysis showed, the greatest effect on the accuracy of the network adjustment comes from the distortion by the aerophoto-camera objective.

c- The analysis show that the atmospheric refraction had caused slight distortion of coordinates of the block phototriangulation points.

d- The analysis of the results of the study evidently prove that systematic and random errors should be carefully considered when constructing photogrammetric network.

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