# SURFACE REFLECTION FROM INCLINED TERRAIN 

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## ABSTRACT

A technique to determine the ground reflection point from an inclined terrain of a line-of-sight terrestrial microwave link is outlined. Once located, the ray path parameters: transmit and receive angles, power focusing factor and optical path length can be calculated. These parameters are used in the determination of the propagation medium transfer function.

## INTRODUCTION

The simplest mechanism to produce deep and selective fading on line-of-sight terrestrial microwave links is by the destructive interference of two rays of almost equal magnitudes. Therefore, it has been believed for a period of time that only reflection from water surfaces can produce a strong enough reflected ray to interfere with the direct ray between the two terminal antennas, and that ground reflections, specially from rough terrain, have negligible effects. However, recent experimental and theoretical results tend to indicate that ground reflections on overland paths may be the cause of some deep selective fading [1]. The proposed mechanisms assume that the amplitude of the direct ray must be reduced to a level comparable to that of the ground reflected ray through one or more of the following processes:
1- Atmospheric refractive multipath : in this case the ground reflected ray does not greatly alter the already frequency selective nature of this phenomenon.
2-Defocusing and,
3- Antenna decoupling.
In the last two cases, the ground reflected ray will add frequency selectivity to the nonselective nature of these two phenomena producing in-band amplitude and phase distortions.
In order to assess the effects of the ground reflected rays, a modification to an earlier refractive multipath model was proposed [2]. In this modification, a specular reflected ray from either a smooth or a rough terrain was incorporated [3]. In this paper we outline a technique to determine the reflection point from an inclined terrain and consequently all ray path parameters.

## THEORETICAL MODEL

The model utilizes the modified refractive index profile to simplify the analysis while taking into consideration the spherical nature of the earth.
Two cases are considered:

## CASE I: The reflection point is at mean sea level:

The modified refractivity profile is assumed to be linear near ground and consequently the rays follow arc of circles. The path geometry is shown in Figure (1) and the governing equations are:

$$
\begin{gather*}
H_{T}=R\left(\cos \theta_{S}-\cos \theta_{T}\right)  \tag{1}\\
H_{R}=R\left[\cos \left(\theta_{S}+2 \theta_{e}\right)-\cos \theta_{R}\right]  \tag{2}\\
L_{1}=R\left(\operatorname{SIN} \theta_{T}-\operatorname{SIN} \theta_{S}\right)  \tag{3}\\
L_{2}=R\left[\operatorname{SIN} \theta_{R}-\operatorname{SIN}\left(\theta_{S}+2 \theta_{e}\right)\right] \tag{4}
\end{gather*}
$$

where:
$\mathrm{R} \quad$ is the radius of curvature of the ray $=-1 /(\mathrm{dm} / \mathrm{dh})$.
$\theta_{\mathrm{T}}, \theta_{\mathrm{R}}$ are the transmit and receive angles.
$\theta_{\mathrm{S}}$
is the grazing angle with respect to a horizontal ground.
$\theta_{e} \quad$ is the ground inclination at the reflection point.
$\mathrm{H}_{\mathrm{T}}, \mathrm{H}_{\mathrm{R}}$ are transmitter and receiver antenna heights above mean sea level.
$\mathrm{L}_{1}, \mathrm{~L}_{2}$ are the distances of the reflection point from the transmitter and receiver antennas.


Figure 1. Path geometry for reflection point at mean se level.


Figure 2. Inclination of earth versus the distance of reflection point $L_{1}$ from the transmitter for $L=45 \mathrm{~km}$.W $\Delta \mathrm{N}=-40 \mathrm{~N}$-units $/ \mathrm{km}$ and $\mathrm{H}_{\mathrm{R}}=90 \mathrm{~m}$.

The solution of the simultaneous equations (1) to (4) are subject to the conditions that $\left(\mathrm{L}_{1}+\mathrm{L}_{2}\right)$ must be equal to the link length and that Snell's law applies at the reflection point. Therefore, only a proper combination of $\theta_{\mathrm{e}}$ and $\mathrm{L}_{1}$ can lead the reflected ray to hit the receiver. Figure (2) shows typical results for a path length of 45 km and different antenna heights. It is obvious that for equal antenna heights the reflection point from a flat ground must be at mid path.

CASE II: The reflection point above mean sea level
A more realistic approach is to assume that the reflection point is above mean sea level as shown in Figure (3). In this case it is required to determine the height $\Delta \mathrm{H}$ of the reflection point on the inclined path above mean sea level and the distance $\Delta \mathbf{X}$ from the reflection point to the point at which the ray would have hit the mean sea level. In this case the governing equations are:

$$
\begin{gather*}
H_{T}=R\left(\operatorname{Cos} \theta_{S}-\operatorname{Cos} \theta_{T}\right)  \tag{5}\\
H_{R}=R\left[\operatorname{COS}\left(\theta_{S 1}+2 \theta_{e}\right)-\operatorname{CoS} \theta_{R}\right]+\Delta H  \tag{6}\\
\Delta X=R\left(\operatorname{SIN} \theta_{S 1}-\operatorname{SIN} \theta_{S}\right)  \tag{7}\\
\Delta H=R\left(\operatorname{COS} \theta_{S}-\operatorname{CoS} \theta_{S 1}\right)  \tag{8}\\
\Delta H=\Delta Z \operatorname{TAN} \theta_{e}  \tag{9}\\
\Delta H=\Delta X \operatorname{TAN} \theta_{S}  \tag{10}\\
L_{11}=R\left(\operatorname{SIN} \theta_{T}-\operatorname{SIN} \theta_{S}\right)  \tag{11}\\
L_{22}=R\left[\operatorname{SIN} \theta_{R}-\operatorname{SIN}\left(\theta_{S 1}+2 \theta_{e}\right)\right]  \tag{12}\\
L=L_{11}+L_{22}-\Delta X \tag{13}
\end{gather*}
$$

For the sake of simplicity in the presentation of the results, the length $L_{1}$ is employed in the graphs where:

$$
\begin{gather*}
\mathrm{L}_{1}=(\text { distance of the reflection point from the } \\
\text { transmitter })-\Delta Z \tag{14}
\end{gather*}
$$

$\mathrm{L}_{1}$ represents the intersection of the inclined terrain, with the mean sea level.


Figure 3. Path geometry for reflection point above mean sea level.


Figure 4. Transmit angle versus inclination of earth for $\mathrm{L}=45 \mathrm{~km}, \Delta \mathrm{~N}=-40 \mathrm{~N}$-units $/ \mathrm{km}$ at $\mathrm{H}_{\mathrm{T}}=\mathrm{H}_{\mathrm{R}}=90 \mathrm{~m}$.

Figures (4), (5) and (6) give the transmit, receive and surface angles versus the earth inclination angle $\theta_{e}$ for different values of $L_{1}$. For $L_{1}$ nearer to the transmitter, solution can be obtained for a wider range of earth inclination. Also for $L_{1}$ below the mid path length the curvature of the curves are different than those above it. Note that all the curves intersect at a point where
$\left(\mathrm{L}_{1}+\Delta \mathrm{Z}\right)=22.5 \mathrm{~km}$ (mid path), at a value of $\theta_{\mathrm{e}}=0$.


Figure 5. Receive angle versus inclination of earth for $\mathrm{L}=45 \mathrm{~km}, \Delta \mathrm{~N}=-40 \mathrm{~N}$-units $/ \mathrm{km}$ at $\mathrm{H}_{\mathrm{T}}=\mathrm{H}_{\mathrm{R}}=90 \mathrm{~m}$.


Figure 6. Surface angle above mean sea level versus inclination of earth for $\mathrm{L}=45 \mathrm{~km}, \Delta \mathrm{~N}=-40 \mathrm{~N}$ units $/ \mathrm{km}$ at $\mathrm{H}_{\mathrm{T}}=\mathrm{H}_{\mathrm{R}}=90 \mathrm{~m}$.

The amplitude of the reflected ray can be calculated from the power focusing factor given by [4],

$$
\begin{equation*}
P=\frac{\operatorname{Cos} \theta_{T}}{L \operatorname{SIN} \theta_{R}} \frac{d \theta_{T}}{d L} \tag{15}
\end{equation*}
$$

For case I this gives:

$$
\begin{gathered}
\frac{1}{P}=L R \operatorname{SIN} \theta_{R}\left(1+\frac{\operatorname{TAN} \theta_{T}}{\operatorname{TAN} \theta_{R}} \frac{\operatorname{SIN}\left(\theta_{S}+2 \theta_{e}\right)}{\operatorname{SIN} \theta_{S}}\right. \\
\left.\frac{\operatorname{TAN} \theta_{T} \operatorname{COS}\left(\theta_{S}+2 \theta_{e}\right)}{\operatorname{SIN} \theta_{S}}\right)
\end{gathered}
$$

For case II it becomes:

$$
\frac{1}{P}=A[B+C(D-E)]
$$

where

$$
\begin{aligned}
A & =L R \operatorname{SIN} \theta_{R} \\
B & =1+\frac{\operatorname{TAN} \theta_{T}}{\operatorname{TAN} \theta_{R}}
\end{aligned}
$$

$$
\begin{gathered}
C=\frac{1}{\operatorname{TAN} \theta_{S} \operatorname{Cos} \theta_{S I}}-\frac{\operatorname{TAN} \theta_{S I}}{\operatorname{Cos} \theta_{S} \sin \theta_{S}}+\frac{1}{\operatorname{Cos} \theta_{S} \operatorname{SIN} \theta_{s} \cos } \\
D=\frac{\operatorname{TAN} \theta_{T}}{\operatorname{TAN} \theta_{R}}\left[\operatorname{SIN}\left(\theta_{S I}+2 \theta_{e}\right)-\operatorname{SIN} \theta_{S I}\right] \\
E=\operatorname{TAN} \theta_{T}\left[\operatorname{COS} \theta_{S I}+\operatorname{Cos}\left(\theta_{S I}+2 \theta_{e}\right)\right]
\end{gathered}
$$

The results for case II shown in Figure (7) indie that the inclined terrain focuses the reflected por towards the receiver. That power is reduced as angle $\theta_{\text {S1 }}$, with which the ray hits the grou decreases. Note that the four curves coincide at point where $\mathrm{L}_{1}+\Delta \mathrm{Z}=22.5 \mathrm{~km}$ and the es inclination is equal zero degree.
Finally, the expression for the optical path length be derived in both cases from the relation :

$$
L_{\text {opt }}=\int_{\theta_{T}}^{\theta_{R}} m R d \theta
$$

giving for case I:

$$
L_{\text {opt }}=R\left(m_{T}+\operatorname{Cos} \theta_{T}\right)\left(\theta_{T}-\theta_{S}\right)+R\left(m_{R}+\operatorname{Cos} \theta_{R}\right)\left(\theta_{R}-\theta_{S}-2 \theta_{G}\right)-L
$$

where $\mathrm{m}_{\mathrm{T}}$ and $\mathrm{m}_{\mathrm{R}}$ are the modified refractive indir at the transmitter and the receiver heights respective and for case II:
$L_{\text {ox }}=R\left(m_{T}+\operatorname{Cos} \theta_{T}\right)\left(\theta_{T}-\theta_{S I}\right)+R\left(m_{R}+\operatorname{Cos} \theta_{R}\right)\left(\theta_{R}-\theta_{S I}-2 \theta_{G}\right)-L$
Both the amplitude and the delay of the reflected ray will be modified if the reflection coefficient for the ground is taken into consideration.


Figure 7. Power focusing factor versus inclination of earth for $\mathrm{L}=45 \mathrm{~km}, \Delta \mathrm{~N}=-40 \mathrm{~N}$-units $/ \mathrm{km}$ at $H_{T}=H_{R}=90 \mathrm{~m}$.


Figure 8. Proposed earth profile.

## WORKED EXAMPLE:

A proposed earth profile is suggested as shown in Figure (8). This profile consists of 7 segments with different lengths and inclinations. It is required to determine which of these segments will contribute a ground reflected ray that reaches the receiver.
For each of these segments, it is important to determine the distance $\mathrm{L}_{1}$ at which the inclined terrain in this segment would hit the mean sea level. With the knowledge of $L_{1}$ and $\theta_{e}$ of each segments and the procedure outlined before, the ray path parameters can be determined.

For the proposed profile the calculations have shown that only three rays can reach the receiver along with the direct ray. Tables (1) and (2) summarize the ray path parameters of these rays.
Figures (9) and (10) represent the fade level and group delay versus frequency for the medium. The antenna gain is incorporated in the dashed curve. The absence of the selectivity in the second case is due to the fact that the antenna discriminates against ray 2 that has an almost equal amplitude as that of the direct ray with a relatively large transmit and receive angles.


Figure 9. Fade level versus frequency for the proposed profile. 1- No antenna gain. 2- With antenna gain.

Table 1. Results of the calculations of the worked example.

| Segment <br> No. | $\mathrm{L}_{1}$ <br> km. | $\theta_{\mathrm{e}}$ <br> Degree. | $\theta_{\mathrm{T}}$ <br> Degree. | $\theta_{\mathrm{R}}$ <br> Degree. | $\theta_{\mathrm{S} 1}$ <br> Degree. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | - | - | - | - | - |
| 2 | 2.0 | -1.0 | 1.98468 | 0.25436 | 1.96855 |
| 3 | 1.8 | 0.01 | 0.29045 | 0.30017 | 0.13448 |
| 4 | - | - | - | - | - |
| 5 | 30.0 | -0.001 | 0.30473 | 0.30371 | 0.15436 |
| 6 | 40.0 | -0.005 | 0.30322 | 0.29830 | 0.15488 |
| 7 | 33.0 | 0.02 | 0.28777 | 0.30740 | 0.12679 |

Table 2. Results of the calculations of the worked example.

| Segment <br> No. | $\Delta \mathrm{H}$ <br> m. | $\mathrm{L}_{1}+\Delta \mathrm{Z}$ <br> km. | $\mathrm{L}_{\mathrm{o}}$ <br> m. | P <br> dB | Solution |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | - | - | - | - | - |
| 2 | 7.04497 | 1.59635 | 45025.9 | -32.896 | Yes. |
| 3 | 3.72418 | 23.2656 | 45030.1 | -36.417 | Yes. |
| 4 | - | - | - | - | - |
| 5 | 0.13293 | 22.4308 | - | - | NO |
| 6 | 1.53893 | 22.1275 | - | - | NO |
| 7 | 3.12297 | 41.9861 | 45029.7 | -36.723 | Yes. |



Figure 10. Group delay versus frequency for the proposed profile.
1- No antenna gain.
2- With antenna gain.

## CONCLUSIONS

Due to inclination in the path profile several grou reflected rays can reach the receiver caus interference.
Depending on the path profile, the proposed mo can estimate the number and ray-path parameters of interfering rays. Additional interfering rays may present if elevated inversion layers exist. The over transfer characteristic can thus be computed.

## REFERENCES

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