SCATTERING OF MICROWAVES BY BONE FRACTURES

Nour H. Ismail
Department of Electrical Engineering, Faculty of Engineering,
Alexandria University, Alexandria, Egypt.

ABSTRACT

Microwaves provide an interesting tool for the noninvasive detection of biological media. In this paper, the radiative transfer theory is applied and developed to the biological media at the microwave frequency range. The reflected intensities from a multi-layered medium with planar and rough interfaces are obtained. As an illustration, the relative reflected intensities from a fat, muscle and bone medium are obtained as functions of frequencies and incident angles in the cases of a planar bone interface and a bone fracture.

1. INTRODUCTION

In active remote sensing, theories have been applied to determine the reflected, scattered and transmitted fields or intensities. The radiative transfer theory and applications in active remote sensing for multi-layered media have been studied [1-4]. Basic equations for passive remote sensing to biological media were discussed by Ismail et al. [5]. In section 2 the radiative transfer equations are developed to calculate the reflected intensities from a multi-layered biological medium. As an application of the obtained result, the reflected intensities from a fat, muscle and bone medium with planar interfaces are obtained in section 3. Section 4 is devoted to numerical calculations which are set up to determine that intensities in the case of bone rough interface. Discussion and conclusion are presented in section 5.

2. METHOD AND FORMULATION

Consider a multi-layered medium with different dielectric constants separated from each other by planar interfaces at $z = 0, d_1, d_j, \ldots, d_{N-1}$ as shown in Figure (1). At microwave frequencies, the wavelength is much larger than the cell dimensions in biological media. Therefore, the scattered effect can be neglected and only the absorption effect may be considered [6].

The radiative transfer equations within the j^{th} layer become [4]

$$\frac{dI_j^+(z)}{dz} = -Ka_j I_j^+(z) \quad (1a)$$

$$\frac{dI_j^-(z)}{dz} = Ka_j I_j^-(z) \quad (1b)$$

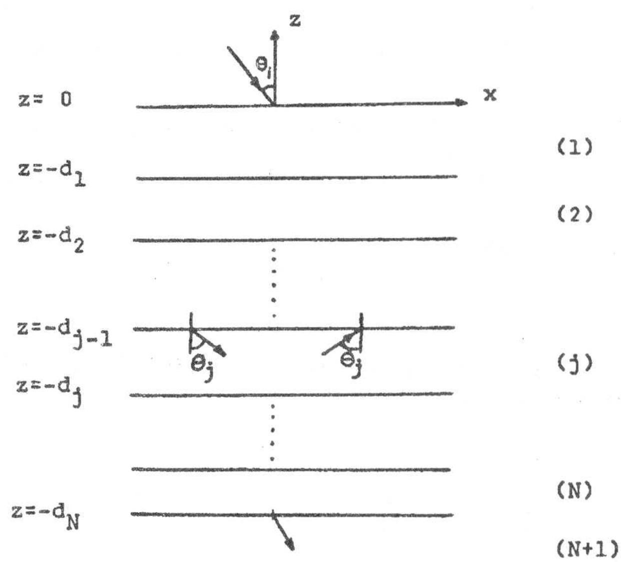

Figure 1. Geometry of the Theoretical Model

where I_j^+ and I_j^- are the upward and downward Stockes parameters, respectively, including both vertical and horizontal polarizations [7]. Ka_j is the absorption coefficient divided by the cosine of the angle θ_j. The boundary condition at $z = -d_{j-1}$ is

$$I_j^-(-d_{j-1}) = T_{j,j-1} I_{j-1}^-(-d_{j-1}) + R_{j,j-1} I_j^+(-d_{j-1}) \quad (2a)$$

and at $z = -d_j$ it is

$$I_j^+(-d_j) = T_{j,j+1} I_{j+1}^+(-d_j) + R_{j,j} I_j^-(-d_j) \tag{2b}$$

where T and R represent 4X4 transmission and reflection coefficient matrices for planar interfaces. Explicit expressions of these coefficients are given by Ulaby et al.[8]. The solutions for (1a) and (1b) have the following forms

$$I_j^+(z) = e^{-Ka_j(z+d_j)} I_j^+(-d_j) \tag{3a}$$

$$I_j^-(z) = e^{Ka_j(z+d_{j-1})} I_j^-(-d_{j-1}) \tag{3b}$$

with $-d_j < z < -d_{j-1}$.

Substituting (2a) and (2b) in (3a) and (3b), we get

$$I_j^+(z) = e^{-Ka_j(z+d_j)} [T_{j,j+1} I_{j+1}^+(-d_j) + R_{j,j} I_j^-(-d_j)] \tag{4a}$$

$$I_j^-(z) = e^{Ka_j(z+d_{j-1})} [T_{j,j-1} I_{j-1}^-(-d_{j-1}) + R_{j,j-1} I_j^+(-d_{j-1})] \tag{4b}$$

Let $z = -d_{j-1}$ in (4a) and $z = -d_j$ in (4b), yielding

$$I_j^+(-d_{j-1}) = e^{-Ka_j(d_j-d_{j-1})} [T_{j,j+1} I_{j+1}^+(-d_j) + R_{j,j} I_j^-(-d_j)] \tag{5a}$$

$$I_j^-(-d_j) = e^{-Ka_j(d_j-d_{j-1})} [T_{j,j-1} I_{j-1}^-(-d_{j-1}) + R_{j,j-1} I_j^+(-d_{j-1})] \tag{5b}$$

Now, we can relate the intensities at the (j-1)th and the jth layer interfaces as

$$L_j \begin{bmatrix} I_j^+(-d_{j-1}) \\ I_{j-1}^-(-d_{j-1}) \end{bmatrix} = M_j \begin{bmatrix} I_{j+1}^+(-d_j) \\ I_j^-(-d_j) \end{bmatrix} \tag{6}$$

where

$$L_j = \begin{bmatrix} U & 0 \\ e^{-Ka_j(d_j-d_{j-1})} R_{j,j-1} & e^{-Ka_j(d_j-d_{j-1})} T_{j,j-1} \end{bmatrix} \tag{7a}$$

and

$$M_j = \begin{bmatrix} e^{-Ka_j(d_j-d_{j-1})} & e^{-Ka_j(d_j-d_{j-1})} R_{j,j} \\ 0 & U \end{bmatrix} \tag{7b}$$

U is the unit matrix.

Therefore, the intensities at the interface $z = -d_{j-1}$ can be written in terms of those at the boundary $z = -d_j$ as

$$\begin{bmatrix} I_j^+(-d_{j-1}) \\ I_{j-1}^-(-d_{j-1}) \end{bmatrix} = L_j^{-1} M_j \begin{bmatrix} I_{j+1}^+(-d_j) \\ I_j^-(-d_j) \end{bmatrix} \tag{8}$$

In order to relate the intensities at $z = -d_{j-1}$ interface to those at $z = -d_{j+1}$ interface we let $j=j+1$ in (8). Generally, the relation between the intensities at the (j-1)th interface to those at the Nth interface is given as

$$\begin{bmatrix} I_j^+(-d_{j-1}) \\ I_{j-1}^-(-d_{j-1}) \end{bmatrix} = \prod_{\iota=j}^{N} L_\iota^{-1} M_\iota \begin{bmatrix} I_{N+1}^+(-d_N) \\ I_N^-(-d_N) \end{bmatrix} \tag{9}$$

Equation (9) is similar to that obtained by Karam [when the scattering effect is ignored.

3. RELATIVE REFLECTED INTENSITIES FROM THREE LAYERED BIOLOGICAL MEDIUM

Consider a biological medium consisting of fat, muscle and bone with planar interfaces as shown in Figure (2)

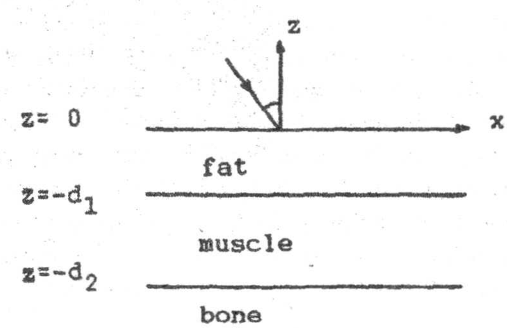

Figure 2. A Three Layers Biological Medium

The total reflected intensity in air, $I_r(0)$, is given as

$$I_r(0) = R_{00} I_0^-(0) + T_{01} I_1^+(0)$$

where $I_0^-(0)$ is the incident intensity and $I_1^+(0)$ is the upward intensity in fat at $z = 0$. $I_r(0)$ can be obtained setting $j = 1$ and $N = 2$ in (9). Also, $I_3^+(0) = 0$ because the bone layer is a half space below $z = -d_2$. Therefore (9) becomes

$$\begin{bmatrix} I_1^+(0) \\ I_0^-(0) \end{bmatrix} = \prod_{\iota=1}^{2} L_\iota^{-1} M_\iota \begin{bmatrix} 0 \\ I_2^-(-d_2) \end{bmatrix}$$

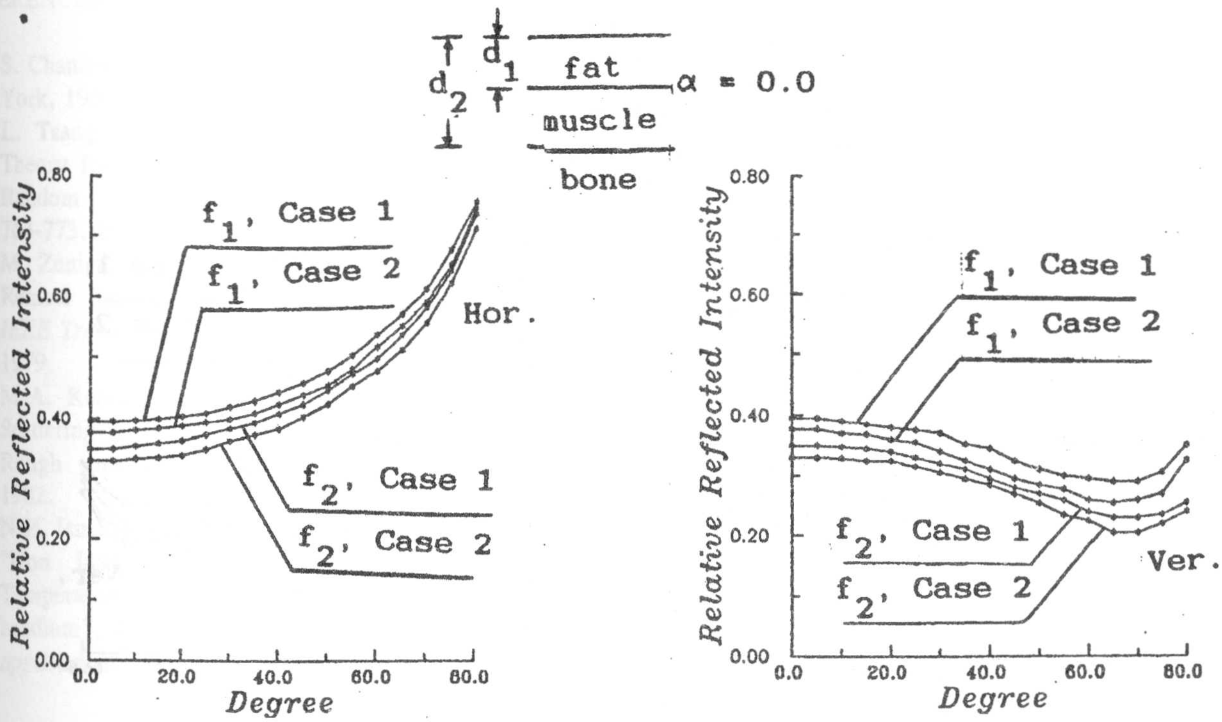

Figure 3. Relative reflected intensities against incident angles for horizontal and vertical polarizations for the planar bone interface case

Case 1: $d_1 = 2$ mm, $d_2 = 12$ mm. Case 2: $d_1 = 5$ mm, $d_2 = 15$ mm.
$f_1 = 0.915$ GHz $f_2 = 2.45$ GHz.

It is seen that $I_1^+(0)$ can be obtained in terms of $I_0^-(0)$ and substituting the result of (11) in (10), we get the relative reflected intensities $(I_r(0)/I_0^-(0))$. Figure (3) shows the relative reflected intensities versus the incident angles at 0.915 GHz and 2.45 GHz for different layer thicknesses. Vertical and horizontal polarizations are considered.

4. RELATIVE REFLECTED INTENSITIES FROM A ROUGH INTERFACE

In this section the bone interface is considered as a rough interface. This case is similar to bone fractures. The same analysis presented in section 3 is followed except that the reflection and transmission coefficients of bone planar interface is multiplied by the shadowing factors [8,9]. The shadowing factor for reflected intensities is given by [8] as

$$S_r = \frac{1}{1+2f(u_i)} \tag{12}$$

where

$$f(u_i) = \frac{1}{2}[\sqrt{2/\pi}\,(\alpha/u_i)e^{-u_i^2/2\alpha^2} - \mathrm{Erfc}(u_i/\sqrt{2}\,\alpha)] \tag{13}$$

$u_i = \cot\theta_i$, θ_i is the incident angle at the bone interface obtained by Snell's law. Erfc is the complementary error function. α is called the root mean square surface slope. While the shadowing factor for transmitted intensities is given as

$$S_t = \frac{1}{1+f(u_t)+f(u_i)} \tag{14}$$

where

$$f(u_t) = \frac{1}{2}[\sqrt{2/\pi}\,(\alpha/u_t)e^{-u_t^2/2\alpha^2} - \mathrm{Erfc}(u_t/\sqrt{2}\,\alpha)] \tag{15}$$

$u_t = \cot\theta_t$, θ_t is the transmitted angle in the bone layer. In the case of greenstick fracture (incomplete breaks occurring only in the resilient bones of children [10]) $\alpha=1.0$. The relative reflected intensities versus the incident angles for both vertical and horizontal polarizations at 0.915 GHz and 2.45 GHz are illustrated in Figure (4).

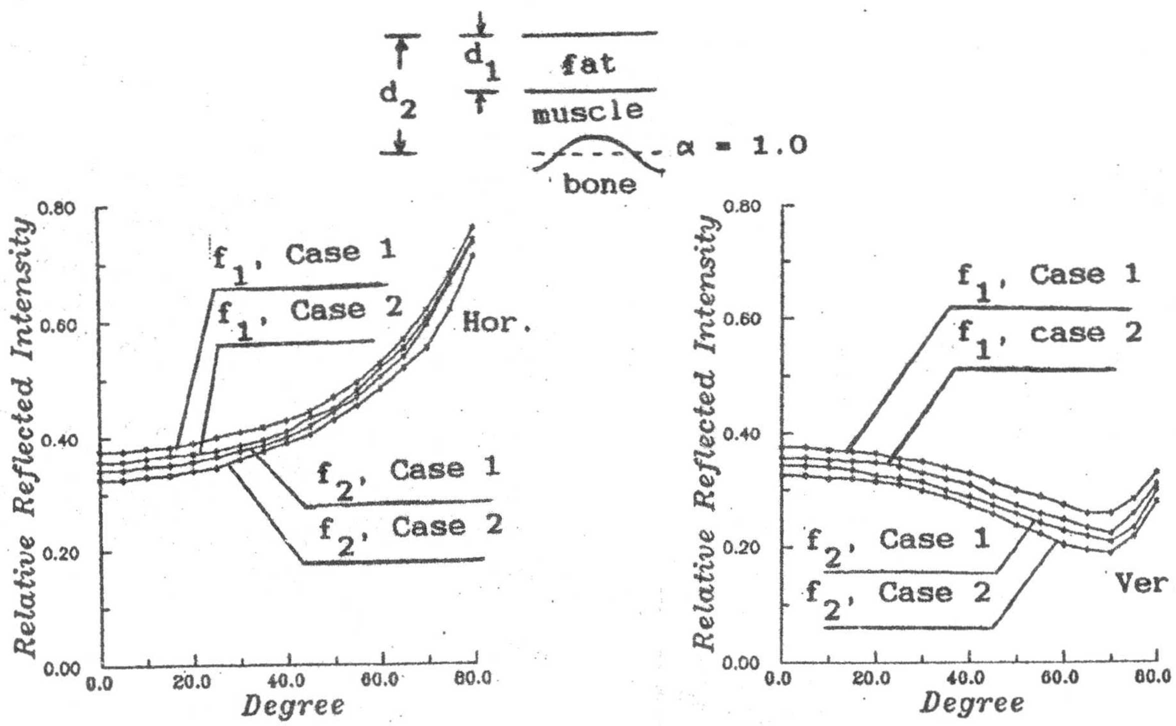

Figure 4. Relative reflected intensities against incident angles for horizontal and vertical polarizations for the rough bone interface case with $\alpha = 1.0$.
Case 1: $d_1 = 2$ mm, $d_2 = 12$ mm. Case 2: $d_1 = 5$ mm, $d_2 = 15$ mm.
$f_1 = 0.915$ GHz $f_2 = 2.45$ GHz.

5. DISCUSSION AND CONCLUSION

The reflection problem from a multi-layered biological medium with planar and rough interfaces has been solved. Figure (3) shows the relative reflected intensities as a function of incident angles at two different frequencies for both vertical and horizontal polarizations with $\alpha = 0.0$. It is observed that those intensities decrease as frequency increases. Also, they are sensitive to the thickness of fat and muscle layers. Figure (4) shows the intensities obtained from a rough bone interface with ($\alpha = 1.0$). From Figure (3) and Figure (4) it is found that the intensities decrease as the bone interface roughness increases. This observation is shown in Figure (5). The bone fracture types such as compression, spiral and greenstick are characterized by certain values of α, [10]. Therefore, the method discussed in this paper can be applied to get a group of curves for the relative reflected intensities at different values of α ($0 < \alpha \leq 1$) to discriminate bone fracture types. Finally, this approach is useful to sense the layer thickness and the bone interface roughness (bone fractures).

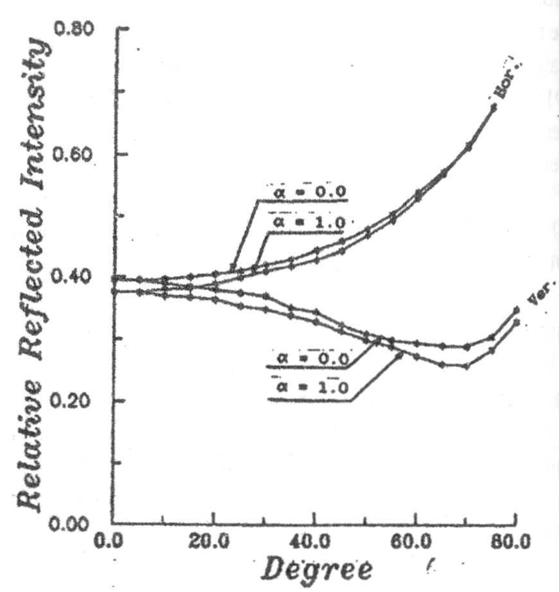

Figure 5. Relative reflected intensities against incid[ent] angles for both vertical and horizontal polarizations f=0.915 GHz, $d_1 = 2$ mm and $d_2 = 12$ mm.

REFERENCES

[1] S. Chandrasekhar, *Radiative Transfer*, Dover, New York, 1960.

[2] L. Tsang and J. A. Kong, "Radiative Transfer Theory for Active Remote Sensing of Half-Space Random Medium", *Radio Science*, Vol.13, 763-773, 1978.

[3] M. Zuniga, T. Habashy and J. A. Kong, "Active Remote Sensing of Layered Random Medium", *IEEE Trans. Geosci. Elect.*, Vol.GE-17, 296-302, 1979.

[4] M.A. Karam and A.K. Fung, "Propagation and Scattering in Multi-layered Random Media with Rough Interfaces", *Electromagnetics* 2, 239-256, 1982.

[5] N.H. Ismail, A.K. Aboulseoud and S.E. El-Khamy, "Non Invasive Method for Measurement of Temperature Emission from an Inhomogeneous Medium", *Journal of Electromagnetic Wave and applications*, Vol.5, No.8, 885-895, 1991.

[6] C.C. Johnson and A.W. Guy, "Nonionizing Electromagnetic Wave Effects in Biological Materials and Systems", *IEEE Proc.*, Vol. 60, 682-718, 1972.

[7] A. Ishimaru, *Wave Propagation and Scattering in Random Media*, Vol. 1, Academic Press, New York, 1978.

[8] F.T. Ulaby, R.K. Moore and A.K. Fung, *Microwave Remote Sensing (Active and Passive)*, Vol.2 BOOK-Mart Press, Inc., NJ, 1986.

[9] L. Tsang and J.A. Kong, *Theory of Microwave Remote Sensing*, John Wiley & Sons, New York, 1985.

[10] J.C. Adams, *Outline of Fractures*, Williams Clowes Limited, London, 1983.