PARAMETRIC ADAPTIVE-INTERACTIVE 2-D MESH GENERATOR FOR MULTIPLY CONNECTED DOMAINS

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ABSTRACT

A new mesh generator capable of dividing 2-D multiply connected domain into triangles with prescribed density is proposed. The method adopts a triangulation technique. The amount of input data is minimized. In addition, the method dispenses with the blocking technique regardless the complexity of the domain. This means that creation of transient zones is no longer needed . The dimensions of the problem are reduced by one, since the new elements are generated along the boundary only, and they are mainly independent of the previously generated ones. A minimum storage capacity is needed. The user can interact continuously with the generation process using many parameters. The heights of the triangles seem to be the most important ones. The method tries to create new triangles as close as possible to the equilateral ones. The generated mesh is independent of the orientation of the domain. The problems of discontinuities, creating triangles with high aspect ratio, and omitting regions near sharp boundaries are recovered. The new nodes and new elements are created simultaneously one by one. By its nature the method suits axi-symmetric domains best.

INTRODUCTION

Introducing a new mesh generator implies that, it possesses the following properties; high reliability, ease of checking the results, utilizing the user's own experience, near linear gross rate and minimal execution time. Finally the mesh generator must act friendly. That means, it permits the user to communicate with it without referring to the user's manual. This is achieved by letting a list of the procedures used in generating the mesh to appear on the screen when needed.

Any mesh generation method falls in one of the following classes:-

1. Coordinate Transformation.
2. Blending functions.
3. Automatic Triangulation.

In most cases, the domain is firstly partitioned into a series of regions with nodes placed on the region boundaries. Secondly, the mesh generator is called to act upon each of the regions. Thirdly, the regions are connected to form the whole domain. Finally, a smoothing algorithm is employed to fix the final locations of the created nodes.

Of all the meshing methods named above, triangulation techniques seem to be the only ones available for automatically generating a mesh without user's interface. They have the following disadvantages; (1) much input data, (2) more execution time, (3) producing poorly graded and poorly shaped meshes, (4) sometimes they fail in some regions.

Taking into account the fact that only a valid mesh is required for an adaptive analysis, makes it plausible to consider the triangulation techniques. Since using a smoothing technique such as the Laplacian method, and an adaptive analysis such as the h-version and the p-version makes it possible to convert a poorly graded mesh into a well graded one.

In the sections that follow, a triangulation technique is proposed. It has, as we claim, the following advantages:

1. No blocking is needed regardless the complexity of the domain and the intensity distribution.
2. Elimination of transient zones.
3. Independencey of orientation.
4. Parametric dependency that optimizes the benefit of the user's own experience.

MESH GENERATION

Assuming we have a simply connected domain with N predefined nodes distributed along its boundary S, with line segment i connecting nodes i and i+1. The procedure to create a valid mesh consists of the following steps :-

A. The distance between all possible pairs of nodes , except for adjacent ones, is checked. If found to be less than a prescribed value TOLERANCE1, then they are combined into one node with average coordinates. That may result in dividing the original boundary S into two separate ones or more. Each of them is acted upon separately and respectively by steps B through E. In what follows, we will denote each domain by S. It is to be understood that this step does not affect predefined nodes.

B. If the angle between lines i and i+1 - nodes i, i+1, i+2 - is less than or equal to a prescribed value θ_1 , then nodes i and i+2 are connected by a straight line. This leads to the formation of a new element with vortices at i, i+1 and i+2. At the same time the number of the nodes of the boundary S is reduced by unity. This process is continued until all angles are searched. This leaves us with a new boundary line having nodes less than or equal to the one we started with. Also, this procedure results in creating new elements (may be none). Regardless new elements are created or not, it is guaranteed that all angles are greater than the prescribed value θ_1. In all cases we denote the newly created boundary by S and the number of its nodal points by N.

C. N isosceles, preferably equilateral, triangles are constructed. Those triangles have line i , i=1, 2,..., N, as bases with the following conditions to be satisfied:-

  1. The sides of the triangles do not intersect one another and do not intersect the boundary S.
  2. If the distance between the vortices of any two adjacent triangles is less than a prescribed value TOLERANCE2, then they are merged into one node. In most cases this operation leads to a reduction in the number of the newly generated nodes, the N′ vortices of the triangles,

  The ratio between the triangle's height and the base's length is to be introduced as a prescribed parameter H_1

D. From step C above, it is found that the N node of the boundary S are augmented by N' (j = 1,2,...,N′) nodes, equal to or less than N. If th angle with node i,from boundary S, as vortex an nodes j and j+1 created in step C above is les than a prescribed angle θ_2, then nodes j and j+ are connected by a straight line. This mean creating a new element and reducing the tot number of nodes involved (N+N') by unity.

E. All nodes from boundary S that do not constitut vertices of the elements generated in D above, ar combined with the vertices of the triangle generated in C to produce new boundary S.

F. Steps A through E are repeated until we get boundary S with less than 5 nodes.

G. If S has 3 nodes then the last element is created. If S has 4 nodes, one of the two following action is to be taken:-

  a. If the length of any of the two diagonals of th quadrilateral forming S is less than a prescribe value TOLERANCE1 then its two end nodes ar combined into one and the quadrilateral formin S degenerates into two straight lines with n elements being created.
  b. If condition a is not fulfilled, then two new elements are generated.

In all cases, the process of creating new elements i ended for the boundary S under consideration.

MULTIPLY CONNECTED DOMAIN

The method treats the case of a multiply connecte domain by converting it into a simply connecte domain. Assume a single exterior domain S, which i always the case, and M interior domains I_j , j = 1,2,...,M. To create a simple domain, M pairs o straight lines are generated by:-

  a. The shortest distance between some node of S node i say, and some node, node j say, of a interior domain, I_k say, is searched.
  b. The nodes of S are renumbered so that

$$\text{node}\ (m) \rightarrow \text{node}\ (m) \quad m <= i$$

$$\text{node}\ (m) \rightarrow \text{node}\ (m + N_k + 2{*}SEG) \quad m >= i$$

where,

N_k number of nodes in I_k and,

SEG number of segments in the line connectin S and I_k

c. The nodes of I_k are to be inserted in S starting with locations (i+SEG).

d. The nodes of the newly created line going from S to I_k are inserted in S starting with location (i+1), while those of the newly created line going from I_k to S are inserted in S in location starting at (i+SEG+N_k).

Steps a through d above decrease the number of the interior domains by one and increase the number of the nodal points of S. Steps a through d are repeated M times until all the M interior domains are absorbed in S and a simply connected domain S is obtained.

The process of creating new pairs of lines which is necessary for absorbing the interior domains in the exterior one, creates duplication in node numbering of the newly generated line. A matter to be taken care of later.

ADAPTIVE ANALYSIS

Taking the method of creating the new elements into account, it seems natural to admit the user to interact with the generation process. Two levels of interaction are admitted. Firstly, after creating a new boundary S, The part of the mesh generated is to be drawn on the screen and the user is capable to modify the locations of the nodes consisting S. He is asked to enter new values for the parameters θ_1 mentioned in B above and H_1 mentioned in C above, to be used in the generation of the next stage. Secondly, after creating the whole mesh the user is admitted to move nodes and delete unwanted triangles (2 or 4 triangles each time).

SMOOTHING THE NET

The mesh is smoothed using the Laplacian technique by which the location of any node is modified so that it is located at the centroid of all the nodes sharing triangles with it. The process is iterative.

GROWTH RATE

The growth rate for the method introduced is independent of the complexity of the domain. This is due to the nature of the method. Figure (1) shows the growth rate of two domains. The first of them is a square with the same number of elements along each of its sides. The second domain is an isosceles triangle with the same number of elements along each of its sides. From Figure (1) it is shown that the growth rate for the two cases introduced is almost linear. A better look to the figure shows that increasing the number of elements decreases the growth rate. Figures (2) and (3) show the mesh generated for calculating the growth rate, with 10 elements along each side.

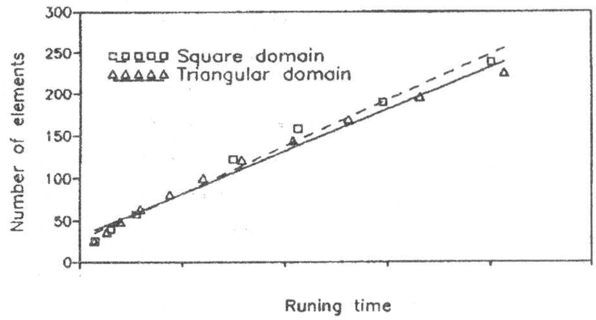

Figure 1. Growth rate.

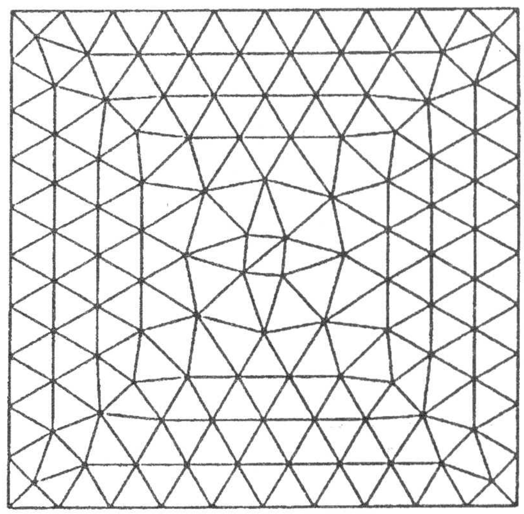

Figure 2. Square domain-10 elements/side.

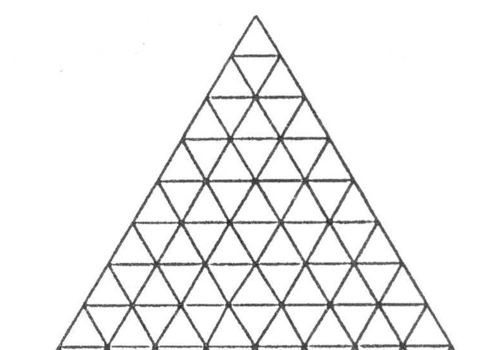

Figure 3. Triangular domain-10 elements/side.

PARAMETRIC EFFECT

To show the power of using parameters to control the mesh generation a T like domain is meshed several times using the same number of elements along its boundary. However, the parameters θ_1 introduced in step B above and the H_1 introduce in step C above are varied. Figures (4-a) through (4-d) show the effect of changing the values of the parameters. From the figures it is clear that changing the parameters θ_1 and H_1, enables one to create either a coarse mesh or a fine mesh without changing the number of the elements along the boundary. The last condition may be a must in some applications due to the boundary conditions, while using a coarse or fine mesh depends on the nature of the problem.

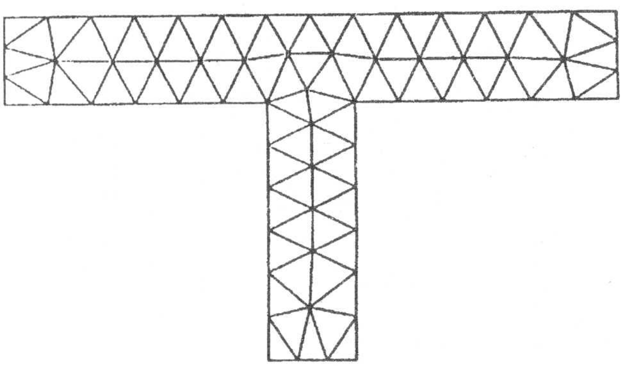

Figure 4-a. $\theta_1 = 90$, $H_1 = 0.86$, $N = 81$
$T = 0.64$ sec, $\sigma = 0.174$.

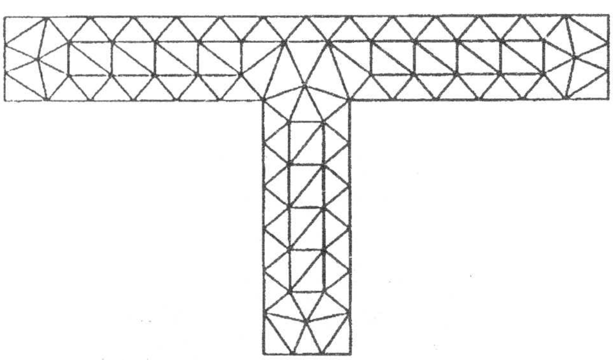

Figure 4-b. $\theta_1 = 60$, $H_1 = 0.86$, "1^{t} level"
$\theta_1 = 60$, H_1 0.70 "2nd level"
$N = 103$ $T = 0.81$ sec, $\sigma = 0.243$

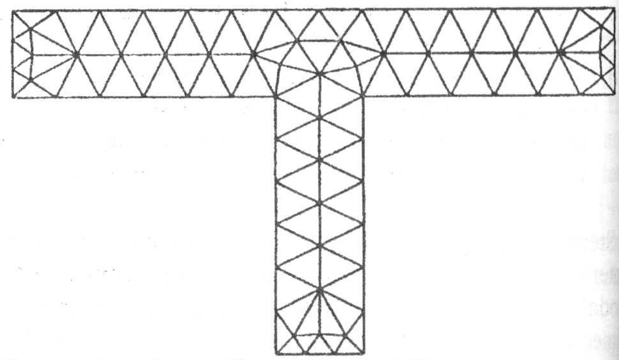

Figure 4-c. $\theta_1 = 60$, $H_1 = 0.70$,,
$N = 103$ $T = 0.92$ sec, $\sigma = 0.24$

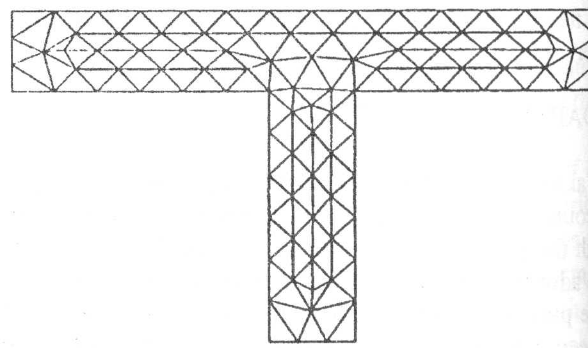

Figure 4-d. $\theta_1 = 99$, $H_1 = 0.55$, "2"^{t} level"
$\theta_1 = 60$ $H_1 = 0.60$ "2nd level"
$N = 157$ $T = 1.35$ sec, $\sigma = 0.369$.

As a measure of the method, we introduce σ define by

$$\sigma = \sqrt{\sum_{i=1}^{3n} (\theta_i - \pi/3)^2}$$

hence, σ is a measure for the deviation of the angl from $\pi/3$. All values of σ given in the figures and a the meshes introduced are crude ones, i.e. n smoothing of any type, Laplacian for example, i carried out. That is a better meshes with smaller valu of σ may be obtained.

ELIMINATION OF TRANSIENT ZONE

In some applications, it may be required to divide t domain into sub-domains, with each of them to b meshed using fine or coarse mesh. In this case, it is usual practice to generate the mesh of each sub-domain separately and use transient zones to overcome the problem of variation in densities. In the proposed method, we dispense with those procedures and treat the whole domain as a single unit. Figures (5-a) through (5-e) show how to treat a domain with different densities of the mesh. Figures(5-a) and (5-b) show the mesh for different values of the parameters θ_1 and H_1, but with the same density over the whole domain. This results in constructing coarse and fine meshes, respectively. Figure (5-c) shows the same domain with one of its legs having fine mesh while the other three legs having coarse mesh. Figure (5-d) shows the same domain with one of its legs having a fine mesh in part of it, while the rest of the domain having coarse mesh. Finally, Figure (5-e) shows the same domain with two of its legs having greater numbers of elements along one side of each leg.

In all cases considered above, the domain is treated as a single unit with no need to generate a transient zone. A matter that (1) reduces greatly the amount of input data; (2) saves user's time; (3) eliminates the need to re-combine the sub-domains and the transients zones into a single domain.

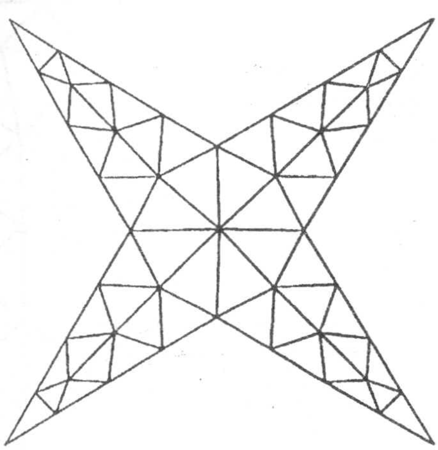

Figure 5-a. $\theta_1 = 60$, $H_1 = 0.80$
$N = 56$ $T = 0.27$ sec, $\sigma = 0.320$.

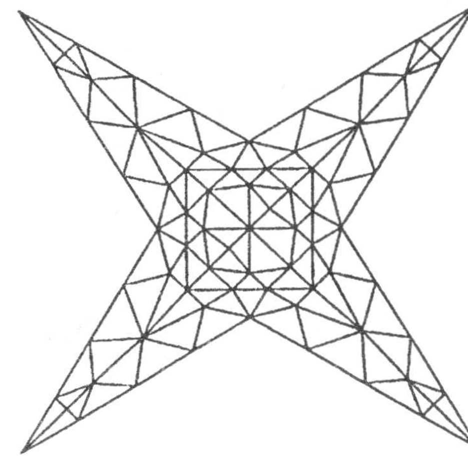

Figure 5-b. $\theta_1 = 20$, $H_1 = 0.50$
$N = 116$ $T = 0.58$ sec, $\sigma = 0.425$.

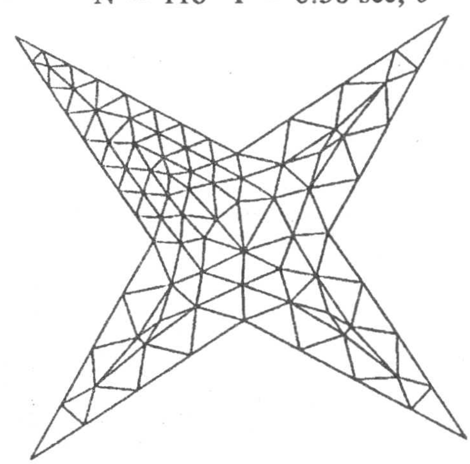

Figure 5-c. $\theta_1 = 60$, $H_1 = 0.60$
$N = 136$ $T = 0.58$ sec, $\sigma = 0.377$.

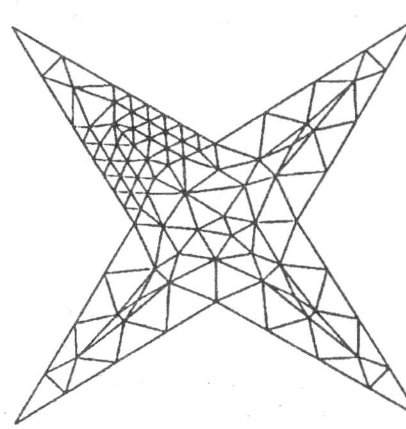

Figure 5-d. $\theta_1 = 60$, $H_1 = 0.86$
$N = 96$, $T = 0.63$ sec, $\sigma = 0.294$.

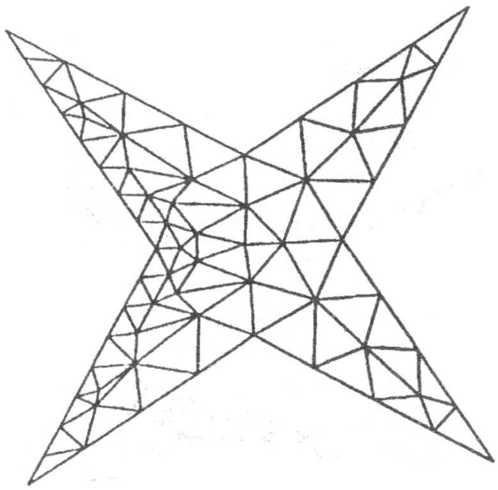

Figure 5-e. $\theta_1 = 90$, $H_1 = 0.65$
$N = 108$, $T = 0.58$ sec, $\sigma = 0.407$.

Figure (6) shows a mesh for square area with higher intensities in one of its corners.

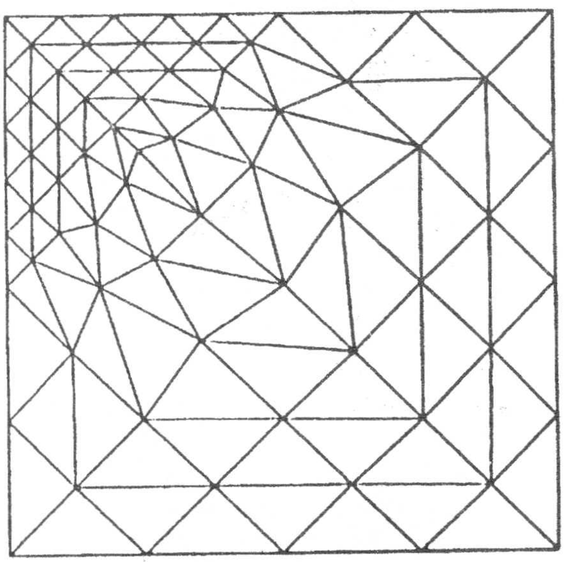

Figure 6. Square domain with fine mesh at one of its corners.

MULTIPLY CONNECTED DOMAINS

It was mentioned before that the method treats multiply connected domains with the same ease by which it treats simply connected domains. Figure (7) shows two square domains with a single opening in each one. Figures (8-a) through (8-c) show a face-lik domain. The meshes are generated by altering th values of the parameters θ_1 and H_1 while keeping th number of elements along the boundary constant Again, the figures show how versatile is the method o using an adaptive mesh generation technique wit parameters that can be varied within the generatio process.

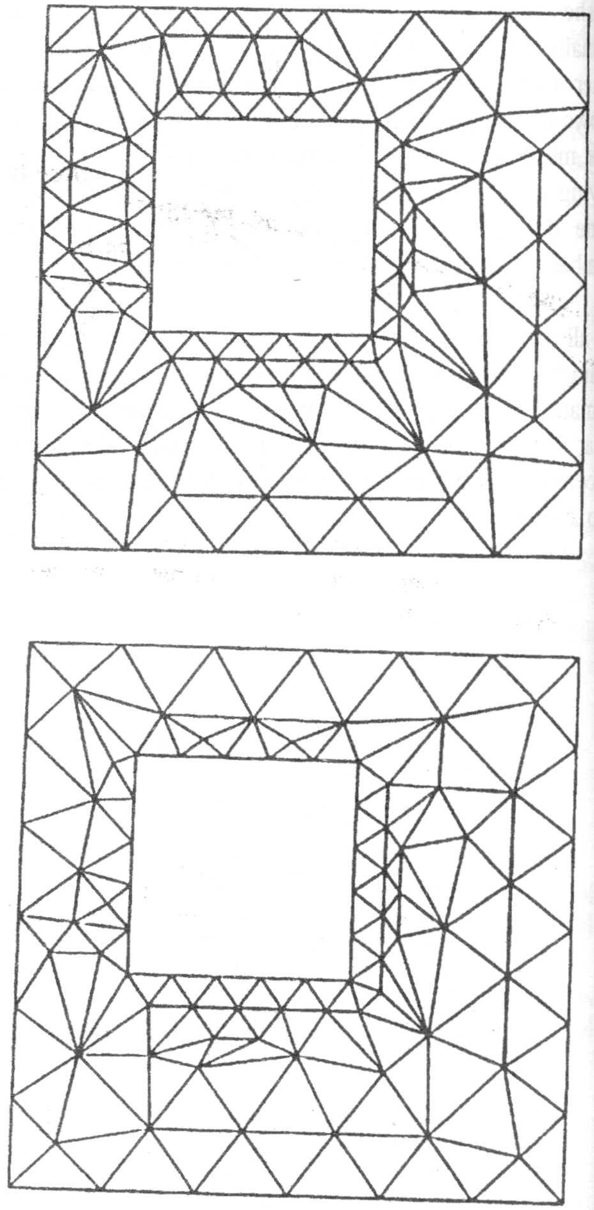

Figure 7. Multiply connected domain.

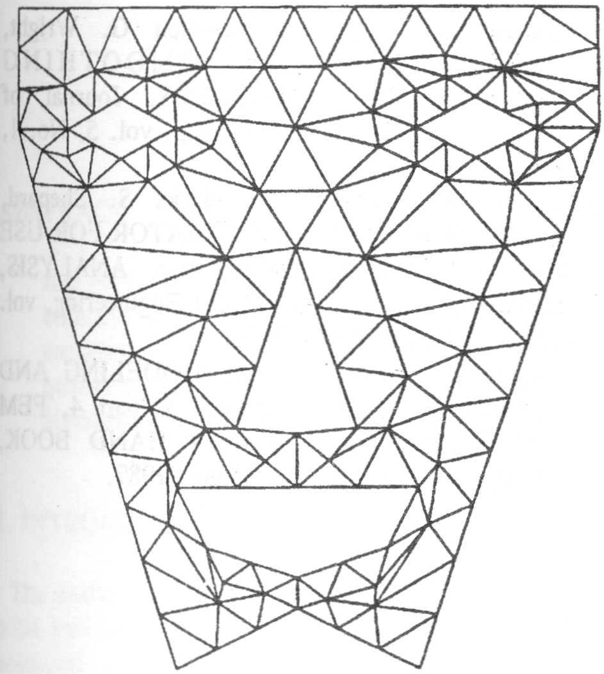

Figure 8-a. Face-like multiply connected domain
$\theta_1 = 99$, $H_1 = 0.99$
$N_o = 204$, $T = 2.50$ sec, $\sigma = 0.388$

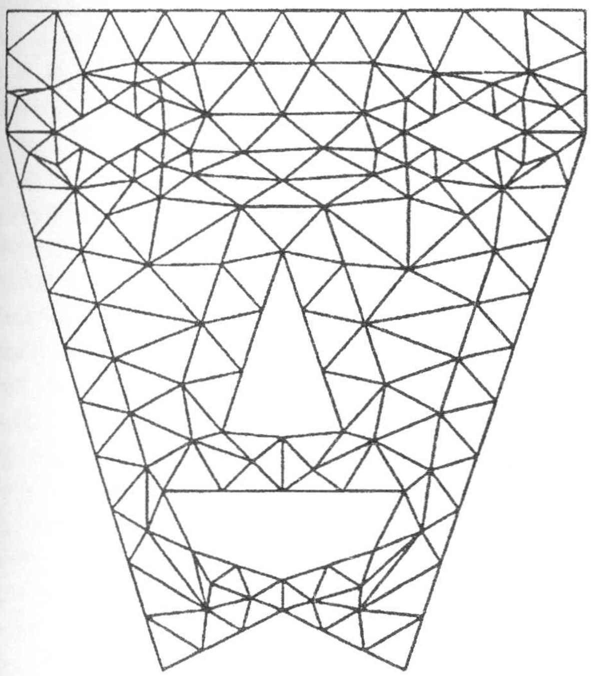

Figure 8-b. Face-like multiply connected domain
$\theta_1 = 90$, $H_1 = 0.86$
$N = 224$, $T = 2.81$ sec, $\sigma = 0.475$

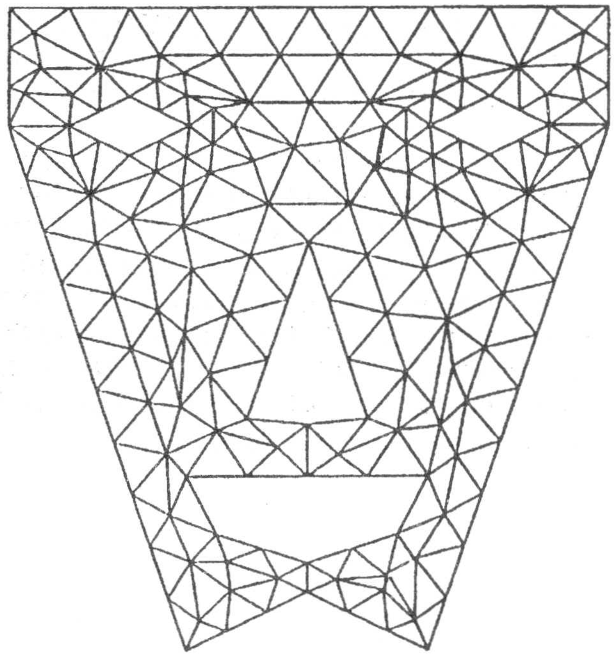

Figure 8-c. Face-like multiply connected domain
$\theta_1 = 70$, $H_1 = 0.65$
$N = 370$, $T = 5.17$ sec, $\sigma = 0.378$.

COMPUTER IMPLEMENTATION

A program is written using TURBO C compiler. A PC is used. It has a CPU 486 with 50 Mhz.

CONCLUSION

A new mesh generating technique is introduced. It has almost linear growth rate. The growth rate decreases with increasing the number of elements. The technique eliminates the need to subdivide the domain before meshing it. Also, it eliminates the need to use a transient zones. Both simply connected and multiply connected domains are treated similarly. The method is flexible and admits the user to control, approximately, the aspect ratio of the elements, the number of the element - fine or coarse mesh.

ACKNOWLEDGEMENT

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