NEUTRON BUILD-UP FACTORS AND ALBEDO DATA IN A DOUBLE-LAYERED SLABS

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ABSTRACT

The concept of the neutron build-up factor and neutron albedo data are important in the analysis of various practical applications. Their use can simplify the calculations in most of these applications. This is due to the simple mathematical form of the neutron flux (knowing the build up factor) which enables carrying out the succeeding calculations analytically. Also the complicated geometry of the problem could be simplified by replacing the double layered slab by a planar source (knowing the albedo-data). A systematic investigation of the effect of the different parameters (the thickness, the type of material of each layer of the slab, the order of arranging the two media in the slab, the type of scattering in each layer and the irradiation geometry) has been carried out using the double S_n-method. It is found that the build-up factor and also the albedo data in a double layered slab are strongly dependent on the c-values of the two media and on the order of arranging the two layers in the slab.

INTRODUCTION

In many practical applications in the fields of radiation shielding, radiation protection, design of fusion devices, neutron therapy, neutron diagnostics and radiogauging, multi-region systems are encountered quite often. In the calculations concerning these applications treating the multi-layered problem, with the given interface and boundary conditions, is a tedious and a complicated problem. The most accurate methods commonly used are the S_n and the Monte Carlo method. They are both numerical methods and need large memory computing machines and much computing times.

At alternative method, which is simpler and still with a high accuracy for treating these calculations is carried out using the build up concept [1,6,7]. The stationary flux at any point in the multi layered slab can be represented in a simple mathematical form and thus the succeeding calculations which are mainly dependent on the flux in the above mentioned fields can be computed analytically. On the other hand information about the reflected neutrons from the multi-layered slabs are essential in many calculations. Considering the characteristics of these reflected neutrons, the calculations of the neutron transport can be greatly simplified. The treatment of the neutron transmission problem of complicated geometries could be accomplished in two steps. In the first step, calculations of the angular distributions and energy spectra of the reflected neutrons when the monoenergetic beam of neutrons is incident on the multi-layered medium at various angles of incidence and various neutron energies are carried out. In the second step, the original problem with its complicated geometry, in which the multi-layered medium would be replaced by a source having the previously determined reflecting properties, can be easily treated.

The objective of this study is to give numerical values for the build-up factors and the albedo data for a large number of different double-layered media that are most important in the practical applications and present them in a suitable form. The double S_n-method is selected for solving the one speed-neutron transport in the double-layered medium. This is due to the less computations and the smaller computing time needed than those in the alternative method (Monte Carlo Method).

A verification of the present code has been carried out by comparison with those in [4,5,9]

BUILD-UP FACTOR

Previous calculations and measurements are scarce [2,8]. The carried out Monte Carlo calculations and the experimental techniques give results that are only useful for the materials and geometries considered. The present results can be used for a variety of materials.

The build-up factor at a certain point (x) is defined as the ratio between the total flux of the uncollided flux at that point, mathematically

$$B(x) = \phi(x)/\phi_o \exp[-(\Sigma_{t_1} x_1 + \Sigma_{t_2} x_2)/\mu_o]$$

where:

ϕ_o = total flux at point (x = 0)

$\phi(x)$ = total flux at point (x)

x_1 and x_2 = thickness of the first layer and second layer respectively

Σ_{t_1} and Σ_{t_2} = total cross-section of the first layer and the second layer respectively

μ_o = the cosine of the angle of incidence of the neutron source

The above equation is valid in case of monodirectional plana source. If the source of neutrons at x = 0 is an isotropic planar source, the exponential function in the above equation is replaced by $E_1(\Sigma_{t_1} x_1 + \Sigma_{t_2} x_2)$ where E_1 is the exponential function of the first order.

The different parameters that affect the values of the build-up factor are the types of the two media specified by the parameter c which is defined as the ratio between the microscopic scattering cross-section and the microscopic total cross-section, the thickness of the two layers, and the order of arranging the two layers. The effect of each of these parameters is studied separately. The calculations are carried out for the case of an isotropic planar source.

The Effect of The Thickness of the Second Layer

The case of a double-layered slab whose first layer is of lower c-value than that of the second layer is first studied. The thickness of the second layer is varied as can be seen in Figure (1), while the thickness of the first layer is kept constant at 3 m.f.p. It can be concluded that the build up factor inside the first layer at shallow depths assumes the value of the build-up factor of a single-layer made of the first layer medium only, then starts to take higher values as it approaches the interface between the two media. This is expected due to the back-scattered neutrons from the second layer.

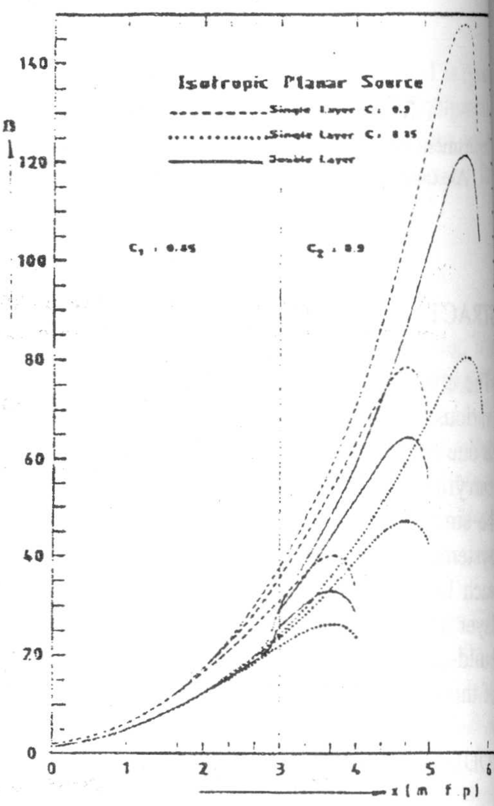

Figure 1. Flux buildup factors versus depth in d layered medium. The first layer has higher c- than the second.

In the second layer, the build-up factor takes h values than if the whole medium was made of th layer only. This increase is enhanced as the thic of the second layer is increased due to the absorbing power of the second layer.

The Effect of The Thickness of the First Layer

The thickness of the first layer is decreased to m.f.p. and then to be 1 m.f.p. as shown in Fi (2,3). It can be concluded that at constant thickn the second layer, the behaviour of the build-up in the first layer at shallow depths has almost the depth dependence as if the whole medium were of the first layer only, but it takes higher values the interface between the two media. As the thic of the first layer decreases, the build-up factor as values of the build-up factor of a single layer ma the second layer medium only.

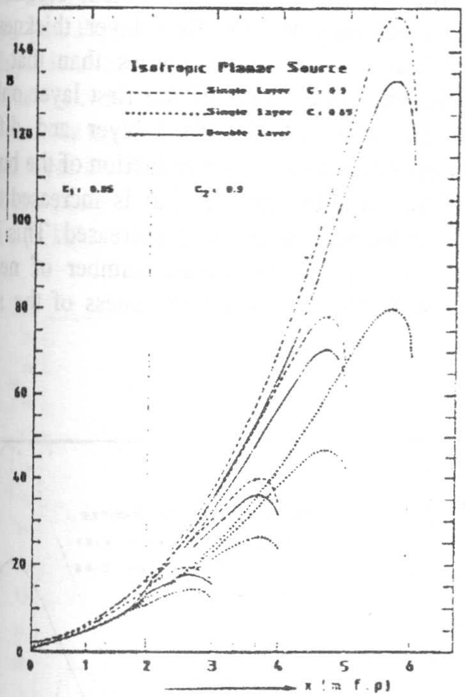

Figure 2. Flux buildup factors versus depth in double-layered medium. The first layer has higher c- value than the second.

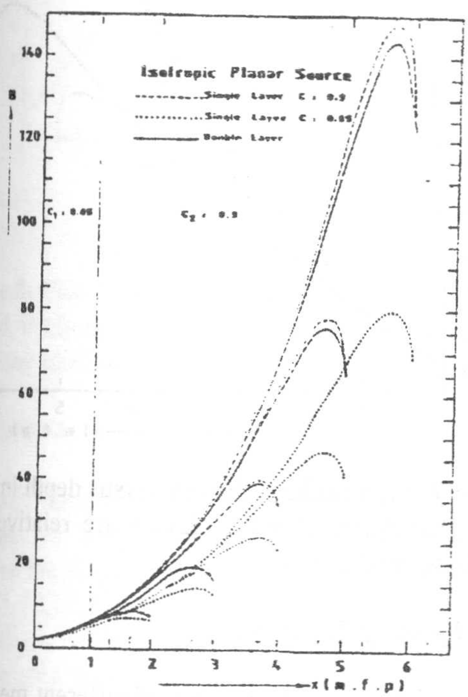

Figure 3. Flux buildup factors versus depth in double-layered medium. The first layer has higher c- value than the second.

The behaviour of the build-up factor in the second layer is noticeably affected by the thickness of the first layer. At very small thickness of the first layer e.g. 1 m.f.p, the build-up factor has almost the same depth dependence as if the whole medium were made of the second layer only. As the thickness of the first layer is increased to be 2 or 3 m.f.p., the build up factor takes lower values.

Order of Arrangement

The above calculations are repeated when the order of arranging the two layers is reversed. The first layer has a higher c-value than the second layer. The results are shown in Figures (4,5, and 6). In the first layer, at constant thickness of the second layer and varying the first layer thickness the build-up factor of the slab takes lower values than that if the whole medium were made of the first layer-medium only. This is due to the higher absorbing power of the second layer which has lower c-value. As the thickness of the first layer increases, the lowerness of the build up factor decreases.

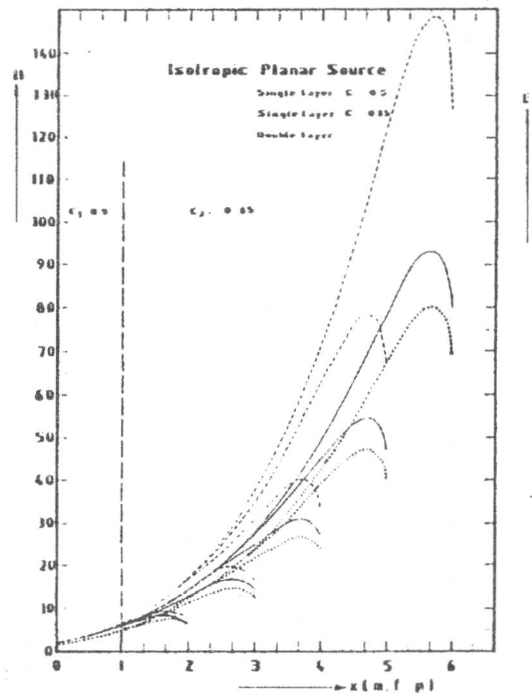

Figure 4. Flux buildup factors versus depth in double-layered medium.The first layer has lower c-value than the second.

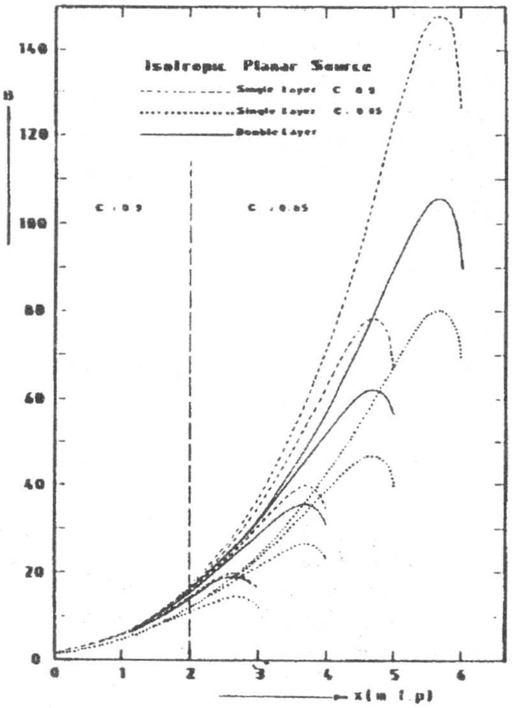

Figure 5. Flux buildup factors versus depth in double-layered medium. The first layer has lower c-value than the second.

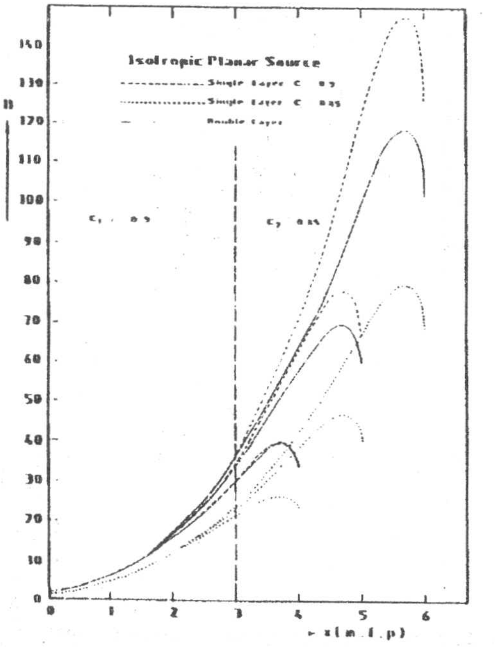

Figure 6. Flux buildup factors versus depth in double-layered medium. The first layer has lower c-value than the second.

Inside the second layer, at constant second la thickness and varying the first layer thickness build-up factor takes lower values than that if whole medium were made of the first layer only constant thickness of the first layer and diffe second layer-thicknesses, the reduction of the build factor of the double-layered slab is increased as thickness of the second layer is increased. This is expected due to the increased number of neutr absorbed with the increased thickness of the sec layer.

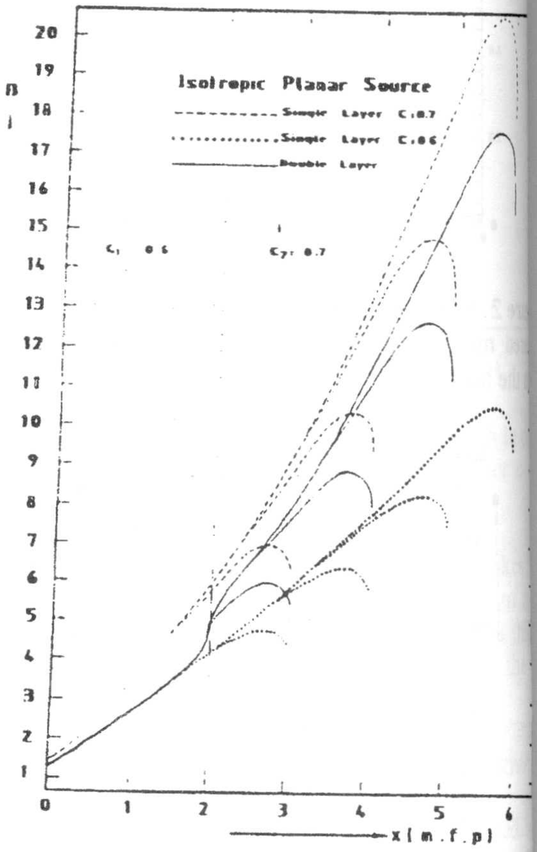

Figure 7. Flux buildup factors versus depth in dou layered medium. The two layers are relatively g absorbing media.

The Effect of The c-Value

Three cases of combinations of different media h been investigated. The first case is that when the layers are relatively good scattering media (high value) (Figure 1,6). The second case is when the layers are relatively good absorbing media (low c-value), (see Figures 7,8). The third case is that when one layer is a relatively good scattering medium while the other is a relatively good absorbing medium. In each case the thickness of the first layer and that of the second layer are varied interchangeably and the effect has been observed. The results are summarized in Figure (9,10).

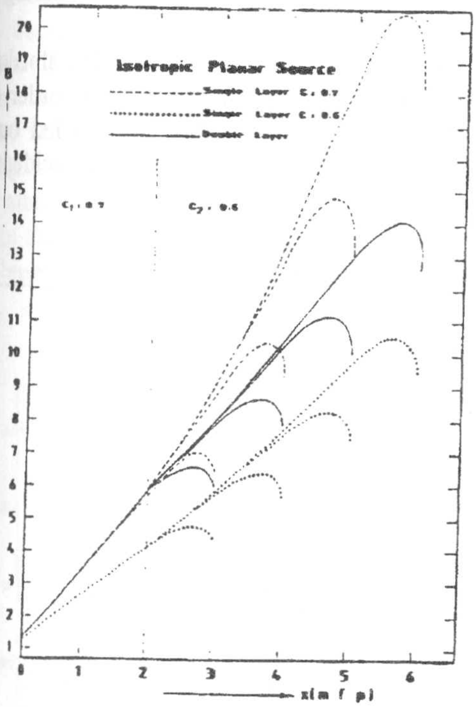

Figure 8. Flux buildup factors versus depth in double-layered medium. The two layers are relatively good absorbing media.

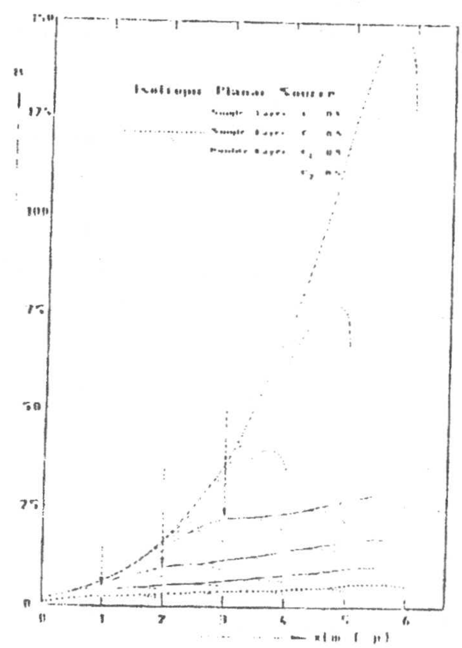

Figure 9. Flux buildup factors versus depth in double-layered medium. The first layer is a good scatterer while the second layer is a good absorber.

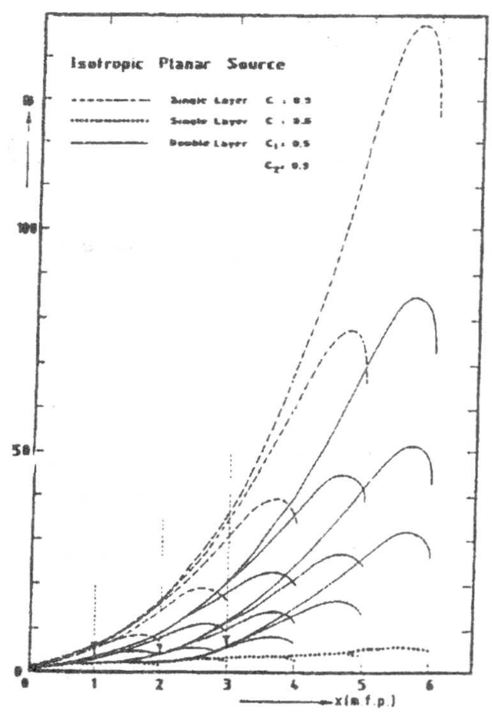

Figure 10. Flux buildup factors versus depth in a double layered medium .The first layer is a good absorber while the second layer is a good scatterer.

3. ALBEDO

Neutron number albedos have been obtained mainly for slabs of a homogeneous medium. Few papers consider the neutron number albedo of double-layered slabs [3,10,11,12].

In realistic energy-dependent problems, the scattering is invariably anisotropic. In order to obtain accurate solutions to these problems using one-speed transport methods, the effect of the anisotropy must be introduced to get a comparatively accurate results.

In plane geometry with anisotropic scattering, the one-speed transport equation takes the form

$$\mu \frac{\partial \phi(x,\mu)}{\partial x} + \phi(x,\mu) = c \int_0^{2\pi} d\alpha \int_{-1}^{1} f(\hat{\Omega} - \Omega)\phi(r,\mu)d\hat{\mu} + Q(x,\mu)$$

where (α) is the azimuthal angle. The angular flux $\phi(x,u)$ and the source $Q(x,\mu)$ are assumed to be independent of the azimuthal angle (α). Except for special cases such as when the medium is moving or consists of a single crystal, $f(\hat{\Omega} - \Omega)$ is a function of $\Omega \cdot \hat{\Omega} = \mu_o$ only, where $\hat{\Omega}$ and Ω are the neutron directions before and after scattering respectively.

Consequently, $f(\hat{\Omega} - \Omega)$ may be expressed as sum of Legendre Polynomials, i.e.,

$$f(\hat{\Omega} - \Omega) = f(\mu_o) = \sum_{l=0}^{\infty} (l + 1) f_l P_l(\mu_o)/4\pi$$

By the orthogonality of these polynomials, one gets

$$f_l = 2\pi \int_{-1}^{+1} f(\mu_o)\, P_l(\mu_o)\, d\mu_o$$

with the normalization condition,

$$f_0 = 2\pi \int_{-1}^{+1} f(\mu_o)\, d\mu_o = 1$$

For isotropic scattering $l = 0, P_o(\mu) = P_o(\mu_o) = 1$ and $f_o = 1$. For linear anisotropic scattering

$$l = 1,\ P_1(\mu) = \mu,\ P_1(\mu_o) = \mu_o,\ f_1 = \hat{\mu}_o$$

where $\hat{\mu}_o$ is the average cosine of the scattering angle.

The albedo is defined as the ratio of the radiation current reflected from a surface to the current incident upon that surface. Mathematically

$$R = \int_{-1}^{0} \phi(0,\mu)\mu . d\mu$$

where

$\phi(0,\mu)$ = the angular flux at the surface.

As a first step in the computations, the neut scattering is assumed to be isotropic. This case represent a large number of materials. As a sec step, the case of linear anisotropic scattering is investigated.

The main two parameters affecting the albedo val are the thickness of each layer and the order of th arrangement. The calculations are carried out for collimated beam of neutrons normally incident on double-layered slab.

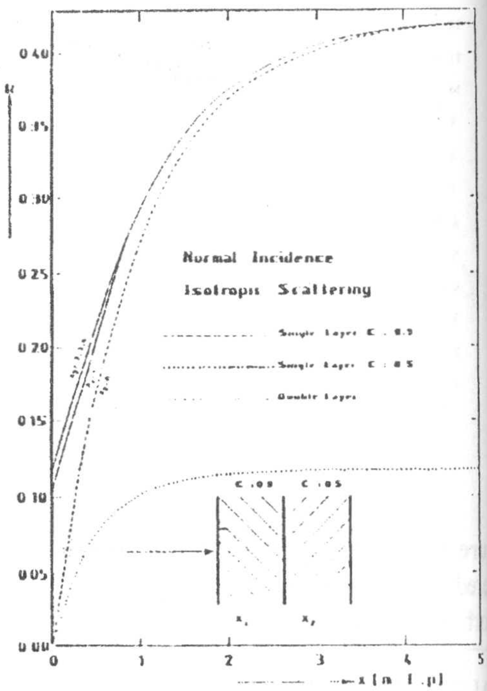

Figure 11. Number albedo for double layered medi Curve parameter is the thickness of the second la The first layer is a good scatterer.

The Effect of the Layers Thicknesses

The effect of the thickness of each laye investigated. In Figures (11-16), the ordinate is albedo value and the abscissa is the thickness of the first layer. Curve parameter is the thickness of the second layer. For comparison the variation of the albedo value with the thickness in case of a single-layered medium is also drawn for the medium of the first layer and that of the second layer. It is observed that the albedo of the double-layered slab increases with the increase of the thickness of the first layer and then reaches the saturation value of the first medium. It is also observed that the rate of the increase of the albedo decreases with increasing the second layer thickness. This is because the backscattering of neutrons is higher for media of high c-values than those of lower values. One also can conclude that changing the kind of the second layer has a large effect on the albedo values as shown in figures (12), (13), where in these figures the first layer is kept the same as in Figure (11), while the second layer is changed.

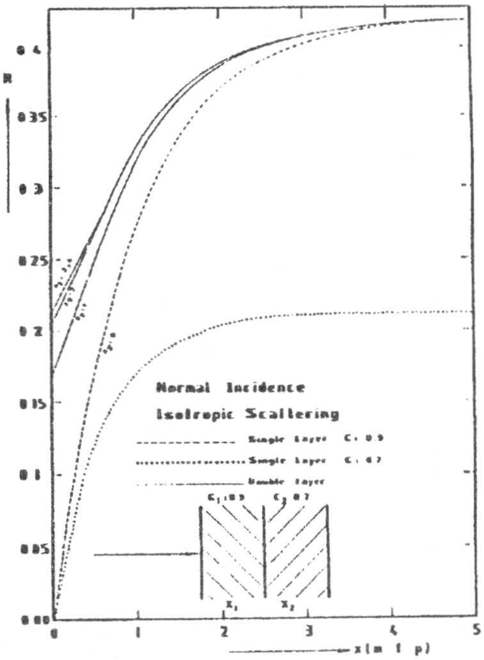

Figure 13. Number albedo for double layered medium. Curve parameter is the thickness of the second layer. The first layer is a good scatterer.

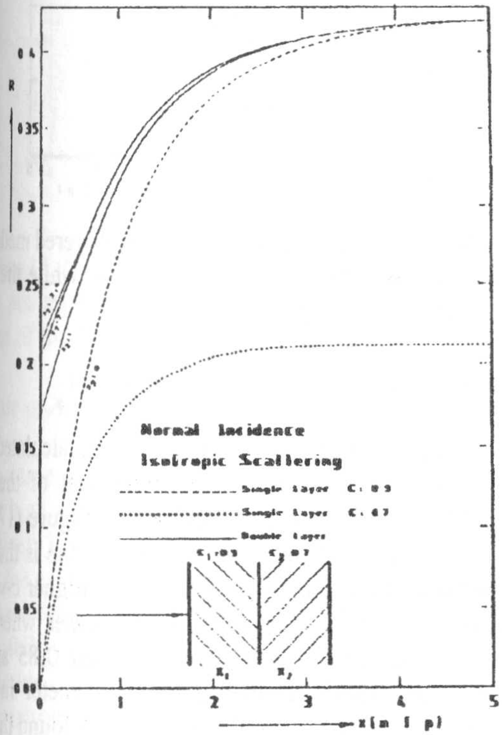

Figure 12. Number albedo for double layered medium. Curve parameter is the thickness of the second layer. The first layer is a good scatterer.

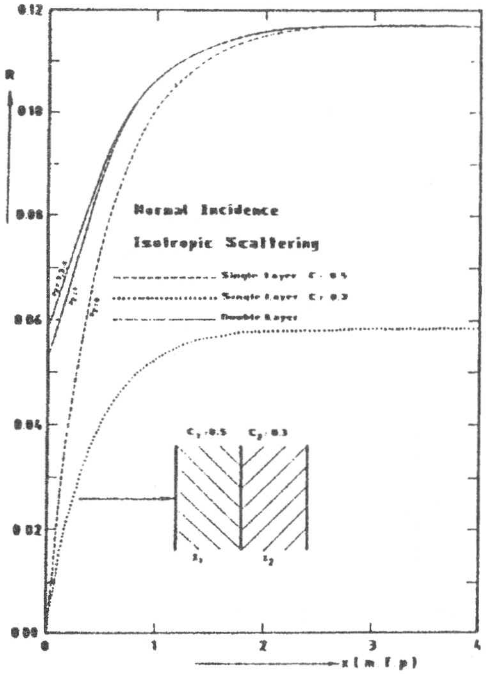

Figure 14 . Number albedo for double-layered medium. Curve parameter is the thickness of the second layer. The first layer is a good absorber.

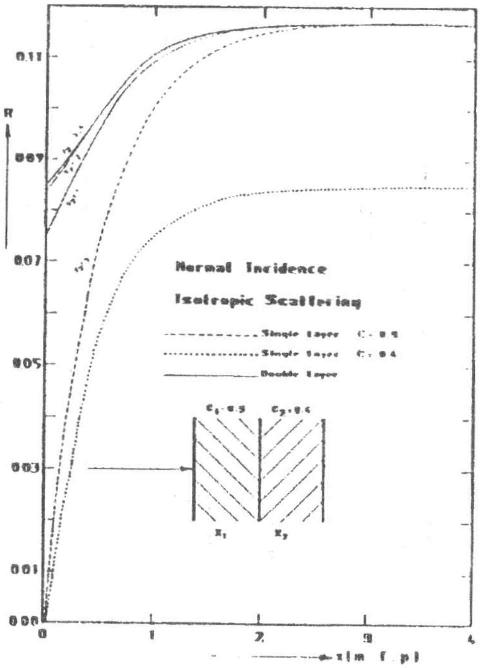

Figure 15. Number albedo for double-layered medium. Curve parameter is the thickness of the second layer. The first layer is a good absorber.

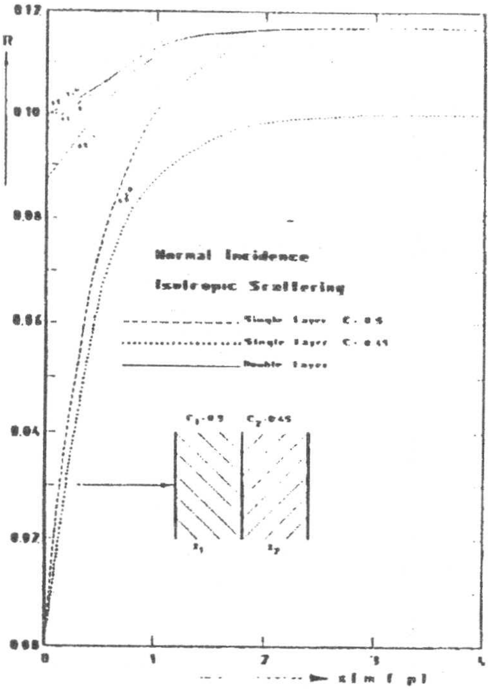

Figure 16. Number albedo for double-layered medium. Curve parameter is the thickness of the second layer. The first layer is a good absorber.

The above calculations are repeated for another lev of the c-values. In Figure (14), (15), (16) the fi layer is of a medium of c = 0.5 while the second la is of a medium of c = 0.3, 0.4, 0.45 respectively. expected the albedo has the same behaviour except t their values are much smaller.

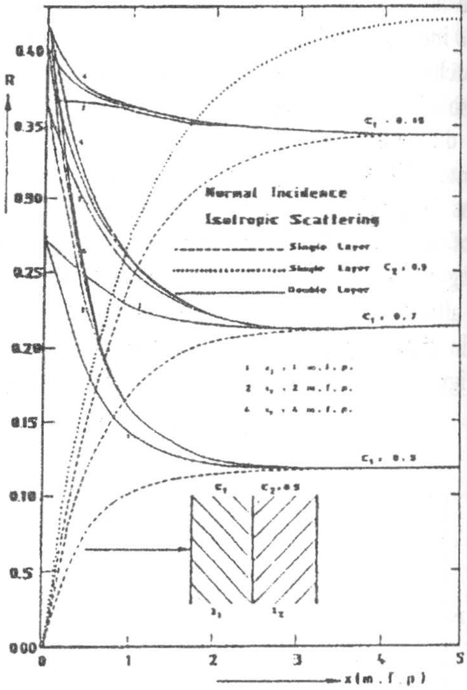

Figure 17. Number albedo for double-layered medi The second layer has the same c-value, while the value of the first layer is varied.

The Effect of the Order of Arrangement

The cases of Figures (11-16), are recalculated with reversing the order of arrangement of the layers. The results are summarized in Figure (17- In this case the medium with lower c-value is the f layer followed by a second medium of higher c-va The three families of curves for the cases where first layer has c-values of 0.5, 0.7 and 0.85 are plotted in the same figure. Curve parameter in e family is the second layer-thickness. It is found that albedo value has a different behaviour. It decre monotonically with the increase of the thickness of first layer until it reaches the saturation value of first layer-medium. This is due to the high absorb power of the first medium. It can be also conclu that the rate at which it reaches this saturation v decreases as the thickness of the second layer decreases. This rate increases as the c-value of the first layer decreases due to the increased absorption as the c-value decreases.

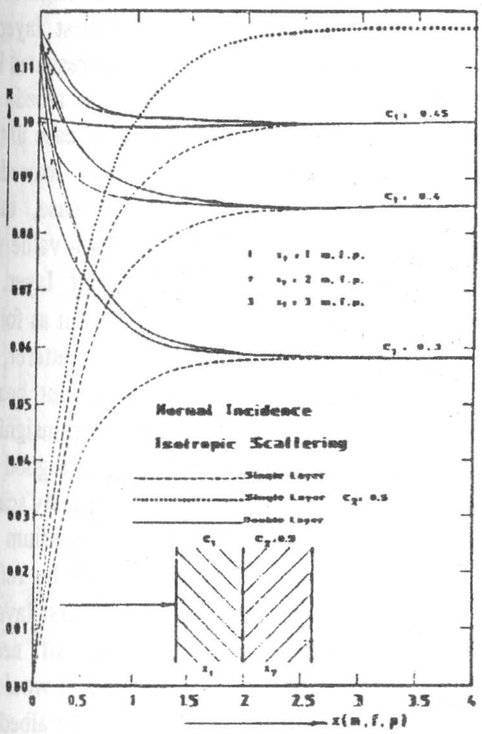

Figure 18. Number albedo for double-layered medium. The second layer has the same c-value, while the c-value of the first layer is varied.

Case of Anisotropic Scattering

The case of a double layered slab with its first layer of a high c-value medium and its second layer of a relatively low c-value medium is considered. In Figure (19) the calculations for the cases where the first layer is characterised by a backward-scattering medium (Z=-0.3) followed by a second layer of a forward scattering medium (Z = 0.3) are represented. For comparison the cases where either one of the two layers is characterized by isotropic scattering or both are also plotted. Two examples are considered, the first where the thickness of the first layer is relatively thin (x = 1 m.f.p.) and the second where the thickness is relatively thick (x = 4 m.f.p.).

In the case , where the second layer is relatively thick (4 m.f.p), it is observed that the albedo starts from the value of the pure second layer , then decreases reaching a minimum value then increases again monotonically until it reaches the saturation value of the first layer . The main contribution to the albedo is due to those neutrons coming from the second layer and escaped collisions in the first layer . As the thickness of the first layer (which is a good back-scatterer) increases further, the beam of neutrons diverges within it and some will be backwardly scattered and hence contribute to the albedo and so the contribution to the albedo at larger thickness of the first layer is not only due to those neutrons coming from the second layer and escaped collisions in the first but also due to those neutrons colliding within the first layer and being backwardly scattered. It should be noted that the depression is less when the scattering inside one of them is isotropic. In the case where the second layer is relatively thin, the effect of the anisotropy is to decrease the rate by which the albedo reaches its saturation-value.

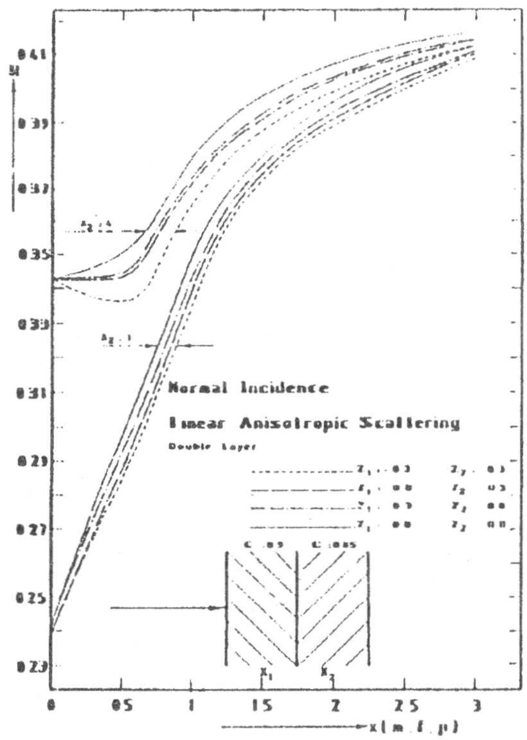

Figure 19. Number albedo for a double-layered medium. The first layer is of a backward scattering medium and the second layer is of a forward scattering medium. Curve parameter is the thickness of the second layer.

The above calculations were repeated with the second layer having different c-values and the results are represented in Figure (20) . The depression in the albedo value at small depths in the first layer of a good back-scattering medium decreases and begins to disappear as the medium of the second layer becomes more absorbing. This is expected since in this case the main contribution would be from the neutrons backwardly scattered from the first layer . The absorption in the second layer is becoming higher and the contribution from it to the albedo is low .

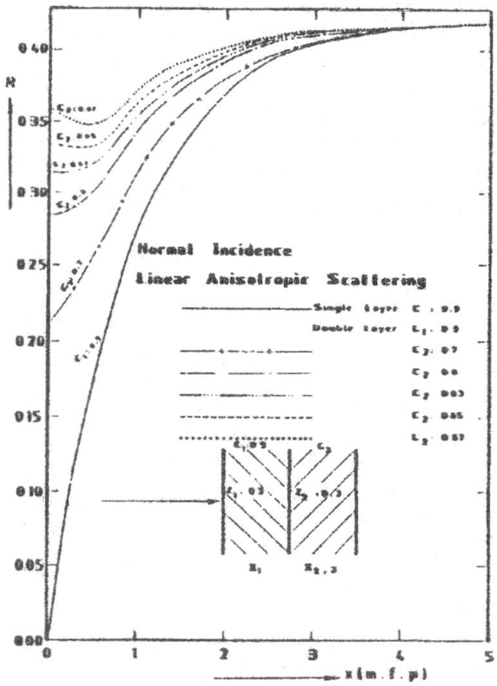

Figure 20. Number albedo for a double-layered medium. The first layer is of a backward scattering medium and the second layer is of a forward scattering medium. Curve parameter is the c-value of the second layer.

For the case of a double-layered slab with its first layer of a relatively low c-value medium followed by a second of a relatively high c-value medium , the albedo values are calculated and are given in Figure (21). In this figure the case where the first layer is of a forward scattering medium (Z= + 0.1) followed b a second layer of a backward scattering medium (Z=-0.1) is plotted. For comparison the case where th two layers are characterized by isotropic scattering i calculated and plotted in the same figure. Th calculations are repeated when the first layer ha different c-values and the results are represented in th same figure. It can be concluded that the albedo take higher values than that for the isotropic case until th thickness of the first layer reaches or exceeds th saturation thickness of the isotropic case, then i decreases and assumes another saturation value whic characterizes the medium of the first layer. Th explanation of this observation may be put as follow Since the first layer is a good forward scatterer, so small thicknesses a the normally incident beam o neutrons will pass through it suffering insignifica divergence and since the second layer is a good bac scatterer, the number of neutrons backwardly scatter is greater than it would be if the medium we isotropic and hence more neutrons will be reflect into the first layer again and since the first layer is good forward scatterer, the number of neutro reaching the surface will be greater than it would be the medium were isotropic. Therefore the albedo w be higher than if the medium were isotropic. But as t thickness of the first layer increases another proc comes into play, namely the absorption process in t first layer. Since the first layer is higher in absorbing power than the second layer, the competiti between the two processes accounts for the pea shown in Figure (21). Below a certain thickness of t first layer, the increase in the albedo due to anisotro of the media is dominant and is higher than t reduction caused by the absorption process in the fi layer. Above this thickness, the reverse is true and t albedo begins to decrease until it assumes the saturat value of the first medium. It should be noted also t the peak in the albedo-thickness curve increases as t c-value of the first layer increases. This is expec since the increase in the c-value, reduces the absorpt in the first layer, and so the number of neutr passing through it to the surface will be larger.

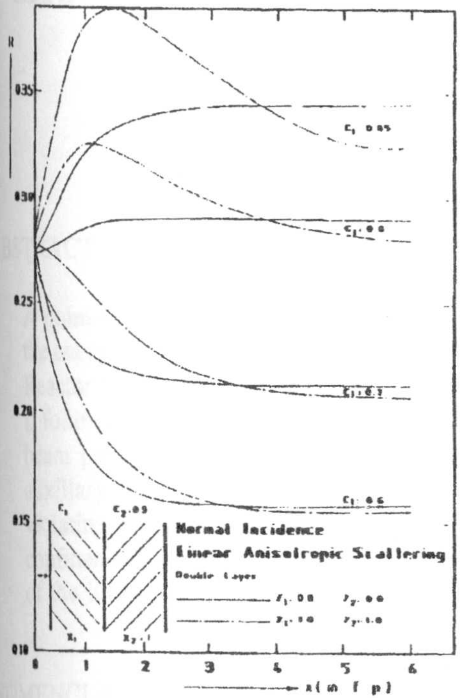

Figure 21. Number albedo for a double-layered slab. Curve parameter is the c-value of the first layer.

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