BENDING OF ISOTROPIC THIN PLATES BY CONCENTRATED EDGE COUPLES USING CONFORMAL MAPPING

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ABSTRACT

In this work the complex variable method of Muschelishvili for solving the biharmonic equation is applied to problems of bending of isotropic thin plates by concentrated edge couples. The results of the method as applied to plate problems by previous authors are presented first. These are then applied to the ellipse by using the conformal mapping to circular plate subjected, respectively, to two bending couples, and to two twisting couples, both applied to the ends of a diameter. Numerical results are presented in the form of graphs. There are two cases (i) Elliptic plate under conformal mapping to circular plate, (ii) Cirular disk plate (special case from (i))

INTRODUCTION

Lurie [1] applied Muschelishvili's method [2] for solving the biharmonic equation to several problems of bending of circular plates. It was mentioned in the same reference that the author has used the method in similar studies earlier in 1928. Lechnitsky [3] further extended the complex variable method to the study of bending of anisotropic plates. An English exposition of Lechntzky's results is given in Sokolnikoff [4].The method of Muschelishvili and Lechnitsky's results is given in Sokolnkiff[4].Using the method of Muschelishvili and Lechnitsky, Morkovin [5] further discussed in detail the bending of an anisotropic clamped elliptic plate. By means of the same method, Friedmann [6] solved several problems related to a large isotropic plate with a circular hole as well as a circular ring plate. Methods using complex variables have been applied to plate problems also by Stevenson [7] and Seth [8].

In this research the complex variable method of Muschelishvili is further applied to study the bending of isotropic homogeneous thin plates, the boundaries of which are subjected to concentrated couples. Plate problems concerning concentrated edge couples have not received much attention. One such problem was discussed by Lechnitsky [3] in which a semi-infinite anisotropic plate is bent by concentrated couples at its edge.

The present method reduces in a simple manner the problems of concentrated edge couples and forces to limiting cases of the first boundary-value problem of the plate theory in which the moment and shear resultant are prescribed along the plate boundary. From the derivation to be presented in this paper it appears that the present theory may as well be applied to more real and practical cases in which couples and forces actually are distributed over small but finite parts of the plate boundary instead of being concentrated at points on the boundary. The method also illustrates the similarity between the plate problems to be considered and their analogous ones in two-dimensional elasticity in which concentrated forces in the plane of a slice are applied at the boundary.

Small deflections will be assumed in the following discussion of bending of thin isotropic plates. We shall consider a free plate with concentrated external couples or forces acting at a finite number of points along its boundary. For convenience, concentrated couples and concentrated forces will be discussed separately. It may be seen that this does not restrict us from loading the plate with both couples and forces at the same time. The middle plane of the plate will be assumed to occupy a region bounded externally by a single closed curve and, in particular,by an ellipse (using conformal mapping this is transformed to a circle).In [9], the solved problem was on a circle and it will be shown that our general case is reduced easily to the special case of a circle.

PRELIMINARY RESULTS

When the middle plane of an isotropic homogeneous thin plate free from lateral load is assumed to be in the complex plane of $z = x + iy$ as shown in Figure (1), the differential equation for its deflection w in a direction perpendicular to the plane has the form:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = 0$$

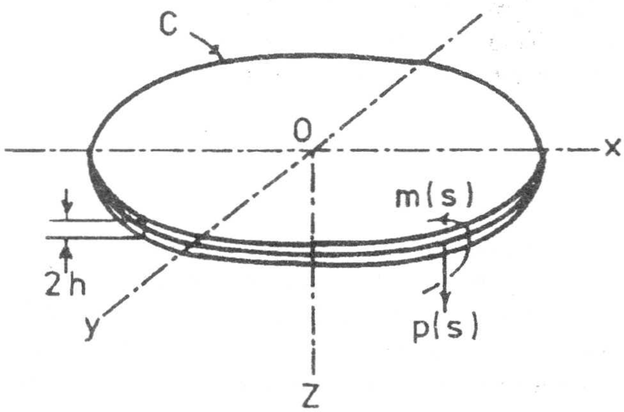

Figure 1. Plate with middle plane is s-plane.

According to Muschelishvi, the solution of this biharmonic equation may be represented in terms of two analytic functions $\phi_1(z)$ and $\psi_1(z)$ of the complex variable z.

$$w = 2\mathrm{Re}\,[\,\bar{z}\varphi_1(z) + \chi_1(z)] \qquad \chi_1(z) = \int \psi_1(z)\,dz \tag{1}$$

We use bars to denote conjugate complex quantities. The bending moments per unit length M_x, M_y, twisting moment per unit length H_{xy} and the shearing forces per unit length Q_x, Q_y may conveniently be represented in terms of $\phi_1(z)$ and $\psi_1(z)$ as follows

$$M_x + M_y = -8D(1+\nu)Re[\dot{\varphi}_1(z)]$$

$$M_y - M_x + 2iH_{xy} = 4D(1-\nu)[\bar{z}\varphi_1'(z) + \psi_1(z)] \tag{2}$$

$$Q_x - iQ_y = -8D\dot{\varphi}_1(z) \quad ,$$

in which $D = 2Eh^3/3(1-\nu^2)$ is the usual flexural rigidity of plate, E, ν and h are Young's modulus, Poisson's ratio and half-thickness of the plate respectively. It is to be noted that H_{xy} has a sign opposite to that of M_{xy} as used by Timoshneko [10].

As shown in Figure (1), the moment and shear resultant per unit length along the boundary C of the plate will be given as functions of the arc length s along C

$$M_\nu = m(s),\, Q_\nu + \frac{\partial H_{\nu s}}{\partial s} = P(s) \text{ on } C, \tag{3}$$

where ν denotes the exterior normal to C. The boundary condition may be shown to have the final form

$$n\phi_1(z) + z\bar{\phi}_1(\bar{z}) + \bar{\psi}_1(\bar{z}) = f_1 + if_2 \text{ (on C)} \tag{4}$$

in which $n = -\left(\frac{3+\nu}{1-\nu}\right)$

and

$$f_1 + if_2 = \frac{1}{2D(1-\nu)} \int_0^s [m(s) + i\int_0^s P(s)ds](dx + idy) \tag{5}$$

The plate problem in which the boundary moment and the shear resultant are prescribed now becomes one of determining the two functions $\varphi_1(\xi)$ and $\psi_1(\xi)$ which satisfy the boundary conditions, equation (4). Once these functions are determined, the deflection, moments and shearing forces at any point in the plate can be computed using equations (1) and (2).

The foregoing results are due to Lechnitsky and may be found in references [3], [4] and [6]

CONDORMAL MAPPING

Let the function

$$z = \omega(\xi)$$

map the boundary C of the given region occupied by the middle plane of the plate in the z-plane into the unit circle γ in the ξ-plane[11]. Curvilinear coordinates (ρ, θ) are thus be introduced into the z-plane which are the maps of the polar co-ordinates in the ξ-plane given by : $\xi = \rho\, e^{i\theta}$

Introducing the mapping function $\omega(\xi)$ into equation (1), where $\omega(\xi)$ is a rational function, we now have the deflection of the plate given by

$$w = 2\mathrm{Re}[\bar{\omega}(\bar{\xi})\phi(\xi) + \chi(\xi)] \quad (6)$$

where

$$\phi(\xi) = \phi_1[\omega(\xi)] = \phi_1(z), \; \chi(\xi) = \int \psi(\xi)\dot{\omega}(\xi)d\xi$$

and

$$\psi(\xi) = \psi_1[\omega(\xi)] = \psi_1(z)$$

The moments and shearing forces at any point in the plate referred to the new coordinates (ρ,θ) may be shown to be related to those referred to the (x,y) coordinates as follows

$$M_\rho + M_\theta = M_x + M_y$$

$$M_\theta + M_\rho + 2iH_{\rho\theta} = e^{2i\alpha}(M_y - M_x + 2iH_{xy})$$

$$Q_\rho - iQ_\theta = e^{i\alpha}(Q_x - iQ_y)$$

in which α is the angle between the x and ρ-directions at the point. As was shown by Muschelishvili [2]

$$e^{i\alpha} = \frac{\dot{\omega}(\xi)\xi}{|\dot{\omega}(\xi)|\rho}, \quad e^{2i\alpha} = \frac{\dot{\omega}(\xi)}{\bar{\omega}'(\bar{\xi})}\frac{\xi^2}{\rho^2}$$

therefore, equation (2) lead to:

$$M_\rho + M_\theta = D(1+\nu)Re[\Phi(\xi)]$$

$$M_\theta - M_\rho + 2iH_{\rho\theta} = 4D(1-\nu)\frac{\xi^2}{\rho^2\bar{\omega}'(\bar{\xi})}[\bar{\omega}(\xi)\Phi'(\xi) + \omega'(\xi)\Psi(\xi)]$$

$$Q_\rho - i\,Q_\theta = -\frac{8\,D\,\xi}{\rho|\omega'(\xi)|}\Phi'(\xi) \quad (7)$$

in which $\Psi(\xi) = \dfrac{\psi'(\xi)}{\omega'(\xi)}, \quad \Phi(\xi) = \dfrac{\phi'(\xi)}{\omega'(\xi)}$

The boundary equation (4) is now transformed into:

$$n\phi(\sigma) + \frac{\omega(\sigma)}{\bar{\omega}'(\bar{\sigma})}\overline{\varphi'(\sigma)} + \bar{\psi}(\bar{\sigma}) = f_1 + if_2 \quad (\text{on } \gamma) \quad (8)$$

in which $f_1 + if_2$ has also been transformed and is now a function of $\sigma = (e^{i\theta})$, namely the value of ξ on the boundary γ of the unit circle.

For a region bounded externally by a single closed curve and mapped onto the unit circle in the ξ-plane, each of the two analytic functions $\phi(\xi)$ and $\psi(\xi)$ can always be represented by a power series consisting only of positive powers of ξ. Furthermore, because of the indeterminacy of functions, we may write.

$$\varphi(\xi) = a_1\xi + a_2\xi^2 + a_3\xi^3 + \ldots$$
$$\psi(\xi) = a_o + \grave{a}_1\xi + \grave{a}_2\xi^2 + \ldots \quad (9)$$

where a_1, a_2 ... , a_o , a'_1 , a'_2, ... are real unknown constants.

Solution of the integro-differential equation

The boundary condition (8) may be reduced to an integro-differential equation for the determination of $\phi(\xi)$ and $\psi(\xi)$. If we multiply both sides of (8) and its conjugate by $\frac{1}{2\pi i}\frac{d\sigma}{\sigma - \xi}$ where $|\xi| < 1$ and integrate over γ, we get by Harnack's theorem [4].

$$n\phi(\xi) + \frac{1}{2\pi i}\int_\gamma \frac{\omega'(\sigma)}{\overline{\omega(\sigma)}}\frac{\overline{\phi(\sigma)}}{(\sigma - \xi)}d\sigma = \frac{1}{2\pi i}\int_\gamma \left(\frac{f_1 + if_2}{\sigma - \xi}\right)d\sigma \quad (10)$$

Once a solution of (10) satisfying the condition $\overline{\psi(0)} = 0$ is obtained, function $\psi(\xi)$ may be calculated by Cauchy's integral formula from (8). The value of $\psi(\xi)$ is determined by forming the conjugate of (8). Since $\omega(\xi)$ is a rational function :

$$\omega(\xi) = \gamma_1\xi + \gamma_2\xi^2 + \gamma_3\xi^3 + \ldots + \gamma_n\xi^n \quad \gamma_1 \neq 0, \gamma_n \neq 0 \quad (11)$$

$$\phi(\xi) = \frac{1}{n}\left[\frac{1}{2\pi i}\int_\gamma \left(\frac{f_1 + if_2}{\sigma - \xi}\right)d\sigma - \sum_{m=1}^{n} K_m\xi^m\right] \quad (12)$$

$$\psi(\xi)=\frac{1}{2\pi i}\int_{\gamma}\left(\frac{f_1-f_2}{(\sigma-\xi)}d\sigma-\frac{\bar{\omega}(\frac{1}{\xi})}{\dot{\omega}(\xi)}\dot{\phi}(\xi)+\sum_{m=1}^{n}\bar{K}_{-m}\xi^{-m}, \qquad (13)$$

and note that $\frac{1}{2\pi i}\int_{\gamma}\overline{\frac{\phi(\sigma)}{(\sigma-\xi)}}d\sigma = 0$

where

$$K_n = a_1 C_n$$

$$K_{n-1} = a_o C_{n-1} + 2 a_2 C_n \qquad (14)$$

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$$K_1 = a_0 C_{n-1} + 2a_2C_2 + \dots\dots\dots + n\ a_n Cn$$

and the constants C_1, C_2,..........,C_n depend on the coefficients of $\omega(\xi)$ and

$$a_1=\left[\frac{1}{n}\frac{d}{d\xi}\left(\frac{1}{2\pi i}\int_{\gamma}\frac{f_1+if_2}{\sigma-\xi}d\sigma\right)\right]_{\xi=0}-\frac{1}{n}\left[\frac{d}{d\xi}\sum_{k=1}^{n}k_m\xi^m\right]_{\xi=0} \qquad (15)$$

$$a_2=\left[\frac{1}{2n}\frac{d^2}{d\xi^2}\left(\frac{1}{2\pi i}\int_{\gamma}\frac{f_1+if_2}{\sigma-\xi}d\sigma\right)\right]_{\xi=0}-\frac{1}{2n}\left[\frac{d^2}{d\xi^2}\sum_{k=1}^{n}k_m\xi^m\right]_{\xi=0} \qquad (16)$$

$$a_3=\left[\frac{1}{6n}\frac{d^3}{d\xi^3}\left(\frac{1}{2\pi i}\int_{\gamma}\frac{f_1+if_2}{\sigma-\xi}d\sigma\right)\right]_{\xi=0}-\frac{1}{6n}\left[\frac{d^3}{d\xi^3}\sum_{k=1}^{n}k_m\xi^m\right]_{\xi=0} \qquad (17)$$

General formulas given in this section enable us to evaluate $\varphi(\xi)$ and $\psi(\xi)$. The bending moment and shearing force may be evaluated from equation (7). It is easy to obtain the results of[9] if $\omega(\xi) = R\xi$.
We shall study a special case by using the following conformal mapping to transform the elliptic plate into a unit circle :

$$z = \omega(\xi) = \gamma_1 \xi + \gamma_3\xi^3 \ (\gamma_1,\gamma_3 \text{ are real numbers}) \qquad (18)$$

with $\gamma_1 = 1.0826b$, $\gamma_3 = 0.0959b$
Substituting equation (18) into equations (12, 13,14, 15, 16 and 17) we obtain:

$$\phi(\xi)=\frac{1}{n}\left[\frac{1}{2\pi i}\int_{\gamma}\frac{f_1+if_2}{\sigma-\xi}d\sigma-(K_3\xi^3+K_1\xi)\right] \qquad (19)$$

$$\psi(\xi)=\frac{1}{2\pi i}\int_{\gamma}\left(\frac{f_1-if_2}{\sigma-\xi}\right)d\sigma-\frac{\bar{\omega}(\frac{1}{\xi})\phi(\xi)}{\dot{\omega}(\xi)}+\frac{K_{-1}}{\xi}+\frac{K_{-3}}{\xi^3} \qquad (20)$$

Since the coefficients of ω (ξ) are real numbers, then $K_1 = K_{-1}$, $K_3 = K_{-3}$ and the even coefficients equal zero because ω (ξ) contains odd coefficients only.

APPLICATIONS

i. Plate subjected to concentrated edge couples

In the absence of shearing forces Q_ν, equation (5) may be written in the form:

$$f_1+if_2=\frac{1}{2D(1-\nu)}\int_0^s[M_\nu+iH_{\nu s}](dx + idy) \qquad (21)$$

In Figure (2), a part of the boundary of the plate between points A and B is shown , of which ds is an element. All the moment vectors are shown in their positive directions as determined by the right-hand screw rule. The moments $M_x(\nu)$ and $H_y(\upsilon)$ represent the x and y-components of the resultant moment per unit length on ds, whose exterior normal is in the ν-direction. The relations between M_ν, $H_{\upsilon s}$ and $M_x(\upsilon)$, $M_y(\upsilon)$ may be readily established from the figure as

$$M_\nu \cos(\nu,y) + H_{\nu s}\cos(\nu, x) = -M_x(\nu)$$
$$M_\nu \cos(\nu,x) - H_{\nu s}\cos(\nu, y) = M_y(\nu)$$

Multiplying the first equation by ds and the second by ids, adding and remembering that $\cos(\nu,y)\, ds = -dx$, $\cos(\nu,x)\, ds = dy$
we obtain:

Substituting this into the integral in equation (21) and applying the resultant to the finite length of arc between A and B , we write

(22)

in which indicates the increase in the value of $f_1 + if_2$ as the boundary is described from A to B.

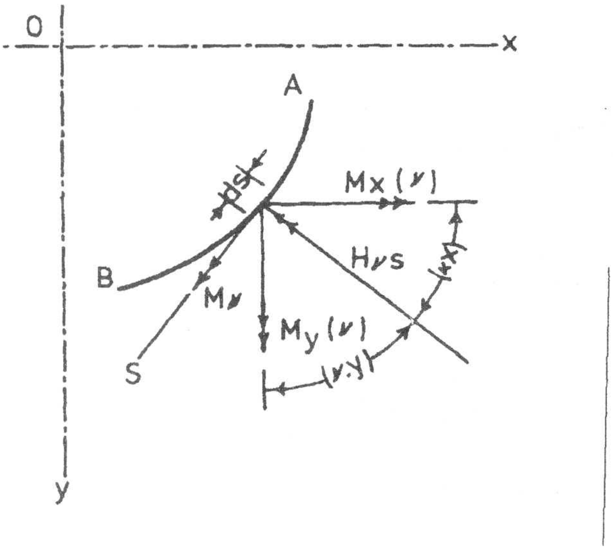

Figure 2. Couples at boundary of plate.

Consider now the case of concentrated couple M^* (the star indicating that its dimension is not that of moment per unit length but simply that of moment) acting at a point P on the boundary and having components M_x^* and M_y^*. We enclose P with a small but finite circular are A-C-B as shown in Figure (3). Applying equation (22) and letting A and B approach P, we get

$$[f_1 + if_2]_p = \frac{1}{2D(1-\nu)}[M_x^* + iM_y^*] = \frac{M^*}{2D(1-\nu)}$$

Observing the physical significance of the integral in equation (22), we see that the limiting procedure adopted is justified.We conclude, therefore, that on crossing the point P along the boundary in the positive direction, the function $f_1 + if_2$ gains a value equal to the concentrated couples at P divided by 2D (1 - ν). With the value of $f_1 + if_2$ assumed to be zero at an arbitrary starting point on a closed boundary, its value at any other point will be equal to the sum of the concentrated couples acting between these points divided by 2D(1-ν). It should be noted, however, that in going from the first point to the second, the positive direction of s is to be followed.

As an example we consider the problem of a plate under conformal mapping as being a circular plate subjected to an arbitrary number,r say of concentrated couples M_1^*, M_2^*, ... M^*_r, on its boundary at the points where $\sigma_1 = e^{i\alpha 1}$, $\sigma_2 = e^{i\alpha 2}$,..., on the unit circle ($0 \leq \alpha_1 \leq \alpha_2 < .... < \alpha_r < 2\pi$), respectively. A similar problem in the two-dimensional elasticity was discussed by Muschelishvili [2]. It is evident that the couples must fulfill the equilibrium condition $\sum_{K=1}^{r} M_k^* = \sum_{K=1}^{r} (M_{kx}^* + iM_{ky}^*) = 0$

According to this and the above conclusion we obtain Table (I) for the values of the function $f_1 + if_2$, on various arcs around the circumference of the plate, assuming its value to be zero on arc $\sigma_r\ \sigma_1$.

Table I. Values of Function ($f_1 + if_2$)

Arc	$2D(1-\nu)(f_1 + if_2)$
$\sigma_r\ \sigma_1$	0
$\sigma_1\ \sigma_2$	M_1^*
$\sigma_2\ \sigma_3$	$M_1^* + M_2^*$
...	...
...	...
$\sigma_{r-1}\ \sigma_r$	$M_1^* + M_2^* ... M^*_{r-1}$
$\sigma_r\ \sigma_1$	$M_1^* + M_2^* ... M_r^* = 0$

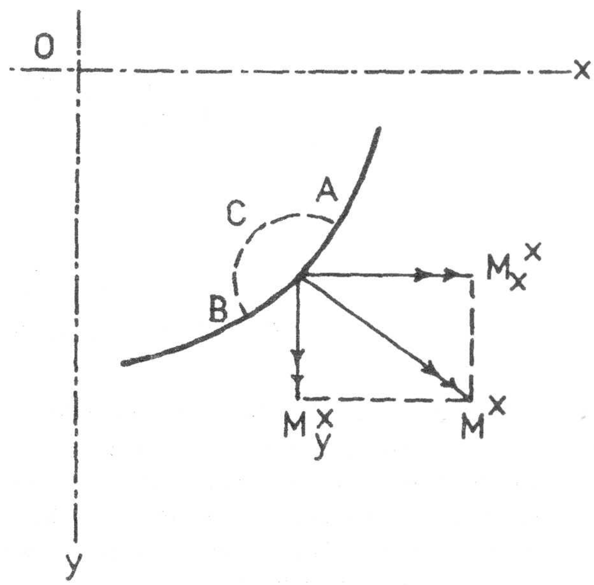

Figure 3. Concentrated couple at the boundary of the plate.

We have the following integrals

$$\frac{1}{2\pi i}\int_{\gamma}\left(\frac{f_1+if_2}{\sigma-\xi}\right)d\sigma=\frac{1}{2\pi i}\int_{\sigma_1}^{\sigma_2}\left(\frac{f_1+if_2}{\sigma-\xi}\right)d\sigma+\frac{1}{2\pi i}\int_{\sigma_2}^{\sigma_3}\left(\frac{f_1+if_2}{(\sigma-\xi)}d\sigma+\ldots+\right.$$

$$\ldots+\frac{1}{2\pi i}\int_{\sigma_r}^{\sigma_1}\left(\frac{f_1+if_2}{\sigma-\xi}\right)d\sigma=\frac{1}{2\pi i}\,\frac{1}{2D(1-\nu)}\left[M_1^{*}\log\frac{\sigma_2-\xi}{\sigma_1-\xi}\right.$$

$$\left.+(M_1^{*}+M_2^{*})\log\left(\frac{\sigma_3-\xi}{\sigma_2-\xi}\right)+\ldots+(M_1^{*}+M_2^{*}+\ldots+M_r^{*})\log\frac{\sigma_1-\xi}{\sigma_r-\xi}\right]$$

$$=\frac{i}{4\pi D(1-\nu)}\sum_{k=1}^{r}M_K^{*}\log(\sigma_k-\xi)$$

$$\frac{1}{2\pi i}\int_{\gamma}\frac{f_1-if_2}{(\sigma-\xi)}d\sigma=\frac{i}{4\pi D(1-\nu)}\sum_{k=1}^{r}\bar{M}_K^{*}\log(\sigma_k-\xi)\quad,$$

where $\bar{M}_k^{*}=M_{kx}^{*}-iM_{ky}^{*}$

Substituting these results into equations (19,20) we obtain :

$$\varphi(\xi)=\frac{1}{n}\left[\frac{i}{4\pi D(1-\nu)}\sum_{k=1}^{r}M_k^{*}\log(\sigma_k-\xi)-(k_3\xi^3+k_1\xi)\right] \tag{23}$$

$$\psi(\xi)=\frac{i}{4\pi D(1-\nu)}\sum_{k=1}^{r}\bar{M}_k^{*}\log(\sigma_k-\xi)+\frac{3C_1(k_3-C_3k_1)\xi}{n(1+3C_3\xi^2)}$$

$$+\frac{i}{4\pi D(1-\nu)n}\left[\sum_{k=1}^{r}\frac{M_k^{*}C_1}{\sigma_k(\sigma_k-\xi)}+\frac{M_k^{*}C_3}{\sigma_k^{3}(\sigma_k-\xi)}-\frac{3C_1C_3M_k^{*}\xi}{(1+3C_3\xi^2)(\sigma_k-\xi)}\right] \tag{24}$$

where the values of K_1 ,K_3 are calculated from equation (14) using equations (15,16, and 17)

It is easy to evaluate the deflection, the moment, and the shearing forces under some conditions : (i)the plate is sujected to two bending couples and (ii)the plate is sujected to two twisting couples.

i. Plate subjected to two Bending couples

In this problem the plate is subjected to two equal and opposite concentrated bending couples each of magnitude M and acting at the ends of a diameter, as shown in Figure (4). Taking

$$M_1^{*}=iM \text{ at } \sigma_1=1 \text{ and } M_2^{*}=-iM \text{ at } \sigma_2=-1$$

and substituting these into equations (23), and (24), we obtain the following analytic functions

$$\varphi(\xi) = \frac{-M}{4\pi D(1-\nu)n}\log\left(\frac{1-\xi}{1+\xi}\right) - \frac{1}{n}(k_3\xi^3 + k_1\xi)$$

$$\psi(\xi) = \frac{M}{4\pi D(1-\nu)}\log\left(\frac{1-\xi}{1+\xi}\right) - \frac{FA_1\xi}{n(1-\xi^2)} + \frac{3C_3A_2F\xi}{n(1+3C_3\xi^2)}$$

$$+\frac{3A_3\xi}{n(1+3C_3\xi^2)}$$

$$= \frac{M}{4\pi D(1-\nu)}\log\left(\frac{1-\xi}{1+\xi}\right) - \frac{FA_1\xi}{n(1-\xi^2)} + \frac{3L_4\xi}{n(1+3C_3\xi^2)} ,$$

where

$$\chi(\xi) = \int \dot{\omega}(\xi)\psi(\xi)d\xi = \gamma_1\chi_1 + \gamma_3\chi_3$$

$$\chi_1(\xi) = \int \psi(\xi)d\xi \quad , \quad \chi_3(\xi) = 3\int \xi^2\psi(\xi)d\xi$$

$$\chi_1(\xi) = \frac{F\xi}{2}\log\left(\frac{1-\xi}{1+\xi}\right) - \frac{FA_4}{2}\log(1-\xi^2) + \frac{L_4}{2nC_3}\log(1+3C_3\xi^2)$$

$$\chi_3(\xi) = \frac{F\xi^3}{2}\log\frac{(1-\xi)}{(1+\xi)} - \frac{FL_4}{2}\log(1-\xi^2) - L_3\log(1-3C_3\xi^2) + L_2\xi^2$$

Using equations (28, 29) into equation (27) to obtain:

$$\chi(\xi) = -\frac{F}{2}(\gamma_1\xi + \gamma_3\xi^3)\log\frac{(1+\xi)}{(1+\xi)} - \frac{F}{2}(\gamma_1 + \gamma_3)(1-\frac{1}{n})\log(1-\xi^2) + \gamma_3L_2\xi^2$$

where

$$a_1 = \frac{(n - C_3)\,F}{(n^2 + nC_1 - 3C_3^2)} \quad \& \quad F = \frac{M}{2\pi D(1-\nu)}$$

$$a_3 = \frac{(n + C_1 - 3C_3)F}{3(n^2 + nC_1 - 3C_3^2)} = \frac{(n + C_1 - 3C_3)a_1}{3(n - C_3)}$$

$$A_1 = \frac{(1+C_3)}{(1+3C_3)} \; \& \; A_2 = \frac{3C_1C_3}{1+3C_3} \quad C_1 = \frac{\gamma_3}{\gamma_1} \; C_3 = \frac{\gamma_1^2 - 3\gamma_3^2}{\gamma_1^2}$$

$$A_3 = C_1(K_3 - C_3K_1) \quad \& \quad A_4 = (1 - \frac{A_1}{n})$$

$$K_1 = a_1C_1 + 3a_3C_3 \; \& \; K_3 = a_1C_3 \text{ and } K_2 = 0$$

$$L_1 = 3(C_3A_2F + A_3) \quad , \quad C_1 = \frac{\gamma_3}{\gamma_1}$$

$$L_2 = \frac{1}{2}\left(\frac{3FA_1}{n} + \frac{3FA_1}{n} + \frac{3A_3}{nC_3} - F\right)$$

$$L_3 = \frac{1}{2C_3n}\left(A_2F + \frac{A_3}{C_3}\right)$$

$$L_4 = 1 - \frac{3A_1}{n} \quad , \quad C_3 = \frac{\gamma_1^2 - 3\gamma_3^2}{\gamma_1^2}$$

and

$$\Phi(\xi) = \frac{1}{\dot{\omega}(\xi)}\left[\frac{F}{n(1-\xi^2)} - \frac{1}{n}(3K_3\xi^2 + K_1)\right] \tag{31}$$

$$\phi'(\xi) = \frac{2\xi}{\dot{\omega}(\xi)}\left[\frac{F}{n(1-\xi^2)^2} - \frac{3\gamma_3F}{n(1-\xi^2)\dot{\omega}(\xi)} + \frac{3(\gamma_3K_1 - \gamma_1K_3)}{n\dot{\omega}(\xi)}\right] \tag{32}$$

$$\psi'(\xi) = -\frac{F}{(1-\xi^2)} - \frac{FA_1(1+\xi^2)}{n(1-\xi^2)^2} + \frac{3L_4\gamma_1(\gamma_1 - 3\gamma_3\xi^2)}{n(\dot{\omega}(\xi))^2} \tag{33}$$

From equation (7) we get:

$$M_\theta = \frac{M}{\pi}[(n+1)\mathrm{Re}I_1 + \mathrm{Re}(I_2 + I_3)]$$
$$M_\rho = \frac{M}{\pi}[(n+1)\mathrm{Re}I_1 - \mathrm{Re}(I_2 + I_3)] \tag{34}$$
$$H_{\rho\theta} = \frac{M}{\pi}\mathrm{Im}(I_2 + I_3)$$

$$I_1 = \frac{\Phi(\xi)}{F}$$
$$I_2 = \frac{\xi^2}{\rho^2\overline{\dot{\omega}}(\overline{\xi})}\left[\frac{\overline{\omega}(\overline{\xi})}{F}\dot{\Phi}(\xi)\right] \tag{35}$$
$$I_3 = \frac{\xi^2}{\rho^2\overline{\dot{\omega}}(\overline{\xi})}\left[\frac{\dot{\omega}(\xi)}{F}\overline{\psi}(\xi)\right]$$

We calculate the deflections , moments, and shearing forces for the elliptic plate under conformal mapping into a circular plate using equations (6,25,26,30,....,35). As a special case, the disk may be studied taking $\gamma_3=0$, the results in [9] are recovered. Calculations are made for points along the radii $\theta = 0$ and $\theta = \pi/2$ as well as along the circumference $\rho = 1$ between the angles $\theta = 0$ and $\theta = \pi/2$. Along the radius $\theta = 0$, we have $\xi = \rho$. Substituting the calculated values of the functions $\varphi(\xi)$ and $\chi(\xi)$ from (25, 26) into equation (6) we get:

$$(w)_{\theta=0} = \frac{M\gamma_1}{2\pi D(1-\nu)}[(\frac{1}{n}-1)\{(1-\rho)+C_3(1-\rho^3)\}\log(1-\rho)$$

$$+(\frac{1}{n}-1)\{(\rho+1)+C_3(1+\rho^3)\}\log(1+\rho)+b_2\rho^2+b_4\rho^4+b_6\rho^6]$$

$$(w)_{\substack{\theta=0\\ \rho=0}} = 0 \tag{36}$$

$$(w)_{\substack{\theta=0\\ \rho=1}} = \frac{M\gamma_1}{\pi D(1-\nu)}[(\frac{1}{n} - 1)(1 + C_3)\log 2 + \frac{1}{2}(b_2 + b_4 + b_6)]$$

where

$$b_2 = \frac{-2K_1}{nF} + \frac{2C_3L_2}{F}$$
$$b_4 = \frac{-2}{nF}(K_3 + K_1C_3) \tag{37}$$
$$b_6 = -\frac{2}{nF}(K_3C_3)$$

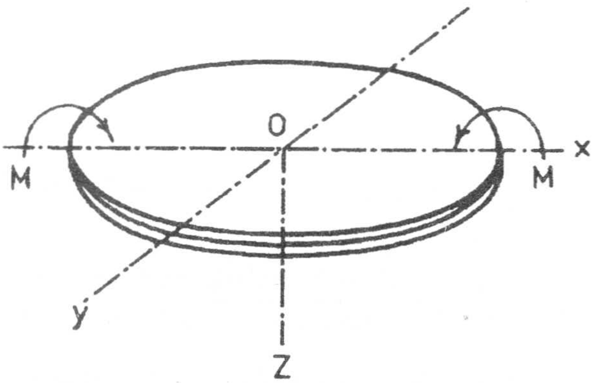

Figure 4. Plate subjected to two blnding couples.

The difference between the two values represents the maximum deflection of the plate. Substituting $\Phi(\xi)$, $\Phi(\xi)$ and $\Psi(\xi)$ from equations (31,32,33) into equation (35)

where $\omega'(\rho) = (\gamma_1 + 3\gamma_3\rho^2)$ we obtain:

$$Re(I_1)_{\theta=0} = \frac{1}{\dot{\omega}(\rho)}\left[\frac{1}{n(1-\rho^2)} - \frac{1}{nF}(3K_3\rho^2+K_1)\right] \tag{38}$$

$$Re(I_2)_{\theta=0} = \frac{2\rho\omega(\rho)}{(\dot{\omega}(\rho))^2}[\frac{1}{n(1-\rho^2)^2} - \frac{3\gamma_3}{n(1-\rho^2)\dot{\omega}(\rho)} - \frac{3(\gamma_1K_3 - \gamma_3K_1)}{nF\omega'(\rho)}] \tag{39}$$

$$Re(I_3)_{\theta=0} = \frac{1}{\dot{\omega}(\rho)}[\frac{-1}{1-\rho^2} - \frac{A_1(1+\rho^2)}{n(1-\rho^2)^2} + \frac{3\gamma_1L_4(\gamma_1 - 3\gamma_3\rho^2)}{nF[\dot{\omega}(\rho)]^2}] \tag{40}$$

Substituting equations (38,39 and 40) into equation (34) to evalute (M_θ),(M_ρ) and $H_{\rho\theta}$, the values of Q_θ and Q_ρ obtain from the third part of equation (7) and by equation (33) we get:

$$(Q_\rho)_{\theta=0} = \frac{-8M\rho}{\pi(1-\nu)(\dot{\omega}(\rho))^2}[\frac{1}{n(1-\rho^2)^2} - \frac{3\gamma_3}{n(1-\rho_2)\dot{\omega}(\rho)} - \frac{3(\gamma_1K_3 - \gamma_3K_1)}{n\dot{\omega}(\rho)F}] \tag{41}$$

$$(H_{\rho\theta})_{\theta=0} = (Q_\theta)_{\theta=0} = 0$$

Similarly, along the radius $\theta = \pi/2$, we have $\xi = i\rho$, and $\omega'(i\rho) = \gamma_1 - 3\gamma_3\rho^2 = \omega'(-i\rho)$

$$(w)_{\theta=\frac{\pi}{2}} = \frac{M\gamma_1}{2\pi D(1-\nu)}[2(\frac{1}{n}+1)(\rho - C_3\rho^3)\tan^{-1}\rho$$

$$+(\frac{1}{n}-1)(1+C_3)\log(1+\rho^2) + b_{22}\rho^2 - b_4\rho^4 - b_6\rho^6]$$

$$b_{22} = \frac{-2}{F}(\frac{K_1}{n} + L_2C_3) \tag{42}$$

To obtain the values of $(M_\theta)_{\theta=\pi/2}$,$(M_\rho)_{\theta=\pi/2}$ and $(H_{\rho\theta})\theta=\pi/2$ using equation (35) to calculate

$$Re(I_1)_{\frac{\pi}{2}} = \frac{1}{\dot{\omega}(i\rho)}[\frac{1}{n(1+\rho^2)} - \frac{1}{nF}(-3K_3\rho^2 + K_1)] \tag{43}$$

$$Re(I_2)_{\frac{\pi}{2}} = \frac{-2\rho^2(\gamma_1 - \gamma_3\rho^2)}{\dot{\omega}(i\rho)^2}[\frac{1}{n(1+\rho^2)^2} - \frac{3\gamma_3}{n(1+\rho^2)\dot{\omega}(i\rho)}$$

$$- \frac{3(\gamma_1 K_3 - \gamma_3 K_1)}{n\,\dot{\omega}(i\rho)F}] \tag{44}$$

$$Re(I_3)_{\theta=\frac{\pi}{2}} = \frac{1}{\omega'(-i\rho)}\left[\frac{1}{(1+\rho^2)} + \frac{A_1(1-\rho^2)}{n(1+\rho^2)^2} - \frac{3L_4(\gamma_1 + 3\gamma_3\rho^2)\gamma_1}{nF(\dot{\omega}(i\rho))^2}\right] \tag{45}$$

and substituting their values taking into account that $(M_\theta)_{\theta=\pi/2}$, $(M\rho)_{\theta=\pi/2}$ and $(H_{\rho\theta})_{\theta=\pi/2}$ in equation (34), as in [9], if $\gamma_3 = 0$, and from the third part of equation (7), and equation (33), we obtain:

$$(Q_\rho)_{\theta=\frac{\pi}{2}} = \frac{8M\rho}{\pi(1-\nu)(\dot{\omega}(i\rho))^2}[\frac{1}{n(1+\rho^2)^2} - \frac{3\gamma_3}{n(1+\rho^2)\dot{\omega}(i\rho)}$$

$$+ \frac{3(\gamma_3 K_1 - \gamma_1 K_3)}{nF\,\dot{\omega}(i\rho)}] \tag{46}$$

$$(Q_\theta)_{\theta=\frac{\pi}{2}} = 0 \;,\quad (H_{\rho\theta})_{\theta=\frac{\pi}{2}} = 0 \tag{47}$$

Finally, along the circumference $\rho=1$, we have $\xi=\sigma$ and $|\,\omega'(\sigma)\,|^2 = \left|\gamma_1^2 + 9\gamma_3^2 + 6\gamma_1\gamma_3\cos2\theta\right|$

$$(w)_{\rho=1} = \frac{M\gamma_1}{2\pi D(1-\nu)}[(\frac{1}{n}-1)\cos^2\frac{\theta}{2}\{1 + C_3(4\cos^2\frac{\theta}{2} - 3)^2\log\cos^2\frac{\theta}{2}$$

$$+(\frac{1}{n} - 1)\sin^2\frac{\theta}{2}\{1 + C_3(3 - \sin^2\frac{\theta}{2})^2\}\log\sin^2\frac{\theta}{2}$$

$$+(\frac{1}{n} + 1)\frac{\pi}{2}\sin\theta\;[1 + C_3(4\cos^2\theta - 1)] \tag{48}$$

$$+2(\frac{1}{n} - 1)(1 + C_3)\log 2 + b_o + b_{222}\cos2\theta]$$

where

$$b_0 = \frac{-2(K_1 + C_3K_3)}{nF}$$

$$b_{222} = \frac{-2(K_3 + C_3K_1)}{nF} + \frac{2L_2C_3}{F}$$

Similarly, for $\rho=1$ we calculate $(M_\rho)_{\rho=1}$, $(M_\theta)_{\rho=1}$ and $(H_{\rho\theta})_{\rho=1}$ from equation (35) to get:

$$Re(I_1)_{\rho=1} = \frac{\gamma_1}{|\dot{\omega}|^2}[\frac{1 + 3C_3(4\cos^2\theta - 1)}{2n}$$

$$-\frac{1}{nF}[K_1 + 9C_3K_3 + 3(K_3 + C_3K_1)\cos2\theta]] \tag{49}$$

$$Re(I_2)_{\rho=1} = \frac{2\gamma_1}{|\omega'(\sigma)|^2}[\frac{1+C_3\cos2\theta}{-4n\sin^2\theta} - \frac{3C_3\left\{3\gamma_3^2(4\cos^2\theta-1) - \gamma_1^2 + 4\gamma_1\gamma_3\right\}}{2n|\dot{\omega}(\sigma)|^2}$$

$$+\frac{3(C_3K_1 - K_3)\left\{\gamma_1^2 + 4\gamma_1\gamma_3 + 3\gamma_3^2 + (\gamma_1^2 + 3\gamma_3^2)\cos(2\theta)\right\}}{nF|(\dot{\omega}(\sigma)|^2}] \tag{50}$$

$$Im(I_2)_{\rho=1} = \frac{2\gamma_1}{|\dot{\omega}(\sigma)|^2}[\frac{C_3\cos\theta}{2n\sin\theta} - \frac{3C_3[(\gamma_1^2 + 4\gamma_1\gamma_3 + 3\gamma_3^2(1 - 4\sin^2\theta)]\cos\theta}{nF\,|\,\dot{\omega}(\sigma)|^2}] \tag{51}$$

$$Re(I_3)_{\rho=1} = \frac{\gamma_1}{|\,\dot{\omega}(\sigma)\,|^2}[\frac{1 + 3C_3(4\cos^2\theta - 1)}{2} - \frac{A_1\cos^2\theta[1 + 3C_3(1 - 4\sin^2\theta]}{2n\sin^2\theta}$$

$$+\frac{3L_4[(\gamma_1^2 - 9\gamma^2)\cos2\theta + 6\gamma_1\gamma_3\sin^2\theta]}{n\,|\,\dot{\omega}(\sigma)\,|^2\,F}] \tag{52}$$

$$I_m(I_3)_{\rho=1} = \frac{\gamma_1}{|\dot{\omega}(\sigma)|^2}\Big[\frac{-(1+3C_3(1-4\sin^2\theta))\cos\theta}{2\sin\theta} - \frac{A_1\cos[1+3C_3(4\cos^2\theta-1)]}{2n\sin\theta} + \frac{3L_4\sin2\theta(\gamma_1^2-9\gamma_3^2-6\gamma_1\gamma_3)}{nF|\dot{\omega}(\sigma)|^2}\Big] \quad (53)$$

Using equations (49,...., 53) into equation (35) we obtain the values of the moment, twisting moments and shearing force at $\rho = 1$ from the third part of equation (7) and equation (33), it is easy to get:

$$(Q_\rho)_{\rho=1} = -\frac{8M\gamma_1}{|\dot{\omega}|^3(1-\nu)}\Big[\frac{-(1+3C_3\cos2\theta)}{4n\sin^2\theta} - \frac{3C_3\{(6\gamma_1\gamma_3-\gamma_1^2)+9\gamma_3^2(4\cos^2\theta-1)\}}{2n|\dot{\omega}|^2} \quad (54)$$

$$-\frac{3(K_3-C_3K_1)\{6\gamma_1\gamma_3+(\gamma_1^2+9\gamma_3^2)\cos2\theta\}}{nF|\dot{\omega}|^2}\Big]$$

$$(Q_\theta)_{\rho=1} = \frac{8M\gamma_1}{|\dot{\omega}(\sigma)|^3}\Big[\frac{3C_3\cos\theta}{2n\sin\theta} - \frac{3C_3\cos\theta[(\gamma_1^2+6\gamma_1\gamma_3)+9\gamma_3^2(4\cos^2\theta-3)]}{2n\sin\theta|\dot{\omega}|^2} - \frac{3(K_3-C_3K_1)(\gamma_1^2-9\gamma_3^2)\sin2\theta}{n|\dot{\omega}(\sigma)|^2F}\Big] \quad (55)$$

When poisson's ratio $\nu = .3$ the corresponding value of n is - 4.714. Some more interesting values for $(w)_{\theta=0}$, $(w)_{\theta=\frac{\pi}{2}}$, $(M_\rho)_{\theta=\frac{\pi}{2}}$, $(M_\theta)_{\theta=\frac{\pi}{2}}$, *and* $(Q_\rho)_{\theta=\frac{\pi}{2}}$ have been computed numerically, the results are plotted in Figures (5,6,7,8, and 9) which include the results of the special case of Yuaw [9].

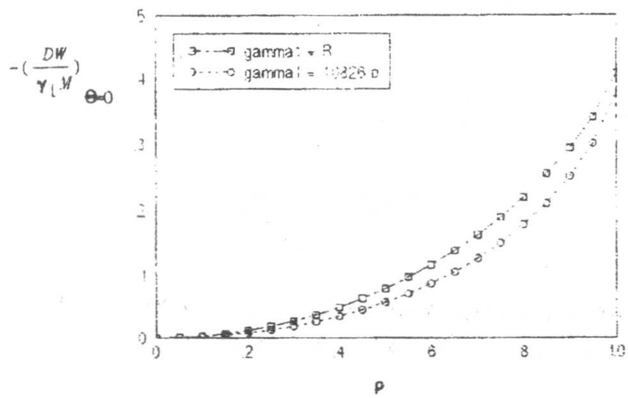

Figure 5. Deflection versus radius (for the case of bending couples).

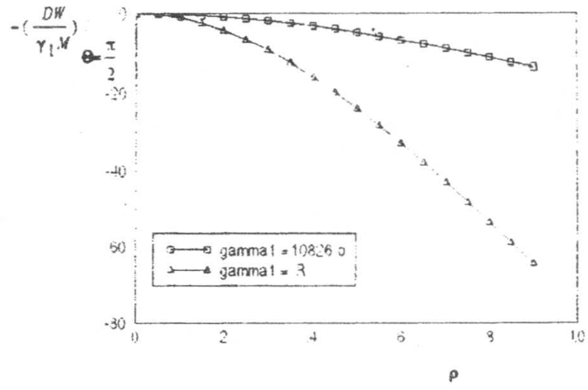

Figure 6. Deflection versus radius (for the case of bending couples).

ii- Plate subjected to two twisting couples

This second problem can be worked out in same manner as the first, we have:

$$M_1^* = M \text{ at } \sigma_1 = 1 \;\&\; M_2^* = -M \text{ at } \sigma_2 = -1$$

as shown in Figure (10) M being the magnitude of each of the two twisting couples applied at the ends of a diameter of the plate. Substituting into equations (19,20) we get :

$$\varphi(\xi) = \frac{-iM}{4\pi D(1-\nu)n}\log\Big(\frac{1-\xi}{1+\xi}\Big) - \frac{i}{n}(K_3\xi^3 + K_1\xi) \quad (56)$$

$$\psi(\xi) = \frac{iM}{4\pi D(1-\nu)}\log\Big(\frac{1-\xi}{1+\xi}\Big) - \frac{iFA_1\xi}{n(1-\xi^2)} + \frac{3iL_4\xi}{n(1+3C_3\xi^2)} \quad (57)$$

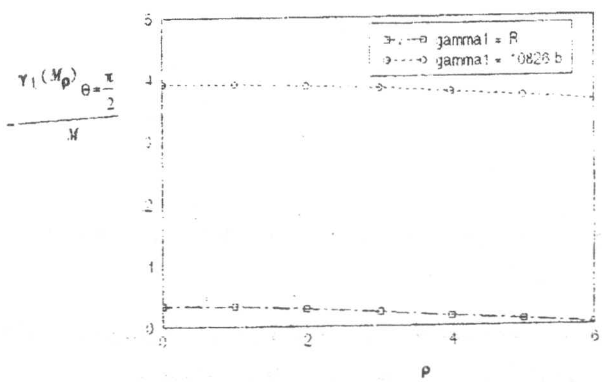

Figure 7. Bending moment versus radius (for the bending couples).

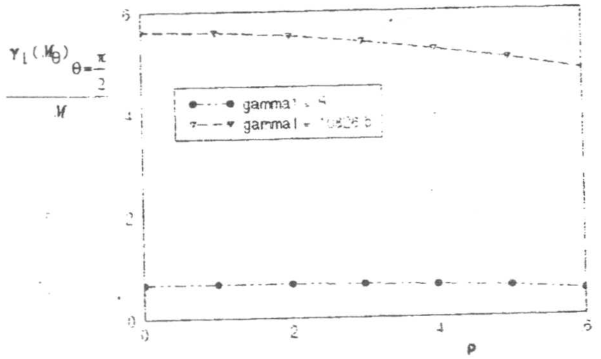

Figure 8. Bending moment versus radius (for the bending couples).

Along the radius $\theta=0 (w)_{\theta=0} = (M_{\theta=0}) = (M_\rho)_{\theta=0} = (Q)_{\theta=0} = 0$. As before, from equation (35), we get:

$$I_m(L_2)_{\theta=0} = \frac{-2\rho\omega(\rho)}{(\dot{\omega}(\rho))^2}\left[\frac{1}{n(1-\rho^2)^2} - \frac{3\gamma_3}{n(1-\rho^2)\dot{\omega}(\rho)} + \frac{3(\gamma_3 K_1 - \gamma_1 K_3)}{nF\dot{\omega}(\rho)}\right] \quad (58)$$

$$I_m(L_3)_{\theta=0} = \frac{1}{\dot{\omega}(\rho)}\left[\frac{-1}{1-\rho^2} + \frac{A_1(1+\rho^2)}{n(1-\rho^2)^2} + \frac{3\gamma_1 L_4(\gamma_1 - 3\gamma_3\rho^2)}{n(\dot{\omega}(\rho))^2 F}\right] \quad (59)$$

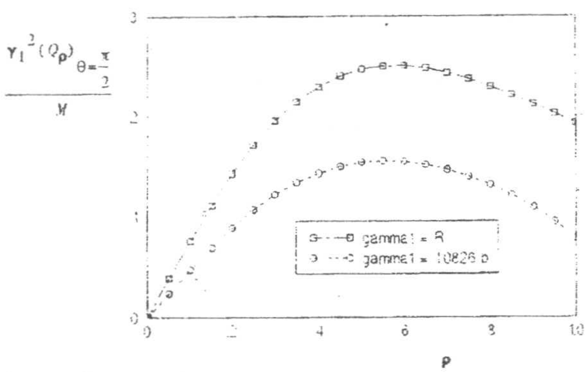

Figure 9. Shearing force versus radius (for the case of bending couples).

Using (58) & (59) into the third part of equation (34) to evalute $(H_{\rho\theta})_{\theta=0}$, we also obtain $(Q_\theta)_{\theta=0}$ as:

$$(Q_\theta)_{\theta=0} = -\frac{8M\rho}{\pi(1-\gamma)n(\dot{\omega}(\rho))^2}\left[\frac{1}{(1-\rho^2)^2} - \frac{3\gamma_3}{(1-\rho^2)\dot{\omega}(\rho)} + \frac{3(\gamma_1 K_3 - \gamma K_1)}{nF\dot{\omega}(\rho)}\right] \quad (60)$$

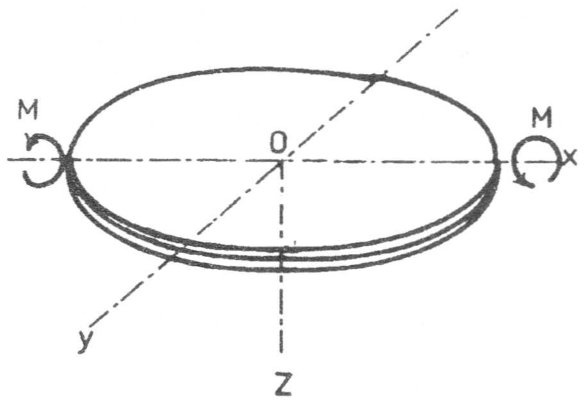

Figure 10. Plate subjected to two twisting couples.

Along the radius $\theta = \pi/2$:

$$(w)_{\theta=\frac{\pi}{2}} = (M_\theta)_{\theta=\frac{\pi}{2}} = (Q_\rho)_{\theta=\frac{\pi}{2}} = 0$$

We obtain the same values when $\gamma_3 = 0$, using equation (35), we get:

$$I_m(L_2)_{\theta=\frac{\pi}{2}} = -\frac{2\rho^2(\gamma_1 - \gamma_3\rho^2)}{(\dot{\omega}(i\rho))^2}\left[-\frac{1}{n(1+\rho^2)^2} + \frac{3\gamma_3}{n(1+\rho^2)\dot{\omega}(i\rho)} + \frac{3(\gamma_3 K_1 - \gamma_1 K_3)}{nF\dot{\omega}(i\rho)}\right] \quad (61)$$

$$I_m(L_3)_{\theta=\frac{\pi}{2}} = -\frac{1}{(\gamma_1 - 3\gamma_3\rho^2)}\left[-\frac{1}{1+\rho^2} + \frac{A_1(1+\rho^2)}{n(1+\rho^2)^2} + \frac{3\gamma_1 L_4(\gamma_1 + 3\gamma_3\rho^2)}{nF\dot{\omega}(i\rho)}\right] \quad (62)$$

Substituting equations (61)&(62) into equation (34) to evaluate $H_{\rho\theta}$ and Q_θ using the third part of equation (7)

$$(Q_\theta)_{\theta=\frac{\pi}{2}} = -\frac{8M\rho}{\pi(1-\nu)(\dot{\omega}(i\rho))^2}\Big[\frac{-1}{n(1+\rho^2)^2} + \frac{3\gamma_3}{n(1+\rho^2)\dot{\omega}(i\rho)} - \frac{3(\gamma_3 K_1-\gamma_1 K_3)}{nF\dot{\omega}(i\rho)}\Big] \qquad (63)$$

Along the circumference $\rho = 1$ between $\theta=0$ and $\theta=\pi/2$ we get:

$$(w)_{\rho=1} = \frac{M\gamma_1}{2\pi D(1-\nu)}\Big[(1+\frac{1}{n})\frac{\pi}{2}\cos\theta[1+C_3(1-4\cos^2\theta)] + \frac{2}{nF}(K_3-C_3K_1)\sin^2\theta - (1+C_3)(1+\frac{1}{n})(\frac{\pi}{2}-\theta) + (\frac{1}{n}-1)\sin\theta[1+C_3(4\cos^2\theta-1)]\log\tan\frac{\theta}{2}\Big] \qquad (64)$$

Substituting into equation (35), the values of $\Phi(\rho)$, $\Phi'(\rho)$ and $\Psi(\rho)$, we have:

$$Re(I_1)_{\rho=1} = \frac{\gamma_1}{|\omega'(\sigma)|^2}\Big[\frac{\cos\theta[1+3C_3(1-4\sin^2\theta)}{2n\sin\theta} + \frac{3}{nF}(K_3 - C_3K_1)\cos 2\theta\Big] \qquad (65)$$

$$Re(I_2)_{\rho=1} = \frac{1}{|\omega'(\sigma)|^2}\Big[\frac{C_3\cos\theta}{2n\sin\theta} - \frac{3C_3\cos\theta[4\gamma_1\gamma_3-\gamma_1^2+3\gamma_3^2(1-4\sin^2\theta)]}{2n\sin\theta\,|\dot{\omega}|^2} - \frac{3\{(C_3K_1 - K_3)(\gamma_1^2 - 3\gamma_3^2)\}\sin 2\theta}{nF\,|\dot{\omega}(\sigma)|^2}\Big] \qquad (66)$$

$$I_m(I_2)_{\rho=1} = \frac{1+C_3\cos^2\theta}{2n\sin^2\theta} + \frac{4\gamma_1\gamma_3+3\gamma_3^2(4\cos^2\theta-1)}{2n\,|\dot{\omega}|^2} - \frac{6(K_3-c_3K_1)[4\gamma_1\gamma_3+(\gamma_1^2+3\gamma_3^2)\cos^2\theta]}{nF\,|\dot{\omega}|^2} \qquad (67)$$

$$Re(I_3)_{\rho=1} = \Big[\frac{1+3C_3(1-4\sin^2)]\cos\theta}{2\sin\theta} + \frac{A_1\cos\theta[1+3C_3(4\cos^2\theta-1)]}{2n\sin\theta}\Big] \qquad (68)$$

$$I_m(I_3)_{\rho=1} = \frac{\gamma_1}{|\dot{\omega}(\sigma)|^2}\Big[\frac{1+3C_3(4\cos^2\theta-1)}{2} - \frac{A_1\cos^2\theta[1+3C_3(1-4\sin^2\theta)]}{2n\sin^2\theta} + \frac{3L_4\{\gamma_1^2-9\gamma_3^2-6\gamma_1\gamma_3\cos 2\theta\}\sin 2\theta}{nF\,|\dot{\omega}(\sigma)|^2}\Big] \qquad (69)$$

Using equations (65,......,69) into equation (34) to evaluate $(M\theta)_{\rho=1}$, $(M_\theta)_{\rho=1}$ and $(H\rho\theta)_{\rho=1}$, by using the values of $(\Phi'(\rho))_{\rho=1}$ into the third part of equation (7) we get|

$$(Q_\rho)_{\rho=1} = \frac{-8M\gamma_1}{\pi(1-\gamma)\,|\dot{\omega}(\sigma)|^3}\Big[\frac{3C_3\cos\theta}{2n\sin\theta} - \frac{3C_3\{\gamma_1^2+6\gamma_1\gamma_3+9\gamma_3^2(1-4\sin^2\theta)\}\cos\theta}{2n\sin\theta\,|\dot{\omega}(\sigma)|^2} + \frac{3(C_3K_1 - K_3)(9\gamma_3^2-\gamma_1^2)\sin 2\theta}{nF\,|\dot{\omega}(\sigma)|^2}\Big] \qquad (70)$$

$$(Q_\theta)_{\rho=1} = \frac{8M\gamma_1}{\pi(1-\gamma)\,|\dot{\omega}(\sigma)|^3}\Big[\frac{1+3C_3\cos 2\theta}{4n\sin^2\theta} - \frac{3C_3\{\gamma_1^2-6\gamma_1\gamma_3-9\gamma_3^2(4\cos^2\theta-1)\}}{2n\,|\dot{\omega}(\sigma)|^2} + \frac{3(C_3K_1-K_3)\{\gamma_1^2+6\gamma_1\gamma_3+9\gamma_3^2(1-4\sin^2\theta)\}\cos\theta}{nF\,|\dot{\omega}(\sigma)|^2}\Big] \qquad (71)$$

Numerical values of some of quantities in equation (62) and $(H_{\rho\theta})_{\rho=1}$, $(M_\theta)_{\rho=1}$ *and* $(Q_\theta)_{\rho=1}$ have been computed for the case in which $\nu = .3$.They are plotted in Figures (11,12,13 and 14). We also evaluate the case of YI. Yuan ($\gamma_3=0$).

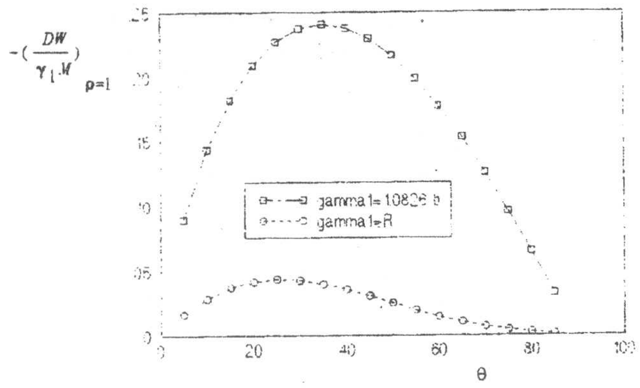

Figure 11. Deflection versus angle (for the case of two twisting couples).

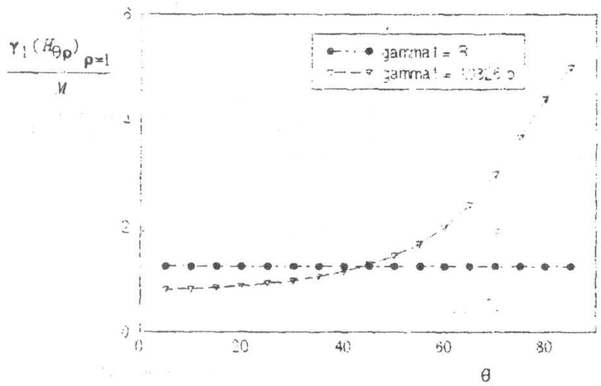

Figure 12. Twisting moment versus angle (for the case of two couples).

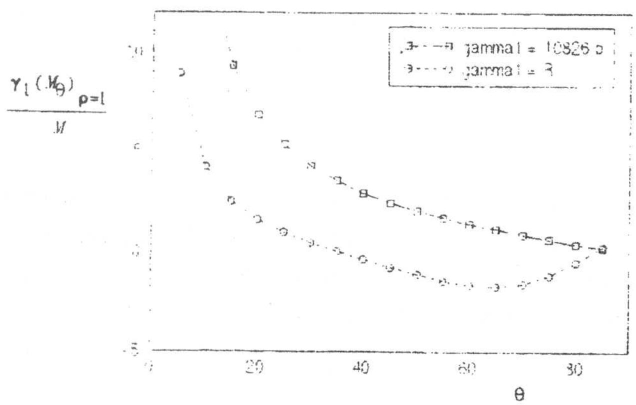

Figure 13. Bending moment versus angle (for the twisting couples).

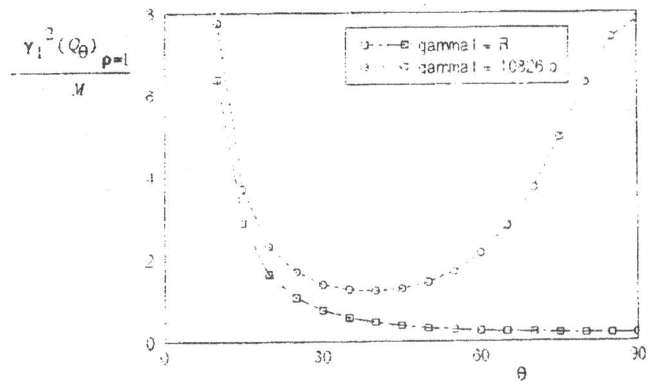

Figure 14. Shearing force angle (for the case of twisting couples).

CONCLUSION

Solving the integro-differential equation, by using a rational, we obtain two analytic functions in the general case. We stuied an ellptic plate as special case. It is easy to apply our results for any shape transformed this shape with conformal mapping to a unit cirle, which subjected to concentrated edge couples. ALso the appliation can be applied to the structure building and mannfactory.

REFERENCES

[1] A.I. Lurie, *Problems Concerning the Bending of a circular Plate*, Journal of Applied Mathematica and Mechanica, Vol. 4. 1940. pp. 93-102 (in Russian with English Summary)

[2] N.I. Muschelishvili, *Some Basic Problems of the Mathematical Theory of Elasticity*, 1971.

[3] S.G.Lechnitsky *Uber Einige Fragen der Biegungstheorie der Dūnnen Platen*, Journal of Applied Mathematics and Mechanics, Vol 2, 1938,pp, 181 - 210 (in Russian with German Summary).

[4] I.S. Sokolnikoff *Mathematical Theory of Elasticity*, Mc GRAW-Hill Boook Company LTD. Bombay-New Delhi, 1956.

[5] V. Morkovin *On the deflection of Anisotropic thin Plates*, Quarterly of Applied Mathematics, Vol. 1, 1943, pp. 116-129

[6] M.M Friedman *Über Einige Fragen der Theorie der Bieguny von Dūnnen Isotropen platten*, Journal of Applied Mathematics and Mechanics Vol. 5. 1941, pp. 93-102 (in Russian with German summary).

[7] A.C. Stevenson *On the Equilibrium of Plates*, Philosophical Magazine, Vol. 33, 1942, pp. 639-661 .

[8] B.R. Seth *Bending of Clamped Rectilinear plates*, Philosphical Magazine, Vol. 38, 1947, pp. 292-297.

[9] YI. YUAN YU *Bending of Isotropic Thin Plates by Concentrated Edge Couples and Forces* Appl. Mech. Vol. 21, 1954. pp. 129 - 139.

[10] S.Timoshenko *Theory of plates and shells*, McGraw-Hill Book Company. Inc, New York, N.Y. 1953.

[11] L.V. Kantorowitsch and V.I.Krylow *Approximate method of higher analysis*, Noordhoff Groningen, 1958.