

# APPLICABILITY OF CRACK LAYER THEORY IN FATIGUE CRACK PROPAGATION OF DIFFERENT MATERIALS

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## ABSTRACT

The applicability of the crack layer theory is discussed. Published data of fatigue crack propagation of polycarbonate, (polyethylene terephthalate), and epoxy resin figure out the good agreement with the theory and the constancy of the specific enthalpy of damage ( $\gamma^*$ ) which considered as a material parameter according to the theory. The Beta-Ti alloy shows good agreement with the theory, with two values of  $\gamma$  each one represent the dominate failure mechanism for a period of crack propagation of the specimen. The application of the theory on crack propagation data of two types of steel, with the necessary approximations, indicates reasonable primarily agreement with the theory. The study points out the need to design several experiments to specify the limitations and the applicability of the theory.

## INTRODUCTION

A critical value of the stress intensity factor  $K_{1C}$  [1] or the energy release rate  $J_1$  [2] is commonly accepted as a measure of fracture toughness of engineering materials.  $K_{1C}$  applies well to brittle materials whose behavior falls within the domain of linear elastic fracture mechanics. Similarly, when the material's behavior falls within the domain of nonlinear fracture mechanics,  $J_{1C}$  applies. This approach has proven useful in materials development and structural design. However, it suffers certain limitations that ought to be addressed in order to further our understanding of the phenomena involved in the resistance of materials to crack propagation.

Most of limitations are due to the lack of a fundamental relationship to link  $K_{1C}$  or  $J_{1C}$  to microstructural process (crack tip damage) resisting crack advance. Recently, the crack layer theory [3] has been advanced to overcome such limitations. The theory formulates fracture toughness as a product of the specific enthalpy of damage as a material parameter and the crack resistance moment as a loading history dependent parameter expressing the amount of crack tip damage required for crack advance. The theory was applied to the analysis of fatigue fracture in epoxy resin [4], (polyethylene terephthalate), [5], polycarbonate [6, 7, 8] as well as the crack

propagation under fixed elongation [9]. The behavior of PMMA under repeated impact loading and sinusoidal fatigue loading [10] which could not be explained by the classical theories of fracture mechanics, found understandable in using crack layer theory.

In this study we present a brief review of the theory together with applications on polymers and metals to find its applicability and limitations.

## THE CRACK LAYER THEORY

The theory considers the system comprising the crack and the accompanying damage (Figure (1)) as a single entity, i.e., a crack layer (CL). The domain adjacent to the crack tip within which the rate of damage growth is positive is defined as the "active zone". The theory describes CL propagation as active zone "movement". This movement is decomposed into translation and deformation of the active zone. Obviously, active zone translation coincides with crack extension. Fracture occurs as alternate sequences of slow and fast processes. The slow process involves damage growth within the active zone to satisfy conditions of local instability at which fast active zone movement (crack excursion) takes place.

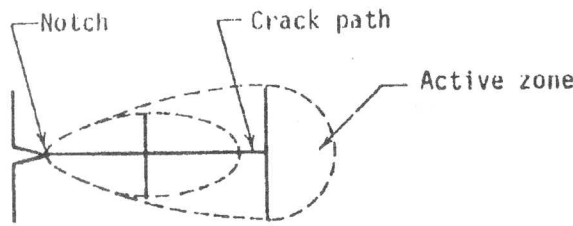


Figure 1.

CRACK PROPAGATION CRITERION

Crack layer propagation is an irreversible process, thus the framework of thermodynamics of irreversible processes is employed. The rate of global entropy  $\dot{S}_i$  may be expressed as;

$$\dot{S}_i = \frac{1}{T} (iX_{tr} + \dot{V} D_{def}) \tag{1}$$

Where,  $T$  is the absolute temperature,  $i$  is the rate of crack extension,  $X_{tr}$  is the corresponding driving force,  $\dot{V}$  is the rate of slow active zone deformation and  $D_{def}$  is the corresponding driving force. The second term is defined as the total energy dissipation rate on slow damage growth leading to crack propagation, i.e.,  $\dot{D} = \dot{V} X_{def}$ .

The driving force for crack extension is derived as;

$$X_{tr} = (J_1 - \gamma^* R_1)$$

where  $J_1$  is the energy release rate,  $\gamma^*$  is the specific enthalpy of damage and  $R_1$  is the CL translational resistance moment.

The specific enthalpy of damage  $\gamma^*$  is a material constant characteristic of the latent energy of material transformation within the active zone from its initial state to a "damaged" state. On the other hand,  $R_1$  expresses the total amount of transformed material associated with crack extension together with instantaneous active zone deformation.

Analysis of thermodynamic stability shows that slow (quasistatic) crack propagation is permissible if the crack layer driving force is non-positive, i.e.,  $J_1 - \gamma^* \leq 0$  [11]. According to the second law of

thermodynamics the global entropy production is always non-negative quantity, i.e.,  $T \dot{S}_i \geq 0$ . For reversible process (or at equilibrium)  $T \dot{S}_i = 0$ . Therefore, non-equilibrium slow crack propagation can occur only if  $\dot{D}$  is sufficiently large to compensate the negative entropy rate associated with crack extension  $(J_1 - \gamma^* R_1)$ .

CRACK PROPAGATION KINETICS

Employing the above argument to equation (1), the following equation is obtained for subcritical crack propagation;

$$i = \frac{\dot{D}}{\gamma^* R_1 - J_1} \tag{2}$$

The denominator describes the energy barrier per unit crack length increment. The numerator is the total energy dissipated on damage (fracture) processes per unit time. As  $J_1$  approaches  $\gamma^* R_1$ , the rate of crack propagation tends to infinity which signals the instability of slow crack growth and the onset of dynamic crack propagation. The transition from slow to fast crack propagation is readily observable and thus  $\gamma^*$  can be determined as  $J_{IC}/R_{IC}$ , where the subscript  $C$  indicates the critical stage.

In fatigue  $i$  becomes  $dl/dN$ , where  $N$  is the number of cycles and the rate of energy dissipation on slow damage processes  $dD/dN$  is expressed as part  $\beta$  of the total irreversible work  $W_i$ . The later quantity is obtained from the load-displacement hysteresis. Thus, equation (2) becomes,

$$\frac{dl}{dN} = \frac{\beta W_i}{\gamma^* R_1 - J_1} \tag{3}$$

It should be noted that  $J_1$  can be replaced by  $G_1$  as the specific case may be.

Alternatively,  $\gamma^*$  and  $\beta$  may be obtained from the entire crack propagation history or from a portion of it if equation (3) is expressed in the form:

$$\frac{J_1}{R_1} = \gamma^* - \beta \left( \frac{dW_i}{dl} \cdot \frac{1}{toR_1} \right)$$

where  $t_0$  is the initial thickness of the specimen. The equation is that of a straight line, the slope of which is  $\beta$  and its intercept is  $\gamma^*$ .

APPLICABILITY OF THE THEORY

The constancy of  $\gamma^*$  as the specific enthalpy of damage were tested under different loading conditions for the polycarbonate [8]. The energy release rate was measured from load-displacement curve from tests conducted on single edge notched specimens at different maximum stress, load ratio, and frequency.  $R_1$  was evaluated as the active zone volume per unit crack advance  $m^3/m^2$ .  $\gamma^*$  was obtained as  $J_{1C}/R_{1C}$  determined at the onset of fast crack propagation. The results obtained clarify that  $J_{1C}$  depends strongly on the loading conditions and the same is true with  $R_{1C}$ . Nevertheless  $\gamma^*$  is constant in accordance with the theory's predictions. The variation in  $\gamma^* = 9.8 \pm 1.4$  cal/g is not unreasonable considering normal experimental errors in this type of test.

Similar analysis has been successfully conducted on polystyrene, another model material that fails by a brittle crazing mechanism. The value of  $\gamma^* = 12 \times 10^6$   $Jm^{-3}$  was obtained using  $G_{1C}$  and  $R_{1C}$  [12].

In another examination of the applicability of the theory,  $\gamma^*$  was determined from the entire crack propagation history, equation (3), instead of the point of crack stability. The material used in this investigation was the Beta Ti-Alloy (Ti-15V-3Cr-3Al-3Sn), which is a material that displays varying mechanisms during the same crack propagation test under the same loading condition [13]. The analysis revealed two stages of fatigue crack propagation of the thin single edge notched specimens used in the investigation. Slip and/or twinning processes apparently dominated the first stage with a specific enthalpy of damage of  $7 MJ/m^3$ . The second stage, approaching critical crack propagation, is dominated by microcracking and is characterized by a specific enthalpy of damage of about  $20 MJ/m^3$ .

Unfortunately, there are no trials for the applicability of the theory on steels. In fact the magnitude of the active zone preceding the brittle crack propagation can not, at present, be evaluated accurately from any experimental observations concurrently with crack propagation. Thus approximate means should be used

to assess the relative magnitude of damage. Wang et al [14] presented an approach to the same theory in their study on Epoxy resin. The approach reformulate equation (3) in the form;

$$\frac{dl}{dN} = \frac{\beta G_1^2}{\mu G_{1C} - G_1} \tag{5}$$

where;  $G_1 = J_1 =$  energy release rate, and the subscript "C" indicates the transition from subcritical to critical crack propagation.  $\mu = R_1/R_{1C}$ , is a damage coefficient.

We applied equation (5) on a rich data of two types of steels published by Sawaki et al [15]. This data covers almost the entire range of stress intensity  $\Delta K$ , from nearly threshold value to final fracture. The materials tested by Sawaki et al were two kinds of low alloy quench and tempered high strength steels (HT 80 and HT 60). The chemical compositions of the materials and their mechanical properties at room temperature as given by the authors [15] are given in Tables (1) and (2) respectively.

Table 1. Chemical composition of the material used (Wt%) (after Sawaki et al [15]).

Steel	C	Si	Mn	P	S	Cr	Mo	Nb	B	V
HT 80	0.15	0.32	1.41	0.018	0.004	0.004	-	0.69	0.035	-
Ht 60	0.12	0.46	1.22	0.012	0.01	0.25	0.25	-	-	0.09

Table 2. Mechanical properties at room temperature (after Sawaki et al [25]).

Steel	$\sigma_{02}$ (MPa)	$\sigma_B$ (MPa)	$\delta$ (%)
HT 80	755	804	24.0
HT 60	531	624	19.4

The data was obtained using compact tension specimens following the ASTM recommended test procedure [16]. In the study of this data we considered the value of crack propagation rate of  $10^{-2}$  mm/cycle as a good representation of the possible measurable critical rate, and thus the corresponding energy release

as a good representation of the possible measurable critical rate, and thus the corresponding energy release rate could be used as an approximation  $G_{IC}$ . Thus using  $\mu = 1$  corresponding to the measured crack speed  $10^{-2}$  mm/cycle and corresponding  $G_{IC}$ , a value of  $\beta$  is obtained for both steel, HT60 and HT80, for different stress ratio. In each case, the phenomenological coefficient,  $\beta$ , is employed as a single fitting parameter to obtain the damage evolution coefficient  $\mu$ . The results obtained for  $\beta$  are tabulated as follows:

Material	Stress Ratio (R)	$\beta \text{ m}^4 \text{ KJ}^{-1} \text{ Cycle}^{-1}$
HT 60	0.05	$2.15259 \times 10^{-11}$
HT 60	0.40	$2.314 \times 10^{-11}$
HT 80	0.05	$5.055 \times 10^{-11}$
HT 80	0.40	$5.250 \times 10^{-11}$
HT 80	0.67	$5.27 \times 10^{-11}$
HT 80	0.80	$3.224 \times 10^{-11}$

The evolution of energy barriers for crack advance, the denominator of equation (5) according to the theory, for each case are shown in Figures (2) to (7), where  $G_I$  is considered as the energy available for crack propagation, the energy required is  $\gamma^* R_1 = \mu G_{IC}$ .

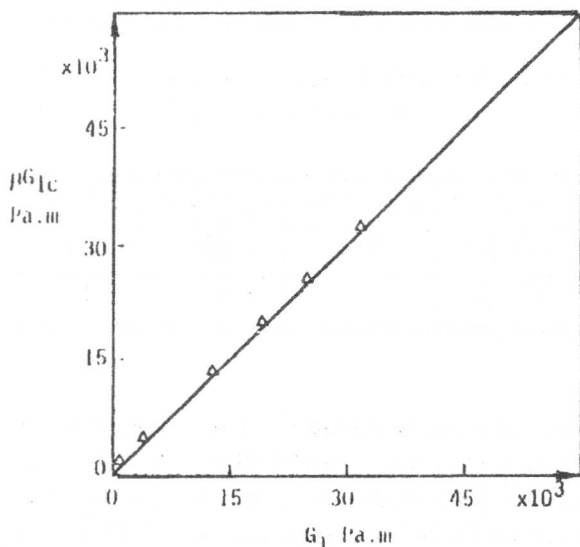


Figure 2. HT 60 - R = 0.05.

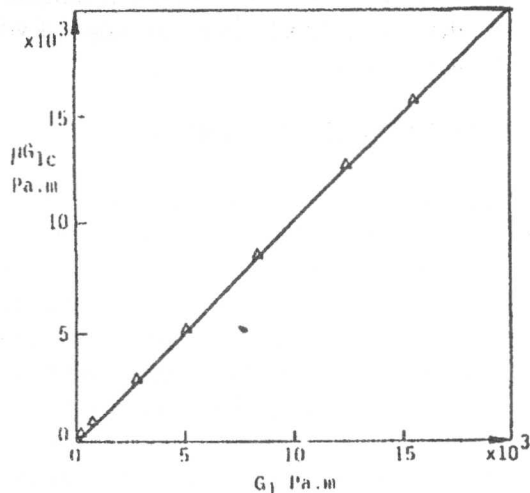


Figure 3. HT 60 - R = 0.4.

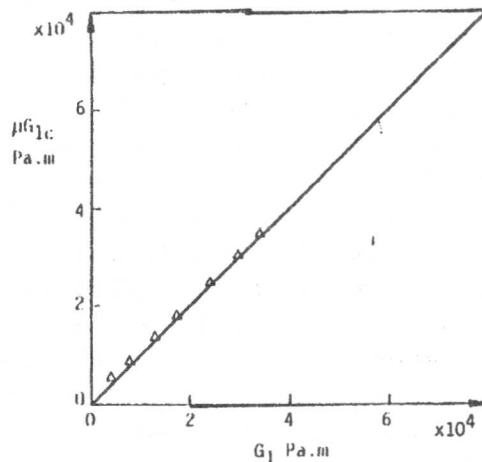


Figure 4. HT 80 - R = 0.05.

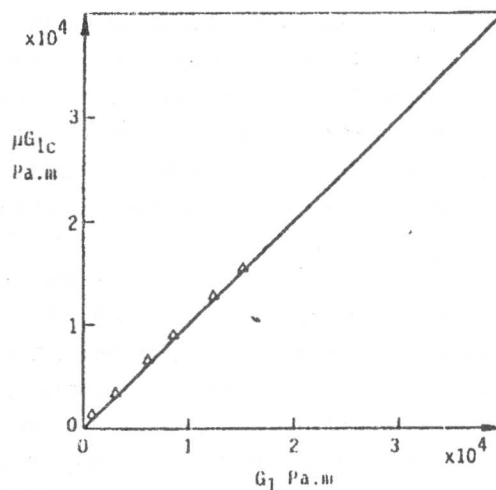


Figure 5. HT 80 - R = 0.4.

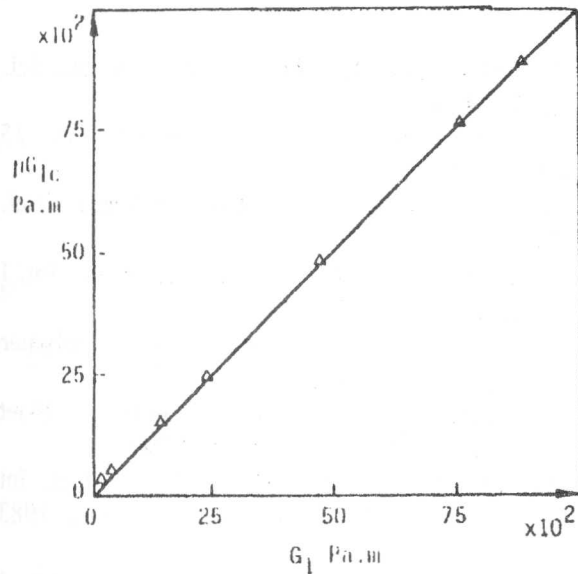


Figure 6. HT 80 - R = 0.67.

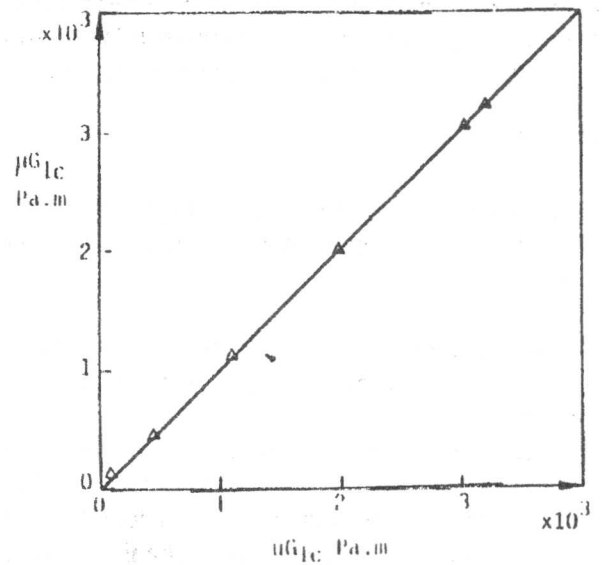


Figure 7. HT 80 - R = 0.08.

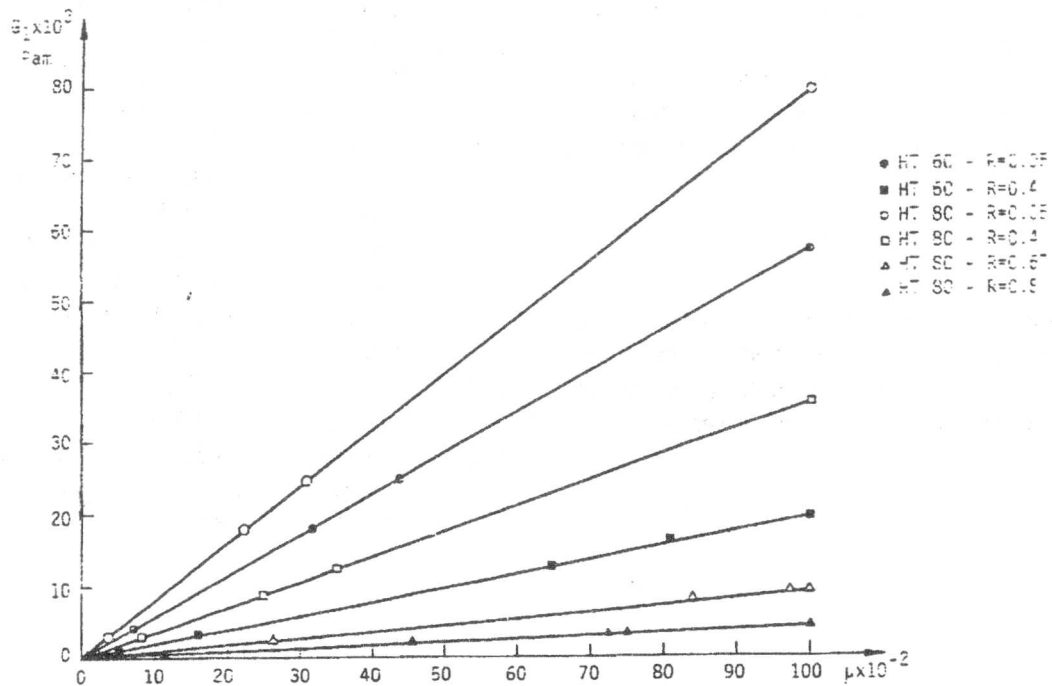


Figure 8.

The correlation between  $G_1$  and  $\mu$  is presented in Figure (8). The strong dependency on the load ratio (R) is clear which means the strong dependency of the critical energy release rate on the loading condition. This dependency is recognized in experiments and in

accordance with the crack layer theory assumptions. The values of the phenomenological coefficient  $\beta$  are practically the same for HT60 with load ratio 0.05, and 0.4. Also for the same load ratios,  $\beta$ , is constant for HT80, but with larger load ratio  $R = 0.67$ ,  $R = 0.8$

this value differs considerably, which may be explained as the existence of different  $\gamma^*$ s or different dominated damage mechanism depending on the load history.

## CONCLUSIONS

The applicability of the crack layer theory is discussed for several materials from which one may conclude;

- 1- A good adaptation of the theory was found for polymers, specially, polycarbonate and polystyrene.
- 2- The tests made on Beta-Ti-alloy indicate good applicability of the theory but the constancy of the specific enthalpy of damage  $\gamma^*$  was found to be valid for the existence failure mechanism which dominate the period of crack propagation.
- 3- The approximation of the theory applied to the data published for two carbon steels indicate promising agreement with the theory. More investigations in metals, specially steels, are needed to verify such results.
- 4- This work highlights the importance of specially designed experiments towards the complete verification of this interesting theory specially in metals.

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## REFERENCES

- [1] R. Griffiths and D.G. Holloway, I. Mater. Sci., 5, 302, 1970.
- [2] S. Yamini and R.J. Young, J. Mater. Sci., 15, 1823, 1980.
- [3] A. Chudnovsky and A. Moet, J. Mater. Sci., 20, 630, 1985.
- [4] J. Bostis, A. Chudnovsky, and A. Moet, Int. J. Fract. 33, 227, 1987.
- [5] J. Krey, K. Friedrich, and A. Moet, Polymer, Vol. 29, August 1988.
- [6] N. Haddaui, A. Chudnovsky and A. Moet, Polymer, Vol. 27, September, 1986.
- [7] M. Bakar, A. Chudnovsky and A. Moet, Int. Conf. Fatigue in Polymers, 29 June, 1983, London.
- [8] M. Bakar, A. Chudnovsky and A. Moet, J. Mater. Sci., 23, 2321, 1988.
- [9] K. Sehanobish, E. Baer, A. Chundovsky, and A. Moet, J. Mater. Sci., 20, 1943, 1985.
- [10] A.I. Gomma, A.H. Hamdy and A. Moet, SPE, ANTEC 90, 1506-1511, Dallas, U.S.A., May, 1990.
- [11] K. Sehanobish, J. Bostis, A. Moet, and A. Chudnovsky, Int. J. of Fracture, 32, 21(1986).
- [12] N. Hadoui, A. Chudnovsky and A. Moet, ACS, 186th National Meeting, August, 1983.
- [13] I. Mostafa, G. Welsh, A. Moet, and A. Chudnovsky, Mater. Sci. and Eng., Sept. 1988.
- [14] X-Wang, K. Sehanobish, and A. Moet, Polymer Composites, Vol. 9, No. 3, June 1988.
- [15] Y. Sawaki, S. Tada, S. Hashimoto, and T. Kawasaki, Int. J. of Fracture, 35, 125, 1987.
- [16] S.J. Hudak, Jr. and R.J. Bucci, "Fatigue crack measurement and data analysis", ASTM-STP 738, 321-356, 1981.