AN EXPERIMENTAL STUDY OF A WELL IN A LEAKY AQUIFER

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ABSTRACT

A horizontal Hele-Shaw model is developed with the aim to solve problem connected with groundwater flow to a well in a leaky aquifer recharged uniformly by vertical percolation, under the steady state condition. Procedures and construction techniques for assembling such a model are outlined. Results are presented in graphical forms using dimensionless parameters to determine the well discharge for various rates of percolation and drawdowns. Special attention has been paid to determine the well radius of influence.

NOTATION

a	half spacing between the horizontal prespex plates
d	diameter of the capillary tube
g	the gravity acceleration
h	piezometric head at a radial distance r from the well
h^*	depth of oil in the overhanging cylinder
h_w	depth of oil in the well
H_0	depth of the original piezometric surface at radius of island at which the well is pumping
K	permeability coefficient
Q_0	discharge into the overhanging cylinder
Q_r	discharge into the capillary tubes
Q_t	discharge into the overflow tube
Q_w	well discharge
r	radial distance from the well
r_w	well radius
R_0	radius of island at which the well is pumping
x	position of the stagnation point from the well
W	rate of vertical recharge
Y	thickness of the leaky aquifer
υ	kinematic viscosity of the oil

INTRODUCTION

Excess surface irrigation, regardless of the method of its application, always results in some percolation of the applied water below the root zone of the farmed area. In fact, this percolation is essential in order to maintain a favorable salt status in the soil in semi-arid and arid regions. In certain cases a substantial quantity of irrigation water must be wasted to prevent increasing of the soil salinity. The farmed areas often rest on a semi-pervious layer through which leakage is infiltrated into a leaky aquifer. Consequently, a rise in the water table is created as shown in Figure (1). This rise depends on the duration and type of variation of the percolation rate, on the size and shape of the area over which this percolation takes place, and on the hydraulic and geometric parameters of the aquifer. In many other cases, area are subjected to high rate of rainfall, artificial recharge or other surface resources which in turn cause rise in the water table. Drainage wells are used to remove such excess water and to maintain the water table at a preassigned depth below ground surface as shown in Figure (1).

The development of theory of pumping from leaky aquifer has taken place analytically, in the transient state, by Hantush and Jacob [3], Hantush ([4], [5], [6], [7]), Neuman and Witherspoon ([11], [12], [13]) and Husman [8]. They report that the flow is essentially horizontal in the leaky aquifer and vertical in the semi-pervious layers (aquitards) and the errors introduced by this assumption are less than 5% when the conductivities of the aquifers are more than 2 orders of magnitude greater than that of the aquitards. They introduce formulae to determine the drawdown curve for different cases of pumping in the transient state.

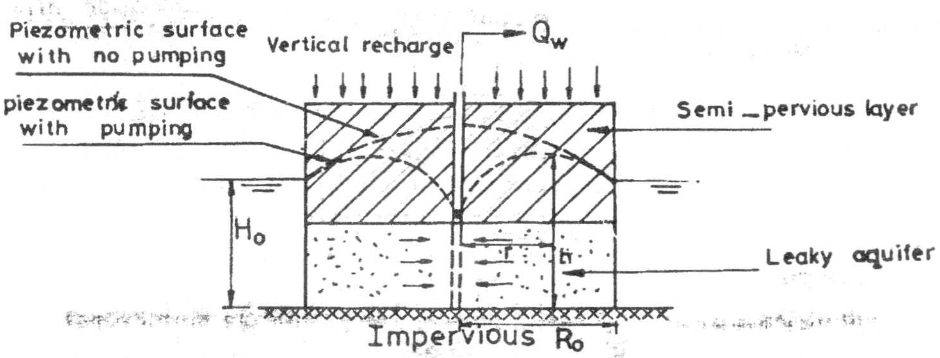

Figure 1. Circular island with central well and vertical recharge.

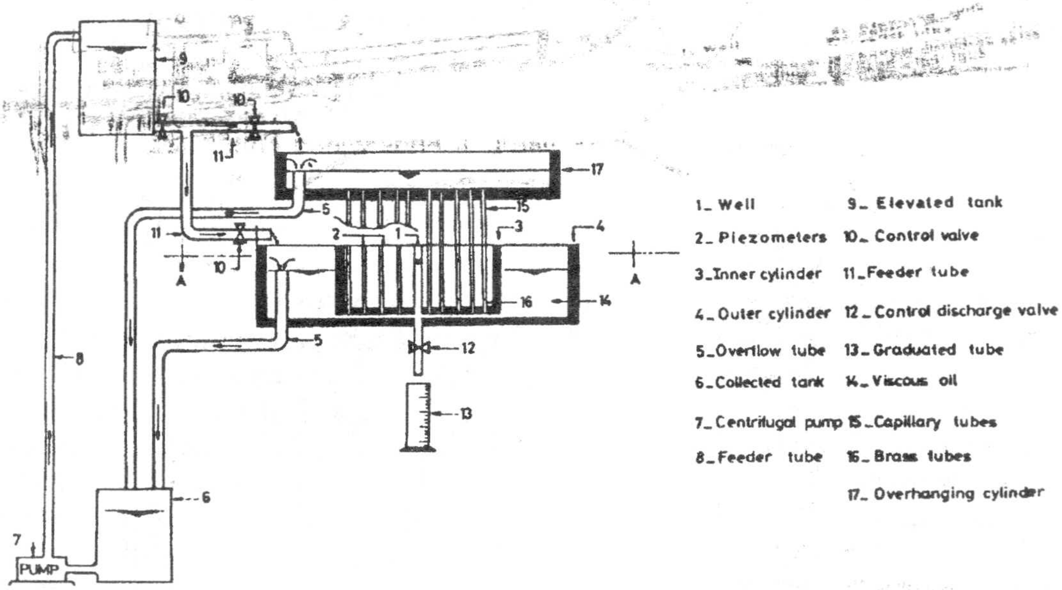

Figure 2. A diagram of the horizontal Hele-Shaw model.

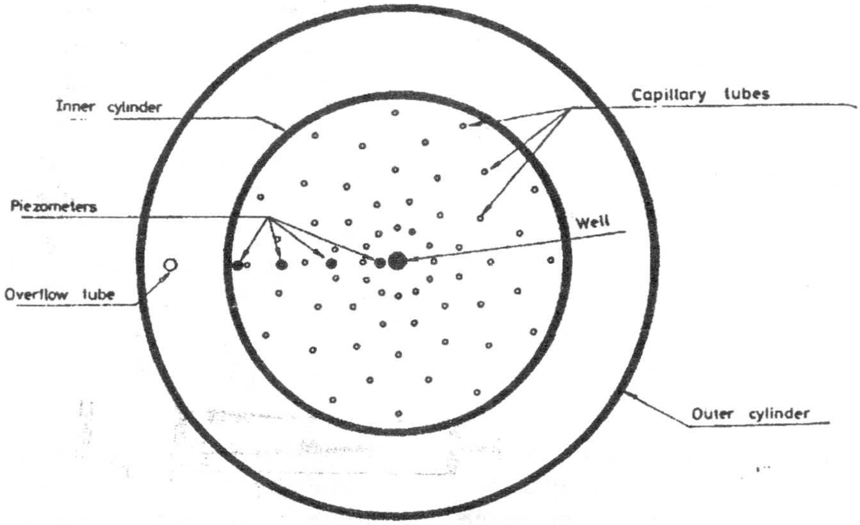

Figure 3. A plan showing the distribution of capillary tubes in the inner cylinder.

After a certain period of pumping, an equilibrium state is established between the well discharge and the recharge from the semi-pervious layer. Kruseman and De Ridder [9] introduced a formula to determine the drawdown of the piezometric head far a case of semi-pervious layer overlaid by unconfined aquifer and the recharge is introduced due to the difference between the phreatic and the piezometric heads.

In our case, the leaky aquifer is only topped by semi-pervious layer and it is recharged vertically due to excess surface water, and the well is pumping in the center of a circular island of radius R_o as shown in Figure (1).

Hele-Shaw model proved to be a versatile tool for investigating groundwater problems. Its analogy to flow in porous media made it superior to a sand model because of the ease of recording data and the avoidance of a variable capillary zone. The use of Hele-Shaw model is based on the fact that a two-dimensional laminar flow of groundwater through a porous soil can be expressed by the same differential equation as the laminar flow of a viscous fluid through a narrow interspace between two parallel plates. The main principle of the use of the model was explained already a great many times (Todd [16], Marino [10], Santing [15], Bear [1] and others), therefore, it will not be discussed again in the present paper. Horizontal Hele-Shaw model have been used little to represent horizontal aquifers (Santing [15] and columbus [2]). Rezk and Moghazi [14] developed a horizontal Hele-Shaw model to solve problem of pumping from a well in a horizontal confined aquifer. They showed the excellent accuracy of this model to simulate the flow in horizontal confined aquifers. According to the authors knowledge, no attempt has been made to model pumping from a leaky aquifer with uniform rate of recharge using the Hele-Shaw model.

The main objective of this study is to investigate the effect of the vertical recharge on the hydraulics of a well pumping from a leaky aquifer, at the steady state. The former designed horizontal Hele-Shaw model designed by the authors [14] is re-developed to simulate the problem under investigation.

DESCRIPTION OF THE MODEL

The following assumptions are taken into account in the design of the model; (1) the flow of groundwater is assumed to be built up of a vertical flow through the semi-pervious layer and a horizontal flow in the leaky aquifer; (2) the aquifer is homogeneous and isotropic; (3) the rate of percolation is constant with respect to time and space.

A diagram of the horizontal Hele-Shaw model used to simulate the problem is shown in Figure (2). It consists of two prespex cylinders (3 mm thick). The outer cylinder has a diameter of 70 cms and represents a constant head reservoir to simulate the original piezometric head of the aquifer H_o. This reservoir is fed from an elevated tank. An overflow tube is used to keep desirable constant head in the reservoir during the experiments. The inner cylinder has a radius of 20 cms and represents the radius of the island, R_o, at which the well is pumping. A narrow interspace between the two bottoms of the cylinders was kept constant with the aid of fiber washers (1.5 mm) to represent the thickness of the aquifer, Y. The two bottoms joined together with the washers by brass bolts. A viscous oil (supper 7500-20 w/50) is used to simulate groundwater. The well diameter $2r_w$ is represented by a glass tube (10 mm diameter) connected to the centre of the inner cylinder as shown in Figure (2). Withdrawal of groundwater through the well is initiated by withdrawing certain amount of oil through a discharge tube connected to the bottom centre of the outer cylinder and opposite to the well. The well discharge Q_w is regulated by a control valve and collected into a graduated tube. Four small diameter glass tubes are connected to the narrow interspace, in a one line with the well, to observe the piezometric heads during experiments.

Vertical recharges can be realized in the model by making use of an overhanging prespex cylinder (60 cm diameter) and elevated by about 40 cms above the bottom of the inner cylinder as shown in Figure (2). About 60 plastic capillary tubes (2.5mm diameter) are projected downwards from the overhanging cylinder and connected into the horizontal narrow interspace with the aid of brass tubes (2.5 mm) as shown in Figure (2). The capillary tubes are similarly distributed, over the bottom area of the inner cylinder, to keep the vertical recharge as uniform as possible (see Figure 3). The overhanging cylinder is also fed from the elevated tank and acts as a distributor of oil to the capillary tubes. An overflow tube is used to keep a constant head in the overhanging cylinder which in turn provides a uniform rate of recharge during the experiment. In order to accelerate the filling of the capillary tubes and to prevent the produce of air bubbles, a syringe is used at the start of any experiment.

DESCRIPTION OF THE EXPERIMENTAL WORK

Two main groups of experiments are carried out in the laboratory. The first one is made for well reading h_w higher than the original piezometric head in the aquifer H_o to represent cases of low rate of pumping from the well with high rate of vertical recharge (Figure (4-a)). The second group is made for $h_w < H_o$ to represent cases of high rate of pumping from the well (Figure (4-b)). Three values of H_o have been chosen (5.0, 7.5 and 10.0 cms). The constant head reservoir is fed from the elevated tank until the desired values of H_o is achieved with the aid of the overflow tube. Oil is also supplied from the elevated tank to the overhanging cylinder until a certain depth h^* is reached, using another overflow tube. Quantities of oil supplied through the capillary tubes depend mainly on the depth h^*. The discharge into the overhanging cylinder Q_o and the discharge escaped through the overflow tube Q_t are calibrated. The total rate of recharge through the capillary tubes into the aquifer Q_r equals Q_o-Q_t. Then, the average rate of recharge into the aquifer is equal Q_r divided by the total cross-sectional areas of the recharging tubes:

$$W = Q_r / 15\,\pi d^2 \qquad (1)$$

where d is the diameter of the capillary tube (2.5 mm)

For each value of H_o four different values of h^* are considered, and for each value of H_o and h^* various reading of h_w are recorded as well as the corresponding piezometric heads using the control discharge valve underneath the well. The well discharge Q_w is, then, calibrated using a graduated tube and a stop watch. In each experiment, the surrounding temperature is observed very carefully to determine the corresponding kinematic viscosity of the oil. Consequently, the permeability coefficient k of the aquifer is obtained from the formula (Todd [16]):

$$k = \frac{a^2 g}{3\nu}$$

where

$a =$ half spacing between the two horizontal plates
$g =$ the gravity acceleration
$\nu =$ kinematic viscosity of the oil

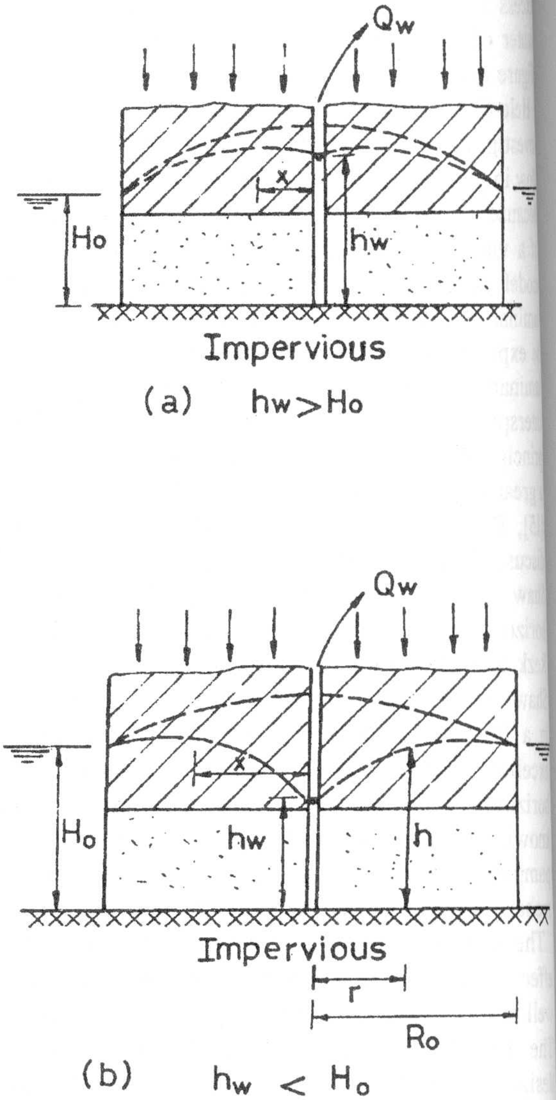

Figure 4. Cases of pumping from the well.

ANALYSIS OF RESULTS

Figures (5) and (6) show the profile of the measu piezometric surface, using group of piezometers, cases of $h_w < H_o$ and $h_w > H_o$ respectively. Different rates of recharges and drawdowns are investigated. It can be seen that the piezometric surface approaches the well tangentially at $r = r_w$, where r is the radial distance from the well. The elevation of the piezometric surface increases as the increase of r and reaches a maximum at the stagnation point (r=x) at which a water divide exists and the effect of pumping from the well vanishes. In other words, the distance x represents the well radius of influence at which all the well discharge is being contributed by vertical recharge. For radial distances greater than the well radius of influence the elevation of the piezometric surface decreases to join the original piezometric head H_o at a distance equal to the radius of island R_o.

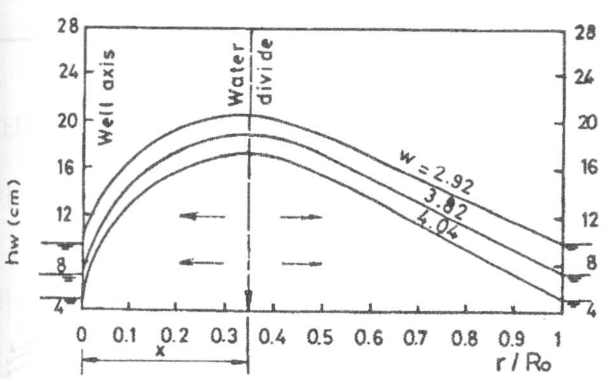

Figure 5. Drawdown curves for $h_w < H_o$.

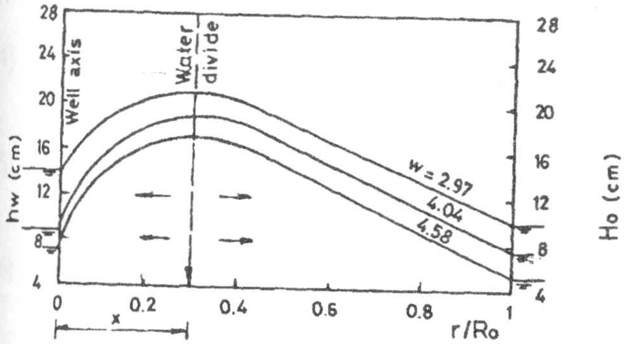

Figure 6. Drawdown curves for $H_w > H_o$.

According to Figure (5) and (6), and according to the studied range of variables, it can be noticed that the position of the stagnation point from the well, x, (the well radius of influence) nearly varies from 0.3 R_o (for $h_w > H_o$) to 0.35 R_o (for $h_w < H_o$), where R_o is the radius of island at which the well is pumping.

For the first group of experiments ($h_w < H_o$), the relationships between the measured well discharge Q_w and the drawdown in the well H_o-h_w are plotted in dimensionless forms for H_o = 5.0, 7.5 and 10.0 cms as shown in Figures (7), (8) and (9) respectively. For the other group of experiments ($h_w > H_o$), the relationship between the well discharge and the value h_w-H_o are plotted in a similar manner as shown in Figures (10), (11) and (12) for H_o = 5.0, 7.5 and 10.0 cms respectively. Figures 5 to 10 are useful to determine the well discharge for various rates of vertical recharges, W, or drawdowns in the well. It can be seen from these figures that the drawdown due to the pumping the well with a constant capacity Q_w will decrease as the recharge W increases. A comparison between the first group of results (Figures (7), (8) and (9)) and the second group (Figures (10), (11) and (12)) for the same W, H_o and the difference between H_o and h_w, shows that Q_w in the first group is bigger than the corresponding value of Q_w in the second group. This conclusion is coincided with the conclusion mentioned in the previous paragraph, where the well radius of influence for the first group ($x \simeq 0.35\ R_o$) is bigger than the well radius of influence for the second group ($x \simeq 0.30\ R_o$).

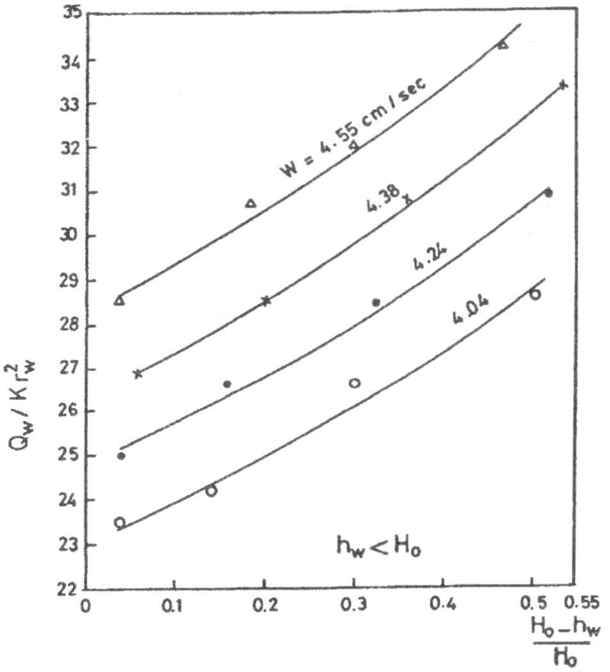

Figure 7. Measured well discharge for H_o = 5.0 cm.

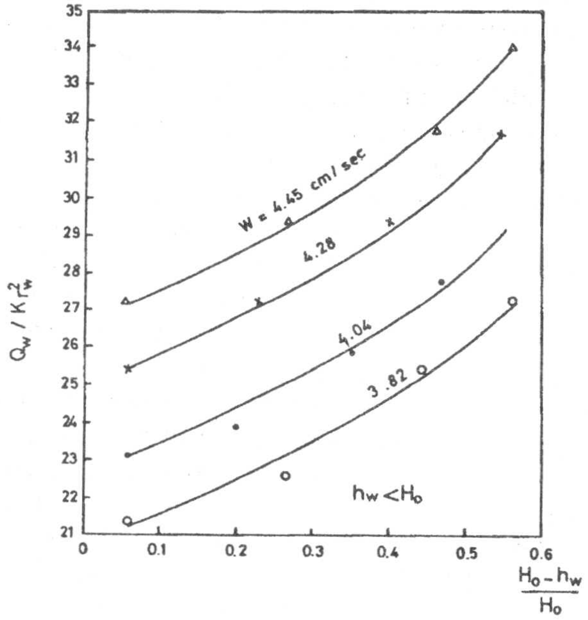

Figure 8. Measured well discharge for H_o = 7.50 cm.

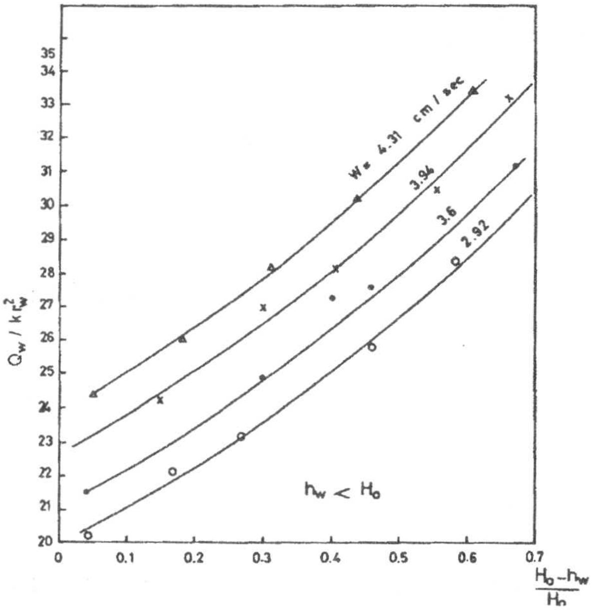

Figure 9. Measured well discharge for H_o = 10.0 (cm).

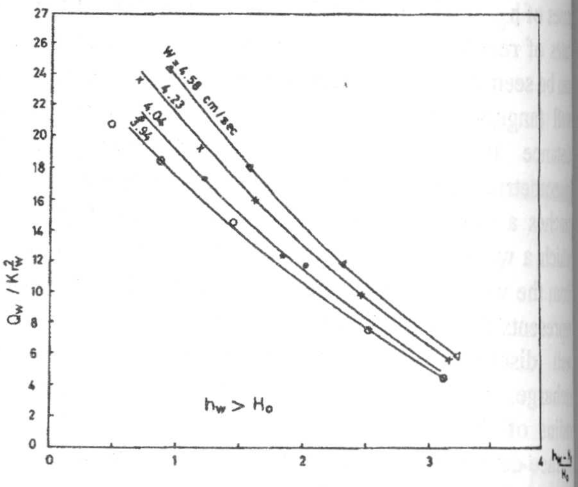

Figure 10. Measured well discharge for H = 5.0

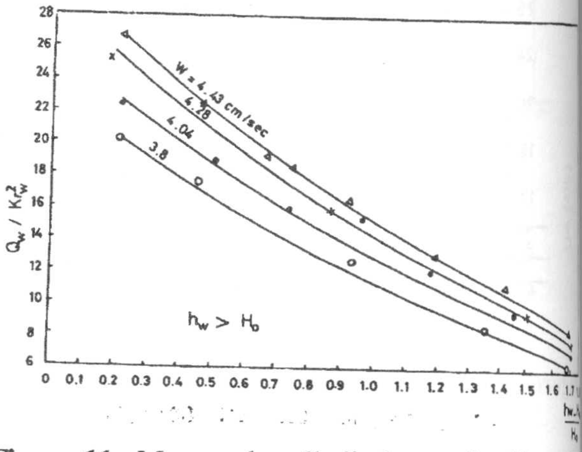

Figure 11. Measured well discharge for H_o = 7 cm.

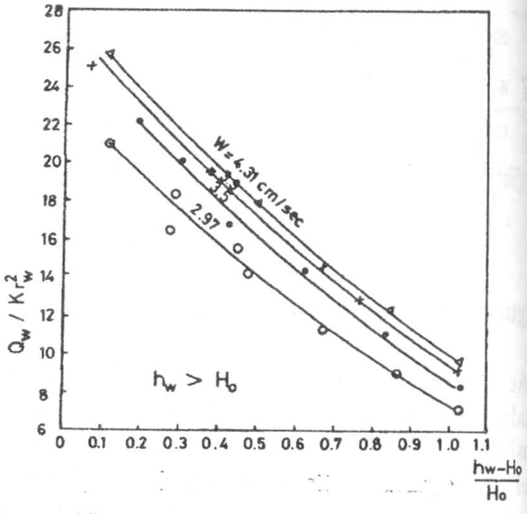

Figure 12. Measured well discharge for H_o = 10.0 cm.

CONCLUSIONS AND RECOMMENDATIONS

1. Using a horizontal Hele-Shaw model, charts have been obtained to determine the discharge of a well pumping from a leaky aquifer recharged uniformly by vertical infiltration under the steady state condition.
2. The radius of influence of a well pumping from a vertically recharged leaky aquifer is estimated to vary between 0.30 to 0.35 the radius of island at which the well is pumping.
3. Profiles of the drawdown curves of the piezometric surface have been observed for various rates of infiltrations and drawdowns.
4. The designed Hele-Shaw model can be easily modified to simulate the effect of evaporation from high groundwater surfaces on the pumping from aquifers.

REFERENCES

[1] Bear, J., "Scales for viscous analogy models for groundwater studies", *J. of Hydraulics Div, ASCE*, vol. 86, HY2, 1960.

[2] Columbus, N., "The Design and construction of Hele-Shaw models", *Groundwater*, vol. 4, No. 2, pp 16-22, 1966.

[3] Hantush, M.S. and C.E. Jacob, "Nonsteady radial flow in an infinite leaky aquifer", Trans. *Amer. Geophysical Union*, vol. 36, pp, 95-100, 1955.

[4] Hantush, M.S., "Analysis of data from pumping tests in leaky aquifers", Trans. *Amer. Geophysical Union*, vol. 37, pp. 702-714, 1956.

[5] Hantush, M.S., "Modification of the theory of leaky aquifers", *J. Geophysical Research*, vol. 65, pp, 3713-3752, 1962.

[6] Hantush, M.S., "Hydraulics of Wells, *Advances in Hydroscience*", vol. 1, Academic Press, pp. 282-432, 1964.

[7] Hantush, M.S., "Flow to wells in aquifers separated by a semipervious layer", *J. of Geophysical Research*, vol. 72, No. 6, pp. 1709-1719, 1967.

[8] Huisman, L., "Groundwater Recovery", Macmillan, 1972, U.K.

[9] Kruseman, G.P. and N.A. De Ridder "Analysis and evaluation of pumping test data", *Int. Inst. for Land Reclamation and Improvement, Wageningen*, 1970, The Netherlands.

[10] Marino, M., "Hele-Shaw model study of the growth and decay of groundwater ridges", *J. of Geophysical Research*", vol. 72, No.4, pp. 1195-1205, 1967.

[11] Neuman, S.P. and P.A. Witherspoon, "Theory of flow in a confined two aquifer system", *Water Resources Research*, vol. 5, pp. 803-816, 1969a.

[12] Neuman, S.P. and P.A. Witherspoon, "Applicability of current theories of flow in leaky aquifers", *Water Resources Research*, vol. 5, pp. 817-829, 1969b.

[13] Neuman, S.P. and P.A. Witherspoon, "Field determination of the hydraulic properties of leaky multiple aquifer system", *Water Resources Research*, vol. 8, pp 1284-1298, 1972.

[14] Rezk, M.A. and Moghazi, H.M., "An experimental study of seepage towards an artesian well using Hele-Shaw model", *Alex. Engg. J.*, vol. 23, No. 2, April C 87-C 93, 1993, Alexandria, Egypt.

[15] Santing, G., "A horizontal scale model based on the viscous flow analogy for studying groundwater flow in an aquifer having storage", *Intl. Assoc. for Scientific Hydrology*", Pub. 43, pp. 105-114, 1958.

[16] Todd, D.K. "Groundwater Hydrology", John Wiely & Sons 1980, U.S.A.