PREDICTION OF OPTIMUM TILT ANGLE FOR PARABOLIC TROUGH WITH THE LONG AXIS IN THE NORTH-SOUTH DIRECTION

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ABSTRACT

A computer model is developed for a parabolic trough oriented north-south to obtain the optimum tilt angle β_{opt} all over the year as well as for each month at any latitude angle (L). Using least squares method, two empirical formulae were obtained for calculating β_{opt}, one for all year around, while the second for each month. A comparison between the proposed incident angle with the well-known value for the polar orientation shows an excellent agreement. The present study shows that monthly adjusted parabolic trough reaches 99.2 % efficiency of that with two-axis tracking trough.

INTRODUCTION

Increasing the efficiency of the parabolic trough becomes a great interest of many researchers, since the parabolic trough is the most appropriate one for producing heat energy at a medium temperature range (100 - 400 °C) [1-8]. One method for increasing the thermal efficiency of any type of collector is to use the optimum tilt angle [9]. This is because the orientation and tilts angle of a solar collector are the two most important factors usually considered in a solar energy system design.

Parabolic troughs can be mounted in several different ways. The most common method is to mount the collector horizontally with the long axis in the east-west direction [10,11,12]. In this case the end losses at any time other than solar noon are substantially high. It can be seen that the period of major collectable solar energy is within about 3 hours period on either side of solar noon [10]. The east-west alignment of parabolic troughs has the advantages that the collectors can be mounted horizontally on the ground with low cost supports and its alignment and tracking are relatively simple. The main disadvantage is the cosine loss at off-noon hours. For moderate concentration (~ 5), the use of parabolic E-W trough with a fixed position (non-tracking) is considered the simplest and cheapest method. In this case, the tilt of the trough about the main axis must be adjusted periodically during the year, roughly, monthly interval [10]. The value of the collector performance lies between 40% to 60% all over the year [6], and the solar fraction reflected from the reflector and received by the collector tube is in order of o.6 [13] (noting that the reflectivity of the surface is considered unity).

Parabolic trough can be also mounted with the long axis in the north south direction. When this orientation is used, tracking is accomplished by rotation of the trough about the long axis trough 180° in a day. The tilt angle is a very important parameter for north-south parabolic troughs. When zero tilt angle for N-S orientation is used, a very poor performance is obtained during winter time (collector efficiency is less than 25% [6]) and a good performance is obtained during the summer time. In general the solar fraction that can be reflected from the reflector surface towards the collector tube for N-S horizontal is higher than that E-W orientation [13] by about 10% when the collection period is all over the day and by about 4% when the collection period is 8 hours a day, during the summer months.

If the tilt angle was selected to be the same value as the latitude angle, the optical daily efficiency of the collector will be is less than 80% at the beginning summer and winter. But, at the beginning of spring and autumn, the daily efficiency can increase up to

99%. Thus the yearly average value is about 99%. [13]. The ideal N-S trough would have adjustable tilt angle. If the tilt angle could be adjusted about 12 times in a year, it would closely approximate the performance of a two-axis tracked trough [10]. To the knowledge of the author no values for the tilt angle have been given before, also no equations are available to calculate such tilt angle at any latitude. This was the goal of the present work.

THEORETICAL ANALYSIS

The general incident angle for a parabolic trough, oriented north south, and tilted with an angle β (see Figure (1)) can be given as [13]

$$\theta = \cos^{-1}\left\{\left[\frac{\sin\phi_3}{\cos\gamma}\right]\left[\cos\alpha\cos\phi\tan\beta + \sin\alpha\right]\right\} \quad (1)$$

where

$$\gamma = \tan^{-1}\left[\frac{\cos\alpha\sin\phi}{\sin\alpha + \cos\alpha\cos\phi\tan\beta}\right] \quad (2)$$

$$\phi_3 = \cos^{-1}\left\{\left[\frac{1}{\cos\beta} - \frac{\cos\phi_1}{\cos\phi_2}\right]\frac{\cos\gamma}{\tan\beta}\right\} \quad (3)$$

$$\phi_1 = \cos^{-1}\left(\frac{\cos^2\beta\cos^2\gamma + \cos^2\phi_2\cos^2\gamma - \sin^2\beta\cos^2\phi_2}{2\cos\phi_2\cos\beta\cos^2\gamma}\right) \quad (4)$$

and

$$\phi_2 = \tan^{-1}(\tan\beta\tan\gamma) \quad (5)$$

The solar fraction -defined as the amount of solar radiation which reaches the absorber tube (provided that the absorber tube has a length equal to the length of the reflector) divided by the total solar beam radiation)- over a period of days starting with day number N_1 and ending with N_2 as counted from January first, can be derived as:

$$F = \frac{\sum_{N=N_1}^{N=N_2} I_b \int_{t_1}^{t_2} \rho\cos\theta\left(1 - \frac{f}{L_1}\tan\theta\right)dt}{\sum_{N=N_1}^{N=N_2} I_b \int_{t_1}^{t_2} dt} \quad (6)$$

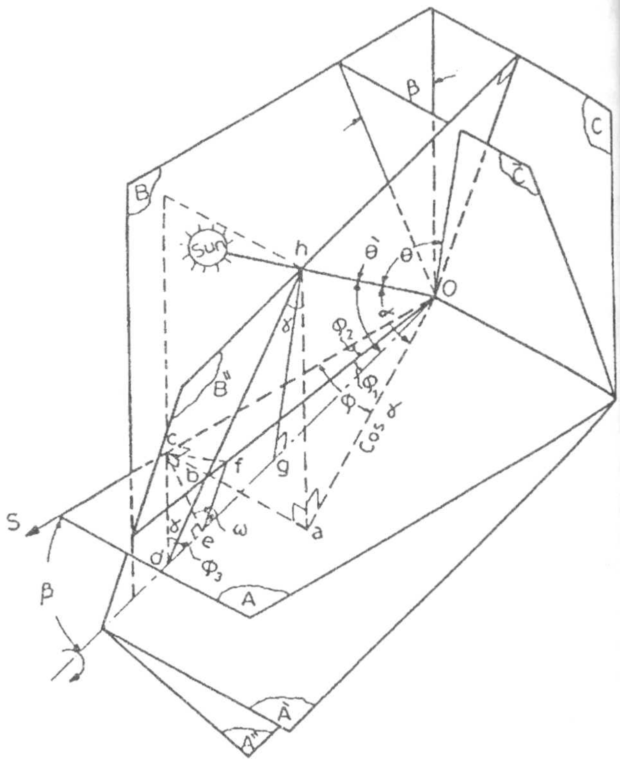

Figure 1. The incident angle for N-S orientation, titled with an angle β.

where ρ is the reflectivity of the collector surface, t_1 & t_2 are the starting and ending time for the specified time interval required for collection in a day, and I_b is the local beam radiation, which depends on the local weather and microclimate and is, therefore, not known a priori. It is possible to calculate I_b out side the earth's atmosphere in order to get universal solution, as [14]

$$I_b = I_o\left[1 + 0.034\cos\left(\frac{2\pi N}{365}\right)\right] \quad (7)$$

where I_o is the solar constant and equal to 1535 W/m^2

In equation (6), the effect of using a glass cover are neglected and t_1 and t_2 are chosen to be 8 and 16 respectively since the solar radiation before and after this range is very small specially during the winter time. However, any other specified time interval can be applied. The effect of shaded length in equation (6) is represented by the term $(1 - f/L_1 \tan \theta)$. This is the case when the incident angle lies in a plane parallel to the plane perpendicular to the aperture area and contains the collector tube (focal plane). As indicated in Ref. [13] the effect of the ratio L_1/f on the amount of the reflected radiation are neglected when this ratio equal to or exceed 10, therefor the ratio L_1 / f has been choose to be 10. A computer program is developed to solve the system of equations [1 to 6] to search for the optimum value of β at a given latitude angle for certain range days. The optimum value for β is the value of β which gives the maximum value for F. In order to get a universal solution in the present analysis not depending on the reflectivity of the collector surface, the value of ρ in equation (6) is chosen to be unity i.e as if we divided the collection factor by ρ.

RESULTS AND DISCUSSION

Before proceeding with the present proposed equation, a check must be done to check the validity of equation (1). Many references [10, 11 and 12] indicted that when the parabolic trough tilted with an angle equal to the latitude angle , the incident angle θ becomes equal to the solar declination angle δ. The declination angle δ, can be found from equation of Cooper [15] as:-.

$$\delta = 23.45 \sin \left[360 \frac{284 + N}{365} \right] \tag{8}$$

If the present system of equation is correct, the value of θ obtained from equation (1) must be equal to the value of δ obtained by equation 8 when applying $\beta = L$ in equation (1). This check had been done and found that the absolute value of θ is equal to the absolute value of δ obtained from equation 8 to the fourth decimal digit. For example for the day number one (N=1) $\theta = 22.97886$ and $\delta = 22.97878$. For the day number 255, $\theta = 1.79626$ and $\delta = 1.7963$. A scanning for the accuracy all over the year showed that the error varying between $\pm 0.002\%$ and $\pm 0.0003\%$ which is invisible.. It is to be noted that it is meant to compare the absolute value of θ with that of δ, since the values of θ are obtained from the inverse of a cosine function which is positive for both negative and positive value of θ. This does not affect any value of the result since we use the cosine function of the incident angle.

Table 1. Yearly optimum tilt angle versus latitude angle.

L	β_{opt}	F at β_{opt}	F $\beta = 0$	F $\beta = L$
0	3.5	.9340	.9330	0.9329
5	8.5	.9340	.9260	0.9329
10	13.5	.9340	.9157	0.9329
15	18.5	.9340	.8988	0.9329
20	23.5	.9340	.8772	0.9329
25	28.5	.9340	.8505	0.9329
30	33.5	.9340	.8200	0.9329
35	38.5	.9340	.7847	0.9329
40	43.5	.9340	.7477	0.9329
45	48.5	.9340	.7090	0.9329
50	53.5	.9340	.6698	0.9329
55	58.5	.9350	.6308	0.9329
60	63.5	.9371	.5934	0.9329

Table 2. The solar fraction versus tilt angle at any latitude angle for January.

β	L						
	0	10	20	30	40	50	60
0	0.898	0.827	0.743	0.651	0.558	0.474	0.441
10	0.953	0.898	0.827	0.743	0.651	0.561	0.474
20	0.987	0.953	0.898	0.827	0.743	0.651	0.561
30	0.985	0.987	0.953	0.898	0.827	0.743	0.651
40	0.950	0.985	0.987	0.953	0.898	0.827	0.743
50	0.894	0.950	0.985	0.987	0.953	0.898	0.827
60	0.819	0.894	0.950	0.985	0.987	0.953	0.898

A selected sample of results obtained are tabulated in Tables (1) through (4). In Table (1) a yearly optimum angle versus latitude angle are tabulated . The corresponding solar fraction and the solar fraction for both horizontal and the polar collector are also presented. The table shows that the optimum tilt angle varies linearly with latitude angle according to the relation

$$\beta_{opt} = L + 3.5 \quad (9)$$

For the polar collector (β=L) it was observed that, the solar fraction F was constant and equal to 0.9329. It does not vary with varying latitude angle. While for horizontal collector the value of F decrease with the increase of latitude angle. This indicates the importance of using large tilt angles as the latitude angle increases. It is to be noted that, The yearly average value for the incident angle cosine is cos θ= cos δ = 0.96 [16]. The discrepancy between the value of the incident angle cosine with the value of F = 0.9329 is due the shaded length effect which is included in equation (6). From Table (1) it is clear that using the optimum tilt angle rather than the polar one improves the solar fraction only by about 0.11%, while the improvement over the horizontal collector which could reach 57% at a latitude angle equal to 60°.

Table 3. The solar fraction versus tilt angle at any latitude angle for July.

β	L						
	0	10	20	30	40	50	60
0	0.901	0.954	0.988	0.984	0.948	0.891	0.816
10	0.830	0.901	0.954	0.988	0.984	0.948	0.891
20	0.747	0.830	0.901	0.954	0.988	0.984	0.948
30	0.655	0.747	0.830	0.901	0.954	0.988	0.984
40	0.563	0.655	0.747	0.830	0.901	0.954	0.988
50	0.460	0.563	0.655	0.747	0.830	0.901	0.954
60	0.058	0.460	0.563	0.655	0.747	0.830	0.901

Despite the fact that the gain using annual tilt angle is rather small, however, it can be further improved by monthly tracking. In order to trace and exactly determine the maximum solar fraction F, a computer print out is given to facilitate such search. The results obtained from equation 6 for January (N_1=1, N_2=31) are plotted in Figure (2) and tabulated in Table (2). Another sample of results obtained for July are presented in Table (3) and Figure (3). In Tables (2) and (3) each column represents the values of F at a fixed latitude angle and variable tilt angle. From this column the optimum tilt angle can be dedicated. For example, from Table (2), at L =0 the maximum value of F lies at a value of β close to 20°. Further calculations were computed to precisely obtain β_{opt} that gives maximum solar fraction. A summary of such calculations for each month are given in Table (4) and Figure (4).

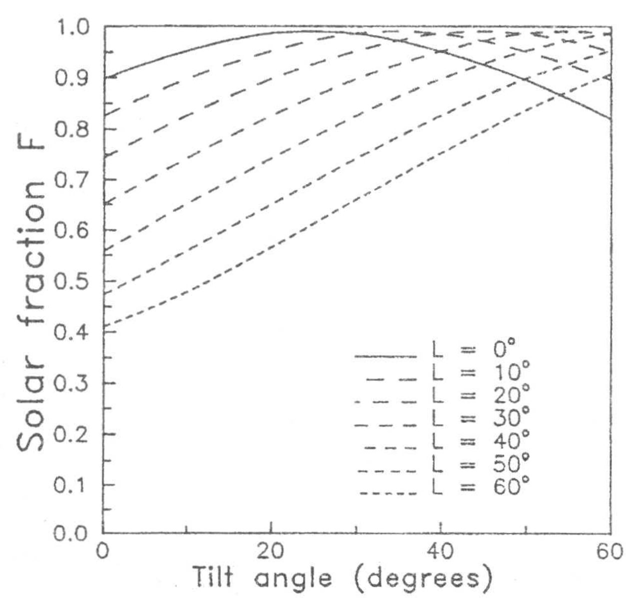

Figure 2. The solar fraction versus the tilt angle for january.

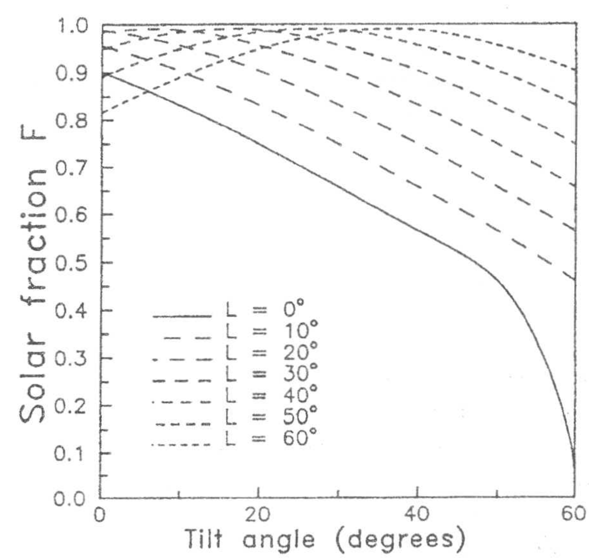

Figure 3. The solar fraction versus the tilt angle for july.

Table 4. Monthly optimum tilt angle and it's solar fraction.

L→	0	10	20	30	40	50	60	
J β F	24 .9917	34 .9917	44 .9917	54 .9917	64 .9917	74 .9917	84 .9917	$\beta_{opt} = L + 24$
F β F	15 .9927	25 .9925	35 .9927	45 .9927	55 .9927	65 .9927	75 .9927	$\beta_{opt} = L + 15$
M β F	2 1.00	12 .9925	22 .9925	32 .9925	42 .9925	52 .9925	62 .9925	$\beta_{opt} = L + 2$
A β F	0 .965	0 .991	8 .9924	18 .9924	28 .9924	38 .9924	48 .9924	$\beta_{opt} = L - 12$, $L > 12$
M β F	0 .91	0 .966	0 .99	7.5 .992	17.5 .992	27.5 .992	37.5 .992	$\beta_{opt} = L - 22.5$, $L > 22.5$
J β F	0 .88	0 .939	0 .99	3.5 .9914	13.5 .9914	23.5 .9914	33.5 .9914	$\beta_{opt} = L - 26.5$, $L > 26.5$
J β F	0 .88	0 .939	0 .980	6 .9916	16 .9916	26 .9916	36 .9916	$\beta_{opt} = L - 24$, $L > 24$
A β F	0 .956	0 .988	6 .9917	16 .9921	26 .992	36 .992	46 .992	$\beta_{opt} = L - 14$, $L > 14$
S β F	0 .992	10 .992	20 .992	30 .992	40 .992	50 .992	60 .992	$\beta_{opt} = L$
O β F	14 .9922	24 .9922	34 .9922	44 .9922	54 .9922	64 .9922	74 .9922	$\beta_{opt} = L + 14$
N β F	23 .9925	33 .9925	43 .9925	53 .9925	63 .9918	73 .992	83 .992	$\beta_{opt} = L + 23$
D β F	26.5 .99	36.5 .99	46.5 .991	56.5 .992	66.5 .992	76.5 .992	86.5 .992	$\beta_{opt} = L + 26.5$

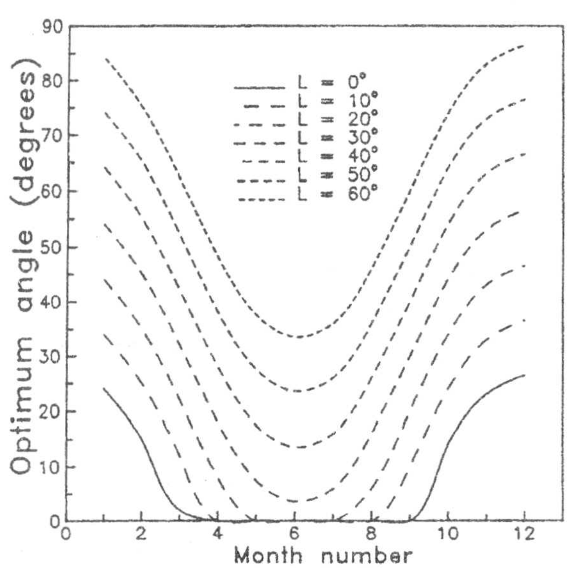

Figure 4. Monthly optimum tilt angle versus latitude angle.

The optimum tilt angle based on monthly period was computed. The results are plotted in Table (4) and in Figure (4). It is clear from the table that the value of optimum tilt angle for the same month varies linearly with the latitude angle. The last column in table 4 gives the linear equation for optimum tilt angle for that particular month. The maximum solar fractions corresponding to the value of β_{opt} are noticed to be constant and equal to 0.992 except for the value of $\beta_{opt} = 0.0$ at which the actual values for β_{opt} are negative. Due to the fact that it is impractical to design a solar collector with negative tilt angle, it is recommended to use the optimum tilt angle equal to zero in such case.. The gain in solar fraction when using a monthly adjustment over the polar one is about 6% . The question that may arise is that : Does it worth to design such variable tilt angle solar parabolic trough?. The answer for such question should be based on a real design and a economic study which is not the scope of the present work. From the data tabulated in Table 4, concerning the equations for the optimum tilt angle, a least square method is used to fit these data, and a general empirical correlation is obtained to find the optimum tilt angle at any specified latitude angle and any month number M as follows:-

$$\beta_{opt}^{month} = L + 21.5013 + 10.6852M - 9.50437M^2 + 1.41244M^3 - 0.0576741M^4 \quad (10)$$

Provided that, if the results obtained from equation 10 are negative, the values of optimum tilt angle should be assigned to zero value. The associated correction coefficient for the given equation is 0.999.

CONCLUSION

The following conclusive remarks can be mentioned:

1- The use of large tilt angle becomes necessary as the latitude angle increases.
2- The optimum yearly tilt angle is not the latitude angle, but grater than the latitude by 3.5°. (see equation 7).
3- The monthly optimum tilt angle is given by equation (8). Using the monthly tilt angle provides a solar fraction equals 0.992 which is greater than the fixed tilt angle (yearly) optimum tilt angle by about 6%.
4- The optimum tilt angle varies linearly with the latitude angle.

NOMENCLATURE

f	= focal length	m.
F	= solar fraction.	
I	= solar radiation	W/m^2
L	= latitude angle	degree..
L_1	= length of the absorber tube.	m.
M	= month number	
N	= day number	
x	= shaded length	m.
t	= time	hr

Subscript

b	= beam	

Greek Symbols

α	= solar altitude angle	degree.
β	= tilt angle	degree.
γ	= rotation angle for N-S, horizontal	degree.
δ	= declination angle	degree.
θ	= incident angle	degree.
ρ	= reflectivity of the collector surface.	
ϕ	= solar-azimuth angle	degree.

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