GRAVITY EFFECT IN PRECISE LEVELLING

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ABSTRACT

Due to the great importance of the effect of gravity on the accuracy of precise levellings, it is necessary to take such effect into consideration in order to refine its observations. Moreover it appears necessary to use real gravity data and height corrections to maintain a high standard of accuracy. This paper shows that correction for gravity effect in precise levelling is a must.

1. INTRODUCTION

The precise levelling is one of the most accurate technique used for measuring absolute heights as well as height differences. In this procedure, the geometric distances between the point on which the staff is set, and the line of sight through the levelling instrument is measured. This line of sight which is tangent to the equipotential surface indicated by the bubble in the spirit levelling tube, is a gravimetric or dynamic system. This means that the precise levelling measurement is a combined result of geometrical and gravitational instruments. The latter is affected by the gravity field of the earth which differs from point to point (Sazhina and Grushinsky 1971; Pick et. al, 1973; and Aguib, 1982).

Heights or height differences obtained from precise levelling, (without effect of gravity field), are not rigorous. The geopotential number has solved the problem of acquiring unique value for the height of each terrain point. Thus a proper treatment for heights is at hand.

2. HEIGHT DEFINITION AND HEIGHT SYSTEMS:

To compare heights, determined by the different observation techniques, some basic concepts and formulae on potential theory of height definition has to be known. A detailed discussion of these theories can be found in most textbooks (e.g. Torge, 1980; Vanicek and Krakiwsky, 1982), hereby only a short account and some recent development are outlined.

2.1 *Geopotential Numbers*

A geopotential number of a point represents the amount of work needed to lift a unit mass along any route on or inside the earth from the geoid, where the geopotential number is zero, to the point in question (e.g. Nassar, 1977; Vanicek, 1980). The reason for such choice is to make the numerical value of the geopotential numbers approximately equal to the heights of the corresponding points above sea level in meters (Youssef, 1988).

For a surface point "i" the potential difference to the geoid (arbitrary point "i_o" with potential W_o) is:-

$$C_i = -(W_i - W_o) = -\left(-\int_0^i g\,dh\right) = -\left(-\int_0^i \bar{g}\,d\bar{h}\right) \quad (1)$$

where:-

g is the actual gravity on the earth's surface measured for each increment along the levelling path from point "0" on the geoid to the terrain point "i",

dh is the differential height increment through the path from "0" to "i"

and $\bar{g}$ and $d\bar{h}$ are the corresponding quantities along the plumb line of "i" (inside the earth), Figure (1).

Practically, there is no continuous profile of "g" and "dh". But, the measurements of both "g" and "dh" can be carried out at discrete points only on the earth's surface along a specific levelling route. Thus, Eq. (1) can not be used to compute the geopotential number "C". Instead, the integral is replaced by summation. The following formula is usually used (Torge, 1986; and Aguib, 1982):

$$C_i = \sum_0^i \bar{g}_{ab} \, \Delta h_{ab} \qquad (2)$$

where:

$\bar{g}_{ab} = 1/2\,(g_a + g_b)$ observed gravity

$\Delta h = h_b - h_a$ observed heights

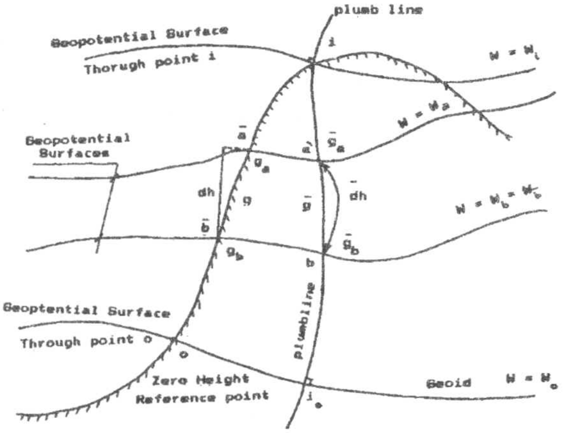

Figure 1. Actual geopotential numbers.

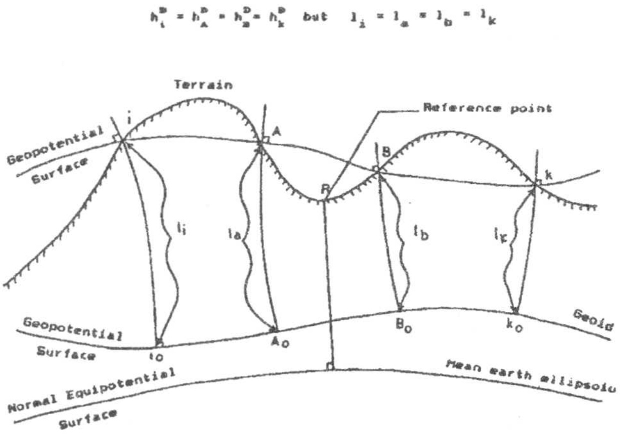

Figure 2. Dynamic height based on actual gravity.

The geopotential number is positive above the geoid and negative below it, constant everywhere on the same geopotential surface and equal zero for the geoid.

2.2 *Dynamic Heights*

The dynamic height h_i^D, of any terrain point "i" is defined as the distance between the geopotential surface through point "i" and the geoid measured along the true plumb line Figure (2). In case of using actual gravity, the dynamic height h^D is given by (Vanicek and Krakiwsky, 1982).

$$h_i^D = \frac{C_i}{G} \qquad (3)$$

Where, G is the gravity on the surface of the reference ellipsoid at the latitude of the point ϕ. and C_i is the actual geopotential number of "i" computed from Eq. (2).

The reference surface (height datum) for the dynamic system of heights is the geoid. It can be seen that h^D is expressed in length units. It does not represent the length "l_i" of the plumb line of "i" between "i" and the geoid. From Eq. (3), it is obvious that h^D is constant for all points located on the same geopotential surface (e.g. $W = W_i$, in Figure (2), whereas the geometrical separations "l" between the two equipotential surfaces $W = W_i$ and $W = W_o$ is generally different at each location i,A,B,K...). Therefore, it can be noticed that the dynamic height does not depict the actual geometric deviations of the physical surface of the earth from the geoid.

2.3 *Orthometric Heights*

The orthometric height h_i^o, based on actual gravity of a point "i" on the terrain, is the distance measured along the true plumb line of "i", Figure (3), between point "i" and the geoid, and is defined as (Krakiwsky, 1965):-

$$h_i^o = \frac{C_i}{\bar{g}_i} \qquad (4)$$

where, C_i is the actual geopotential number of "i" in g.p.u,

and $\bar{g}_i$ is the mean actual gravity along the true plumb line of "i" from the geoid terrain in k gals, for h_i^o in meters.

The mean gravity $\bar{g}_i$ can not be determined rigorously from the theoretical point of view, because the actual mass density distribution with the earth (i.e. along the true plumb line of "i") is not known. Different simplifications are used for this purpose. The **Helmert** orthometric height is in practice the most widely used, which is given by Vanicek and Krakiwsky, (1982) as:

$$h_i^H = \frac{C_i}{g_i^H} \qquad (5)$$

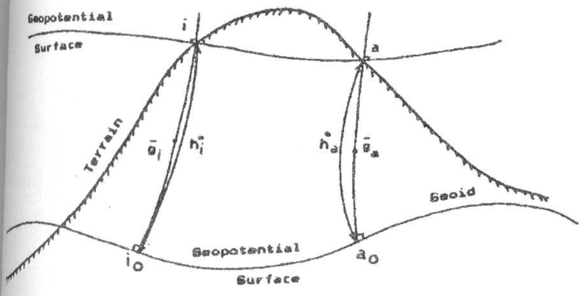

Figure 3. Orthometric height based on actual gravity.

where, g_i^H is the **Helmert** approximation to the mean value of actual gravity along the true plumb line of "i" between the geoid and the terrain. **Helmert** formula for $\bar{g}_i^H$ reads:

$$\bar{g}_i^H = g_i + 0.0424\, h_i \qquad (6)$$

where g_i is the observed surface gravity on the terrain at "i". The unit of the second term is mgals for h_i in meters, and h_i is the observed height of "i" above sea level usually deduced from the precise levelling results before adjustment.

2.4 *Normal Heights*

The normal height h_i^N of terrain point "i" is defined (Krakiwsky, 1965; and vanicek, 1962) as:

$$h_i^N = \frac{C_i}{\overrightarrow{\gamma}_i} \qquad (7)$$

where, $\bar{\gamma}_i'$ is the mean value of normal gravity along the normal plumb line of "i", Figure (4), between the mean earth ellipsoid (point "$\bar{i}_o$"), and the point "$\bar{i}$" (inside the earth below "i") where the normal potential U_i has the same value as the actual geopotential W_i at the corresponding point "i" on the terrain.

Concerning the zero-height reference surface for the system of normal height h^N, there are two alternatives. The first is the approach used in practice where h^N is measured from the physical surface of the earth along the corresponding normal plumb line (to point $\bar{i}_o$, as in Figure (4)). Consequently, the locus of h^N defines the height datum which is a mathematical surface (not generally an equipotential surface) known as "**quasigeoid**". The **quasigeoid** was introduced by **Molodenskii** in the late 1940's (Nassar, 1977). Hence the heights referred to it are known as "**Molodenskii's** normal" $h^{N,M}$ Figure (5).

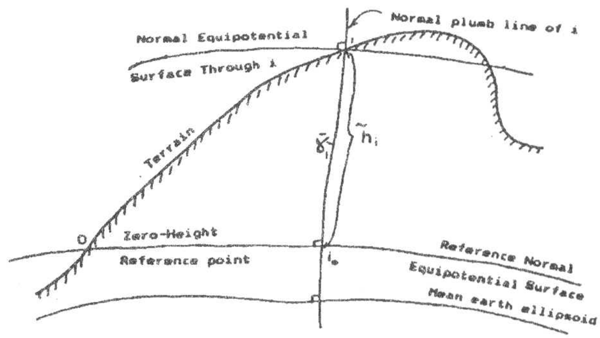

Figure 4. Orthometric height based on normal gravity.

The second alternative is a theoretical approach based on **reckoning** h^N from the surface of the mean earth ellipsoid, as the height datum, along the normal plumb line (to point $\bar{i}$, as in Figure (5). Thus the locus of h^N above the ellipsoid defines another mathematical surface.(not generally an equipotential surface) known as "**telluroid**". This theoretical approach has been followed extensively by **Hirvonen** (1960), and hence the normal heights referring to the ellipsoid and generating the **telluroid** are usually known as "**Hirvonen's** normal heights" $h^{N,H}$ Figure (5).

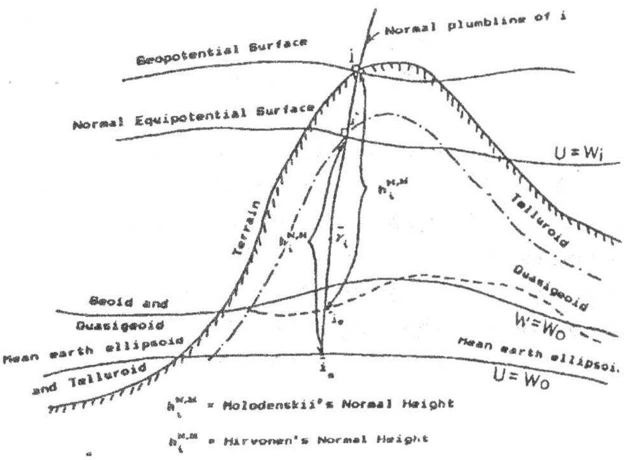

Figure 5. Molodenskii and Hirvonen normal heights based on actual gravity.

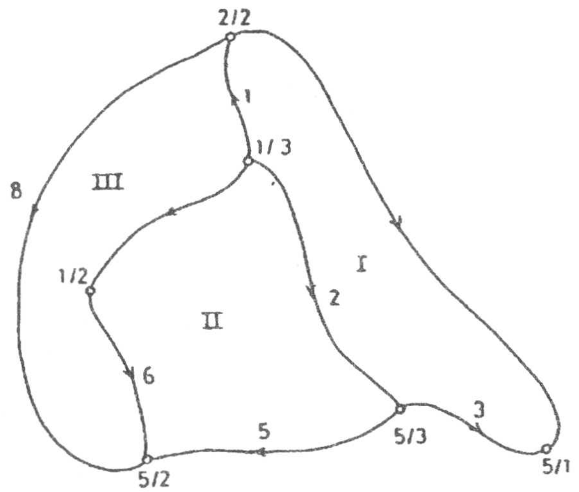

Figure 6. The measurement of Kalabsha levelling network carrying in 1989.

It is worthwhile stating that normal heights are considered in practice as **"scientific heights"**. However, the other two, dynamic and orthometric are known as **"practical heights"**, since they use the geoid as their height datum. In the present paper, the heights will be focussed only on practical height systems, especially the **"Helmert orthometric height"**.

3- OBSERVATIONS AND RESULTS OF GRAVITY FOR KALABSHA LEVELLING NETWORK

The gravity measurements for **Kalabsha** levelling network is chosen to be an example for carrying a numerical application of the effect of gravity field on the precise levelling network. This net as shown in Figure (6) is divided into three loops. The values of observation of the actual gravity, the heights of benchmarks, and the distances between benchmarks, are given in Table (1).

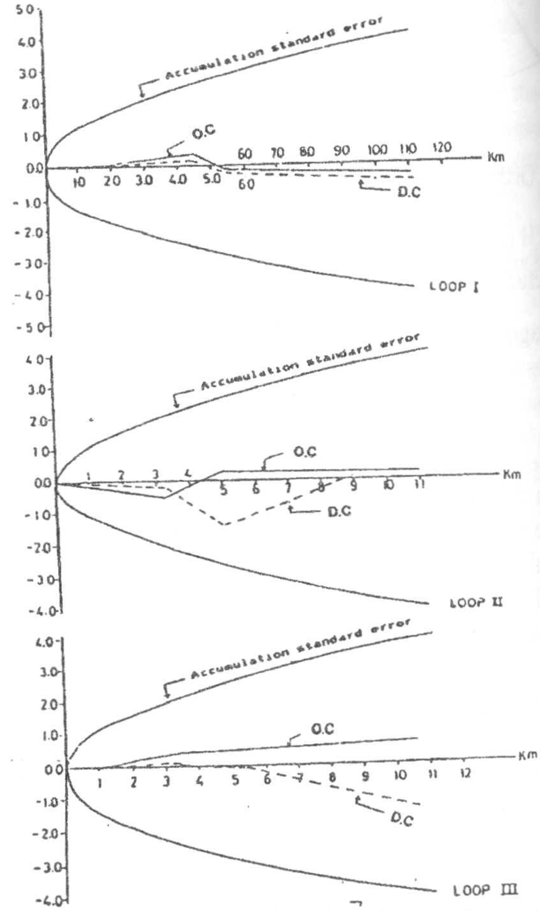

Figure 7. Accumulated orthometric and dynamic corrections for Kalabsha levelling network.

The reference gravity value "G" is usually taken as the normal gravity "γ" value computed on the reference ellipsoid at a certain reference latitude. This latitude is 23° 35′ for the area of **Kalabsha**.

The formula used here to compute γ at ϕ_R is the international formula of, 1967 (Vanicek and Krakiwsky, 1982). This formula was expressed as:

$$G = \gamma_e \, (1 + k_1 \sin^2 \phi + k_2 \sin^2 2\,\phi)$$

Table 1. The observations of actual gravity values for Kalabsha levelling network.

No. of loop	points no	heights of points m	gravity values mgal	diff. heights m	dist km
I	22	211.73	978 791.28		0
	13	210.84	978 789.63	-0.89	1.3
	53	213.47	978 701.56	2.63	4.6
	51	217.39	978 790.60	3.92	5.6
	22	211.73	978 791.28	-5.66	11.1
II	13	210.84	978 789.63		0
	53	213.47	978 791.60	2.63	3.3
	52	229.82	978 786.85	16.35	5.0
	12	212.35	978 788.84	-17.47	7.0
	13	210.84	978 789.63	-1.51	9.0
III	22	211.73	978 791.28		0
	13	210.84	978 789.63	-0.89	1.3
	12	212.35	978 788.84	1.51	3.3
	52	229.82	978 786.85	17.47	5.3
	22	211.73	978 791.28	-18.09	10.6

Table 2. Orthometric and dynamic corrections based on actual gravity.

No. of loop	B.M	h m	g mgal	Δh m	$\bar{g}$ mgal	DC* mm	A** mm	B+ mm	OC_{ij} mm
I	22	211.73	978 791.3		978 790.4	0.0688	-1.4250	14.550	0.37
	13	210.84	978789.6	-0.89	978 790.6	-0.2029	-14.550	1.4290	-0.47
	53	213.47	978791.6	-2.63	978 791.1	-0.3005	-1.4290	1.4730	0.14
	51	217.39	978 790.6	3.92	978 790.9	0.4350	-1.4730	1.4250	-0.04
	22	211.73	987 791.3	-5.66					
II	13	210.84	789.6	2.63	978 790.6	-0.2029	-14.550	14.250	-0.47
	53	213.47	791.6	16.35	978 789.2	-1.2850	-14.250	16.320	0.74
	52	229.82	786.8	-17.3	978 787.9	1.3970	-16.320	14.810	-0.11
	12	212.35	788.8	-1.5	978 789.3	0.1186	-14.810	14.550	-0.14
III	22	211.73	791.3						
	13	210.84	789.6	-0.89	978 790.4	0.06880	-14.250	14.550	0.37
	12	212.35	788.8	1.5	978 789.2	-0.1186	-14.550	14.810	0.16
	52	229.82	786.6	17.4	978 787.9	-1.3970	14.810	16.320	0.13
	22	211.73	791.3	-18.0	978 789.1	1.4240	-16.320	14.250	-0.65

$$DC^{*} = \frac{\bar{g}_{ij} - G}{G} \Delta h_{ij}, \qquad A^{**} = \frac{g_i - G}{g\, h_i}, \qquad B^{+} = \frac{g_i - G}{G} h_j$$

Table 3. Accumulated gravity correction for Kalabsha levelling network.

No. loop	B.M	Distance	$\sigma\ \Delta h$	OC	Acc. OC	Acc. DC
	22	0.0	0.00		0.0	0.0
	13	1.3	1.37	0.37	0.37	0.07
I	53	4.6	2.57	-0.47	-0.10	-0.13
	51	5.6	2.84	0.14	-0.04	-0.43
	22	11.1	4.00	-0.04	0.00	0.0
	13	0.0	0.00		0.00	0.00
	53	3.3	2.18	-0.47	-0.47	-0.20
II	52	5.0	2.68	0.74	0.27	-1.49
	12	7.0	3.17	-0.11	0.16	0.09
	13	9.0	3.6	-0.14	0.02	0.21
	22	0.0	0.00		0.00	0.00
	13	1.3	1.37	0.37	0.37	0.07
III	12	3.3	2.18	0.16	0.50	-0.05
	52	5.3	2.76	0.13	0.66	-1.44
	22	10.6	3.90	-0.65	0.01	-0.02

where: γ_e = 978031.8 mgal ;k_1 = 0.005 3024;
k_2 = - 0.000 0059

which gives G = 978 8660.12 mgal

3.1. *Computation of Orthometric and Dynamic Corrections for Kalabsha Levelling Network*

Firstly, the orthometric correction based on actual gravity "OC", is computed from the following equation:

$$OC = \frac{\bar{g}_{ij} - G}{G} \Delta h_{ij} + \frac{\bar{g}_i - G}{G} h_i - \frac{\bar{g}_j - G}{G} h_j$$

where: $\bar{g}_{ij}$ = 1/2 (g_i + g_j) , $\Delta h = h_j - h_i$;

$\bar{g}_i$, $\bar{g}_j$ are **Helmert** mean actual gravity value at the midpoint of the corresponding plumb line, and computed from Eq. (6), e.g. for point "i" one can write:

$$\bar{g}_i = g_i + 0.0424\ h_i$$

Secondly, the dynamic correction based on actual gravity "DC", is computed as follows:

$$DC_{ij} = \frac{g_{ij} - G}{G} \Delta h_{ij}$$

Also, the accumulated orthometric and dynamic correction based on actual gravity values, are computed.

According to Nassar, (1977) the accumulated standard deviation of the height difference over a distance "S" along a levelling line can be computed from:

$$\sigma_{o,i} = 1.2\sqrt{S_{i,o}(km)} \qquad (8)$$

Where, $S_{i,o}$ is the sum of the lengths of the segments of the levelling line up to the benchmark "i", starting form the initial point "o" of the line.

One can analyses the above formula (8) as follows: The behaviour of random errors is very well known, and statistically follows the **Gauss** probability curve. On the other hand, the behaviour (accumulation or cancellation) of systematic errors is unpredictable. For this reason, the international recommendation is stated as follows (Aguib, 1982): Any kind of systematic error whose magnitude is equal to or larger than 10% of the corresponding accumulated random errors must be taken into account. Dealing with precise levelling and gravity, one can say that if the value of orthometric correction "OC" or dynamic correction "DC" equals or exceeds 0.12 mm/km, it should be taken into account.

The results of this section are presented in both, tabular and graphical forms. The main idea here is to compare both accumulated orthometric and dynamic corrections along the loops with the corresponding accumulated random errors associated with precise levelling.

It can be seen from Table (2), (3) and (Figure 7) that the "DC" ranges between a very small fraction of a millimeter to 1.397 mm, and the "OC" ranges between 0.11 to 0.74 mm. The standardized value of 1.397 mm will be: $1.397/\sqrt{2.0} \cong 1.0$ mm/km for the dynamic correction, and the orthometric correction is:

$0.74 / \sqrt{2.0} = 0.52$ mm/km.

Obviously, This values is about 9 times greater than the significant criterion of 0.12 mm / km for the "DC"; and about 5 times for "OC".

4. CONCLUSION AND RECOMMENDATION

From the results of the analysis as well as investigation of the **Kalabsha** network obtained data, the following points may be concluded:-

The orthometric correction (OC) is seemingly very significant as it causes large distortions in **heights** of benchmarks along levelling routes especially when these routes are large and cover large areas. The (OC) reached an apparent value for **Kalabsha** levelling network.

Moreover, the dynamic correction (DC) is also very significant and reached a considerable value.

Undoubtedly the above results show that the gravity correction applied to precise levelling is of a great effect. Hence, it has to be taken into account in order to arrive to a rigorous and unique definition of heights based on precise levelling, especially for large levelling routes where a change in gravity can be present. Such a step would be highly requested and essential.

Keeping this effect in mind is recommended for high precision routine levelling works, as it enables refinement of their observations to be taken into account more accurately, thereby improving the accuracy of levelling substantially.

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