FEED WATER TEMPERATURE REGULATION IN A MARINE REGENERATIVE STEAM CYCLE

M. Hanafi
Marine Eng. & Naval Arch. Dept.,
Faculty of Engineering, Alexandria University,
Alexandria, Egypt.

M. Mosleh
Ship Engineering Dept.,
Faculty of Engineering, Suez Canal Univ.,
Port Said, Egypt.

ABSTRACT

Dynamic energy balances for complex transient heat transfer process in closed type feed water heaters w.r.t. command and perturbations signals are presented. Digital computational assessment in parametric study of the dynamics of feed water temperature regulation is carried out. Stability problems encountered from large transportation lags and time delays as well as time and frequency domains analysis are investigated.

NOMENCLATURE

A	Circumferntial tube area subjected to heat transfer (m^2)
A_1	Inside peripheral tube area per unit length for one tube (m^2/m)
A_2	Outside peripheral tube area per unit length for one tube (m^2/m)
C	Total capacity of steam vapour in shell, tube wall and shell (KJ/K)
C_{sh}	Thermal capacity of shell (KJ/K)
C_v	Thermal capacity of steam vapour in shell (KJ/K)
C_w	Thermal capacity of tube wall (KJ/K)
c_f	Specific heat of feed water (KJ/kg.K)
c_{sh}	Specific heat of shell (KJ/kg.K)
c_w	Specific heat of tube wall (KJ/kg.K)
d_i	Inner tube diameter (m)
d_o	Outer tube diameter (m)
d_s	Inner shell diameter (m)
d_{so}	Outer shell diameter (m)
F'	Product of kg water per meter length per tube by its velocity = feed water rate (kg/s)
$\bar{F}$	Normal flow of feed water (kg)
$\bar{F}'$	Rate of normal of water (kg/s)
F_m	Mass flow rate of feed water per unit free area (kg/m^2 hr)
F_s	Rate of bled steam flow (kg/s)
FWT	Feed water temperature inlet to the boiler (°C)
$\bar{G}$	Boiler evaporation (kg/hr)
G(S)	Forward path transfer function
H(S)	Feedback path transfer function
h_1	Inside film coefficient of heat transfer ($KJ/m^2.s.K$)
h_2	Outside film coefficient of heat transfer ($KJ/m^2.s.K$)
K	Sensor's gain (-)
K_o	Overall gain = $K_p.K_v/K$ (mm/K)
K_p	Controller's gain (mm/K)
K_v	Control valve gain (-)
LMTD	Logarithmic mean temperature difference between condensing steam and feed water in heater (°C)
M_f	Feed water mass per unit length per tube (kg/m)
M_p	Percentage maximum overshoot of feed water outlet temperature deviation (-)
M_v	Steam mass enclosed in shell (kg)
M_w	Mass of tube wall per unit length per tube (kg/m)
n	Number of tubes of feed water heater (-)
P_b	Bled steam pressure (Pa)
r	$\Delta\bar{F}/\bar{F}$ (-)
S	Laplace operator (1/s)

T	Dead time of sensor (s)
T_1,T_2 and T_{12}	delay times of heat transfer process (s)
t	time (s)
U	Overall heat transfer coefficient ($KJ/m^2.s.k$)
$\bar{U}$	Normal steady state value of overall heat transfer coefficient ($KJ/m^2.s.k$)
v	Feed water velocity through heat exchanger (m/s)
w	Specific weight of water (kN/m^3)
X	Displacement of control valve (mm)
x	Length of feed water heater (m)
λ	Heat lost by steam during condensation (KJ/kg)
Δ...	Change in ... (-)
ρ_f	Density of feed water (kg/m^3)
ρ_{sh}	Density of shell metal (kg/m^3)
ρ_w	Density of tube metal (kg/m^3)
ρ_s	Density of steam enclosed in shell (kg/m^3)
θ_b	Bled steam saturation temperature in steady state (°C)
θ_f	Dynamic feed water temperature outlet of heat exchanger (°C)
$\delta\theta_r$	Steady state temperature rise of feed water through heater (°C)
θ_s	Steam saturation temperature (°C)
θ_w	Dynamic tube metal temperature (°C)
τ	Time delay in controller (s)
τ_d	Time delay in sensor (s)
τ_1	Dead time in the heat transfer process dynamics (s)
τ_v	Time delay in control valve (s)
ω_π	Phase cross-over frequency (rad/s)

INTRODUCTION

One of the complex problems in analyzing the dynamics of control systems is the study of heat exchangers dynamics for temperature regulation.

The heat transfer problem incorporated in the process dynamics comprises a fluid temperature function not only in time but also in the length of heat exchanger; a matter which yields a transportation lag in the plant beside the inevitable transportation lag incorporated in the temperature sensor. In addition to the excessive time delays in both sensor and control valve, the thermal capacities of condensate, bled steam enclosed in shell, tube walls and the shell, together wit inside and outside film coefficients of heat transf the conductivity of tubes will result in conside large time delays characterizing the dynamic beha of the automatic control loop.

Moreover, the existence of multi-perturb namely the variation in the fluid temperature in the heat exchanger and the fluctuation of boiler lo meet the propulsion requirements results in addi complicated complexity to the problem. Dynami heat exchangers are extremely varied dependin whether compressible or incompressible fluids inside or outside the tubes.

After establishing conventional heat transfer sc [1], the evolution of the dynamic analysis of exchangers took place in the branch of pro dynamics in chemical engineering.

Among the pioneer research workers in this were Gould, Cohen, Campbell, Catheron, De Lees, Koppel and Fricke [2-9]. While [5,6,7] ca out experimental research work on different typ heat exchangers, theoretical mathematical studies presented by the others.

Meanwhile, further advances were developed in field by reputed scientists of mechanical and co engineering. Some of the most famous investigato the domain of control of steam installations Oppelt, Quack, Profos, Doležal,Varcop and Isern [10-23] to whom is attributed the merit of the progress achieved in the art of regulation of s power plants components. Additional contribu particularly in mathematical analysis of heat exchan are ascribed to Masubuchi and Yang [24,25].

The aim of this paper is a parametric study fo dynamic analysis of closed type feed water heaters reality, two sources of perturbation affect the co loop. The first one is the variation in feed w temperature inlet to the heat exchanger attributed change in condenser vacuum caused by a transitio ship's course from hot to rather cold regions or versa. The second external disturbance is originate the change in amount of condensate passing thro the heaters caused by boiler load variation to adapt propulsion requirements. The discussion perturbation influences is beyond the scope of study and a separate research should be dedicated analyzing this control system when subjected to aforementioned effectual external disturbances.

The regenerative steam cycle under discussion includes either two or three heaters corresponding to the standard marine feed water temperatures 115.5°C (240 °F) and 160°C (320°F) respectively. The concept of equal temperature rise distribution among the heaters is adopted. The analysis in this paper is restricted to the first low pressure closed type feed water heater-assumed a single pass heater-as an illustrative demonstration of the problem of feed water temperature regulation.

In order not to violate the validity of the linearized obtained model only slight variations in condenser vacuum and amount of condensate are assumed. The vacuum is changed from 0.0478 bar abs. (28" 1/2 Hg vacuum) to 0.0647 bar abs (28" Hg vacuum) due to regional environmental deviations in sea cooling water temperatures.

Part and overload boiler evaporations are considered 90% and 110% respectively of the normal load.

DYNAMIC SYSTEM OF EQUATIONS

Considering the marine regenerative steam power plant shown in Figure (1), with the known control conventions indicated on each system, the temperature control system under discussion in this paper is the low pressure single pass closed type feed water heater enclosed with the dotted line,

Control schemes often used for heat exchangers are demonstrated in Figure (2). The scheme adopted in this research is that shown in Figure (2-a). The pneumatic controller illustrated in Figure (3) and based on the flapper-nozzle concept with double seat valve and minor feedback having P_1 control property is used [26]. A concise description of this control valve is as follows: The increase in feed water temperature outlet will cause the increase in gas temperature inside the bulb and consequently the increase of its pressure. This will lead to the expansion of Bourdun tube and the augmentation of the gap between the flapper and nozzle. The operating air pressure under the primary diaphragm will decrease pulling downwards the double seat valve decreasing the air leakage from vent. The control air pressure will increase and the steam valve operated by this air is partially closed. Concerning the minor feedback the air pressure under the feedback below will increase and the nozzle approaches the flapper plate diminishing the air gap, thus transforming the control property of the controller from I_0 to P_1, whereas the bulb and its tube connection as temperature measuring device incorporates both dead and delay times.

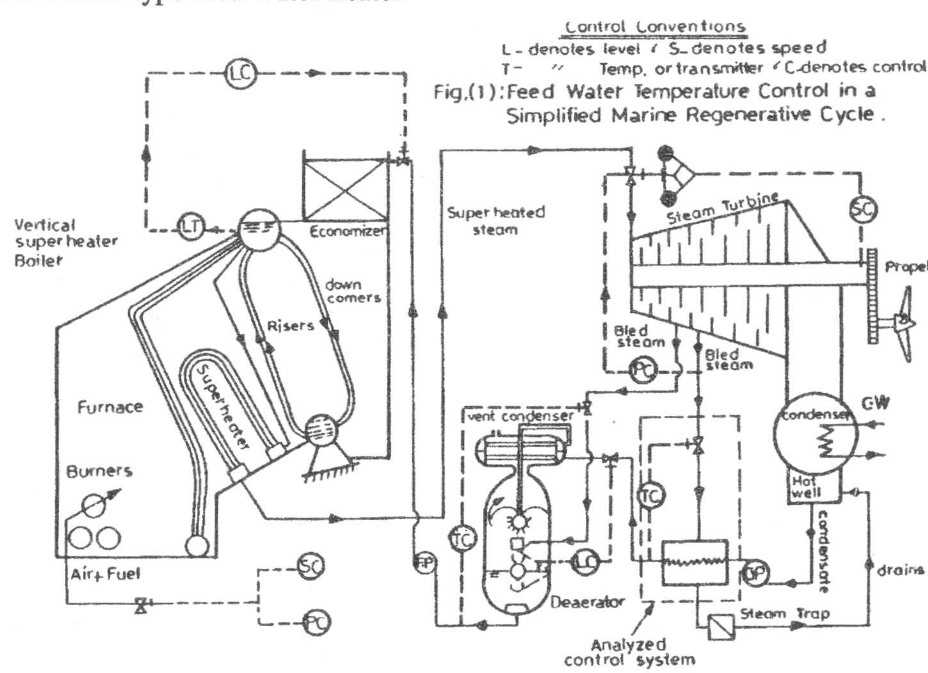


Figure 1. Feed water temperature control in a simplified marine regenerative cycle.

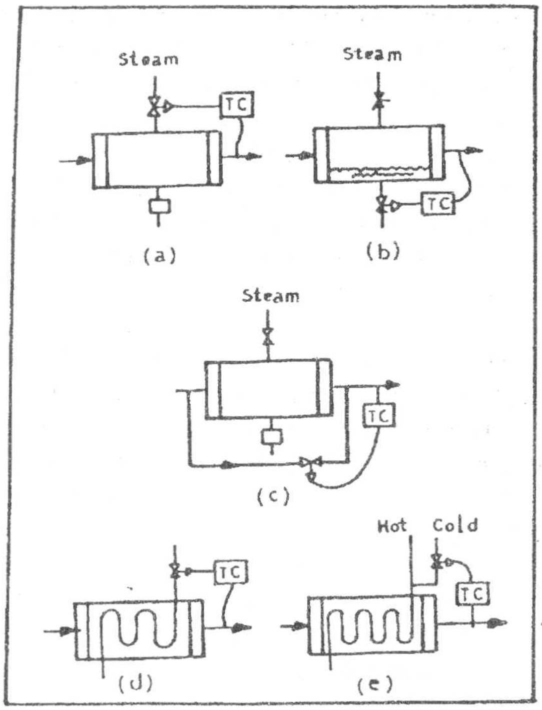


Figure 2. Control schemes for heat exchangers (a) throttle steam flow, (b) throttle condensate (c) bypass method, (d) throttle liquid flow (e) regulate inlet temperature.

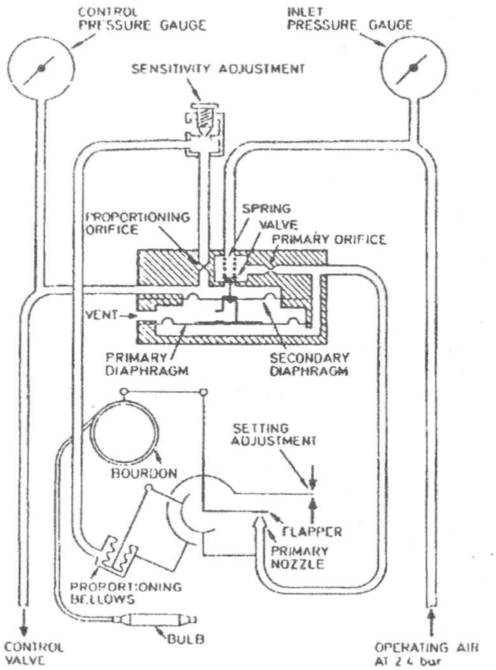


Figure 3. Pneumatic dialset controller.

Materials and dimensions of feed water heate discussion are illustrated in the schematic d shown in Figure (4), [27].

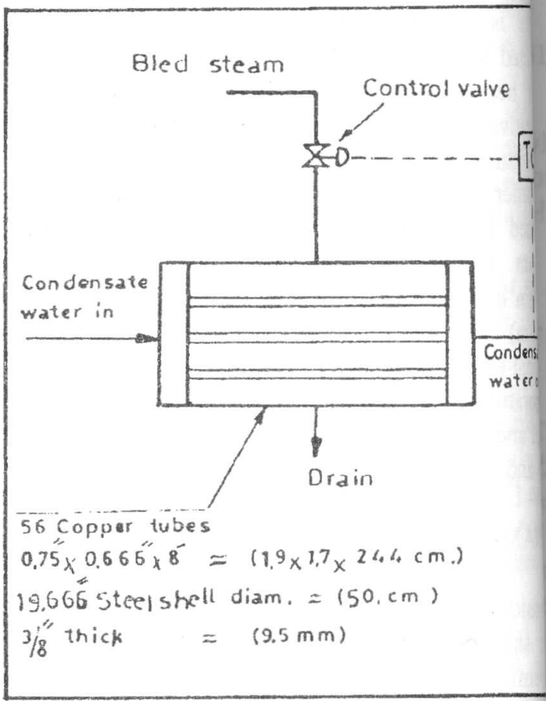


Figure 4. Dimensions of analyzed single-pass water heater.

Concerning the response to changes in temperature θ_s, the usual steady state assumptions axial conduction, no backmixing, and constant properties are made. It is also necessary to assu wall resistance and no capacity in the condensate to limit the equations to second order. If signifi the wall resistance could be split and added to th film resistances, and the condensate film capacity be added to that of the wall [27].

The water temperature θ_f is a function in both t and tube length x, i.e:

$$\theta_f = \theta_f\,(t, x)$$

then

$$\Delta\theta_f = \frac{\partial\theta_f}{\partial t}\Big|_{x=\text{const.}} .\,\Delta t + \frac{\partial\theta_f}{\partial x}\Big|_{t=\text{const.}} .\,\Delta x$$

The energy balance for the water stream is wri for a length dx of a single tube:

Since the rate of heat added to the feed water equals the heat transferred through the tube wall temperature to the water it follows [27]:

$$M_f \cdot \frac{dx}{dt} \cdot c_f \cdot \Delta\theta_f = h_1 (A_1 \cdot dx)(\theta_w - \theta_f) \qquad (3)$$

Substituting equation (2) in equation (3) it follows

$$M_f \cdot c_f (\frac{\partial \theta_f}{\partial t}) dx + F' \cdot c_f \cdot \frac{\partial \theta_f}{\partial x} dx = h_1 A_1 \cdot dx (\theta_w - \theta_f) \qquad (4)$$

where

$$F' = M_f \cdot \frac{dx}{dt} = M_f \cdot v$$

In order to write the energy balance for the wall, the energy stored in the wall equals the difference between the heat transferred from the steam to the wall and the heat transferred from the wall to the feed water:

$$M_w c_w (\frac{\partial \theta_w}{\partial t}) dx = h_2 A_2 \, dx (\theta_s - \theta_w) - h_1 A_1 \, dx (\theta_w - \theta_f) \qquad (5)$$

Introducing the following time constants to equations (4) and (5), it could be written

$$T_1 = \frac{M_f c_f}{h_1 A_1} = R_1 C_1,$$

$$T_2 = \frac{M_w c_w}{h_2 A_2} = R_2 C_2,$$

and

$$T_{12} = \frac{M_w c_w}{h_2 A_1} = R_1 C_2$$

where R_1, R_2, C_1 and C_2 denote thermal resistances and capacities. Equations (4) and (5) are reduced to:

$$T_1 (\frac{\partial \theta_f}{\partial t}) + v T_1 (\frac{\partial \theta_f}{\partial x}) = \theta_w - \theta_f \qquad (6)$$

and

$$T_2 (\frac{\partial \theta_w}{\partial t}) = (\theta_s - \theta_w) - \frac{T_2}{T_{12}} (\theta_w - \theta_f) \qquad (7)$$

According to [2,3,4], the partial differential equations (6) and (7) are converted to ordinary differential equations by taking the Laplace transform with respect to time.

The variables θ_f and θ_w in the following transformed equations represent deviations from the normal values at any point along the exchanger:

$$T_1 S \theta_f + v T_1 (\frac{d\theta_f}{dx}) = \theta_w - \theta_f \qquad (8)$$

$$T_2 S \theta_w = \theta_s - \theta_w - \frac{T_2}{T_{12}} (\theta_w - \theta_f) \qquad (9)$$

Eliminating θ_w gives the first order equation namely:

$$\frac{v}{a} (\frac{d\theta_f}{dx}) + \theta_f = \frac{b}{a} \theta_s \qquad (10)$$

where $a = \dfrac{(T_1 S + 1)(T_{12} T_2 S + T_{12} + T_2) - T_2}{T_1 (T_{12} T_2 S + T_{12} + T_2)}$

i.e.

$$a \simeq a_1 S + a_o$$

resulting from the quotient of numerator by denominator of (a)
where $a_1 = 1$

and $\dfrac{b}{a} = \dfrac{1}{(T_1 T_2) S^2 + (T_1 + T_2 + T_1 T_2 / T_{12}) S + 1}$ or,

$$\frac{b}{a} = \frac{1}{\alpha S^2 + \beta S + 1}$$

The solution of equation (10) for the boundary condition $\theta_f = 0$ at $x = 0$ is the step response of the first order system:

$$\frac{\theta_f}{\theta_s} = \frac{b}{a} (1 - e^{-ax/v}) \qquad (11)$$

The term $x/v = \tau_l$ is the time for water to flow through the tubes, which is the transportation lag. Hence:

$$\frac{\theta_f}{\theta_s} = \frac{1}{\alpha S^2 + \beta s + 1}[1 - e^{-(a_1 S + a_o)\tau_l}] \qquad (12)$$

Bode plots of equation (12) have oscillatory nature particularly at the existence of large transportation lag [2,3,4,27].

To introduce the vapour capacity enclosed in shell, the tube wall and the shell capacity as well, the energy balance for the tubes and shell could be summarized as the difference between heat released from steam and the heat transferred to the feed water is stored in the tube wall shell and vapour capacities, i.e.

$$(C_v + C_w + C_{sh})\frac{d\theta_s}{dt} = (K_v F_s \lambda)X - UA(\theta_s - \frac{\theta_{fi} + \theta_f}{2}) \qquad (13)$$

where,

$$C_v = \frac{M_v}{P}\lambda(\frac{\partial P}{\partial \theta_s}),$$

$$M_v = \frac{\pi}{4}\left[d_s^2 - n.d_o^2\right](x).\rho_s \qquad \text{and}$$

P = The steam pressure in shell approximated to about 3/8 the bled steam pressure [27].

For initial feed water temperature $\theta_{fi} = 0$, the transformation of equation (13) to the Laplace domain yields:

$$(C\ S + UA)\ \theta_s = (K_v\ F_s\ \lambda)\ X + \frac{UA}{2}\theta_f \qquad (14)$$

where, $C = C_v + C_w + C_{sh}$

As discussed before, the transfer function of the sensor has the form:

$$\frac{Ke^{-TS}}{1 + \tau_d S} \qquad (15)$$

A transportation lag of 1 second is assumed due to the location of the sensor distant from the condensate outlet.

The reason for selecting a gain K is to reach error of about 2% and this gain should be nume compensated in the forward path. The transfer fu for the controller could be written as:

$$\frac{K_p}{1 + \tau S}$$

Assuming only friction forces in the control while neglecting its inertia forces, the transfer fu relating the control valve to the con displacements is

$$\frac{1}{1 + \tau_v S}$$

Combining equations (12), (14), (15), (16) an results in the block diagram shown in Figu indicating the transfer functions, unit step resp and control properties of each minor block.

It is emphasized that in the following num computations the role of boiler load and inlet water temperature as external disturbances discarded. Parametric analysis at different ope conditions will be dynamically considered. In concerns the theoretical treatment of the control s response to a change in inlet condensate temperat constant steam temperature [27] could be deduced equation (10) for $\theta_s = 0$ and $\theta_f = \theta_F$ = constan = 0 as:

$$\theta_f = \theta_F . e^{-(a_1 S + a_o)\tau_l}$$

or

$$\theta_f = K_1 . e^{-a_1 \tau_l S}$$

where $K_1 = \theta_F .\ e^{-a_o \tau_l}$

Considering the response of the control syste changes in the amount of condensate water:

If the flow is increased suddenly, there is an imme change in the heat-transfer coefficient throughou exchanger. However, the percentage change in ov coefficient is less than the percentage increase in f and so the exit temperature must drop, and this dr spread out over a time interval of roughly τ_l seco the time for water to flow through the exchanger.

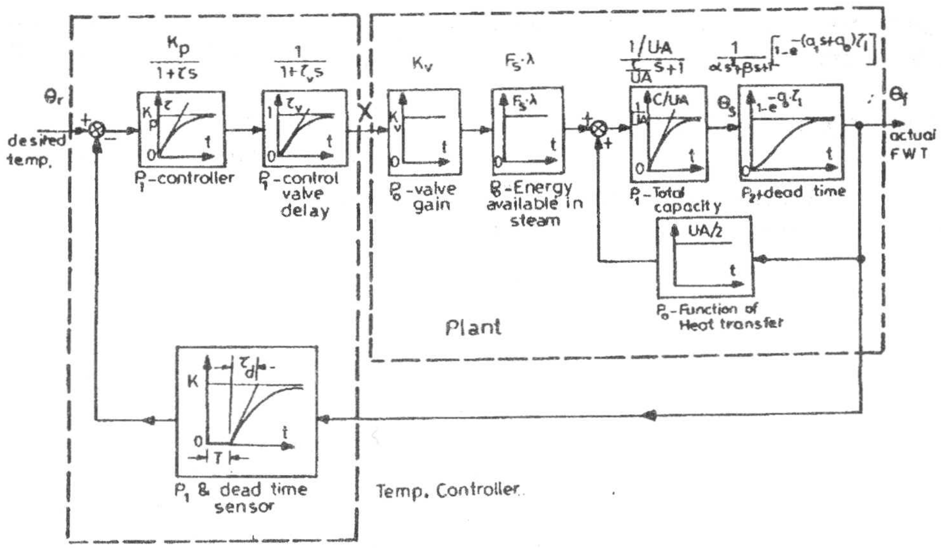


Figure 5. Block diagram for feed-water temperature regulation without disturbances.

If negligible wall capacity and a linearized relationship between the overall coefficient and the flow [8] are assumed,

then $$U = \bar{U}(1+\beta_1 r)$$

where, $r = \frac{\Delta \bar{F}}{\bar{F}}$ = change in flow /normal flow

The value of the factor β_1 ranges between 0 and 0.8 depending on the fraction of the total resistance in the inside film.

The heat balance for the feed water is

$$M_f c_f\left(\frac{\partial \theta_f}{\partial t}\right) + \bar{F}'(1+r)c_f\left(\frac{\partial \theta_f}{\partial x}\right) = \bar{U}A(1+\beta_1 r)(\theta_s - \theta_f) \quad (19)$$

or $$T'\left(\frac{\partial \theta_f}{\partial t}\right) + \bar{v}T'(1+r)\left(\frac{\partial \theta_f}{\partial x}\right) = (1+\beta_1 r)(\theta_s - \theta_f) \quad (20)$$

where: $\bar{v}$ is the normal velocity of water and T' is a time constant equals $M_f c_f/\bar{U}.A$.

Since $$\theta_f = \bar{\theta}_f + \Delta\theta_f \quad (21)$$

where $\bar{\theta}_f$ is a function of length and bearing in mind that the steam temperature is kept invariant i.e $\theta_s = \bar{\theta}_s$. Substituting equation (21) into equation (20) gives,

$$T'\left(\frac{\partial \Delta\theta_f}{\partial t}\right) + \bar{v}T'(1+r)\left(\frac{\partial \bar{\theta}_f}{\partial x} + \frac{\partial \Delta\theta_f}{\partial x}\right) = (1+\beta_1 r)(\bar{\theta}_s - \bar{\theta}_f - \Delta\theta_f) \quad (22)$$

Neglecting the terms $r(\partial\Delta\theta_f/\partial x)$ and $\beta_1\ r\ \Delta\theta_f$ and taking the Laplace transform leads to

$$(T'S+1)\Delta\theta_f + \bar{v}T'\left(\frac{d\Delta\theta_f}{dx}\right) + \bar{v}T'(1+r)\frac{d\bar{\theta}_f}{dx} = (1+\beta_1 r)(\bar{\theta}_s - \bar{\theta}_f) \quad (23)$$

Using the steady-state relationship $\bar{\theta}_s - \bar{\theta}_f = (\theta_s - \theta_{fi})e^{-\tau_1/T'}$, we obtain:

$$(T'S+1)\Delta\theta_f + \bar{v}T'\left(\frac{d\Delta\theta_f}{dx}\right) = -(\theta_s - \theta_{f_i})r(1-\beta_1)e^{-\tau_1/T'} \quad (24)$$

The solution for $\Delta\theta_f = 0$ at $x = 0$ is the transfer function for flow rate changes, i.e.

$$\frac{\Delta\theta_f}{r} = \frac{-(\bar{\theta}_s - \bar{\theta}_{fi})(1-\beta_1)e^{-\tau_1/T'}(1-e^{-\tau_1 \cdot s})}{T'S} \quad (25)$$

At the termination of the mathematical simulation, equations from (18) through (25) which represent the description of the response of the regulated feed water temperature to external disturbances of inlet feed water temperature and boiler evaporation will not be considered in the digital solutions of this work.

THE DYNAMIC MODEL UNDER DISCUSSION

Referring to the block diagram illustrated in Figure (5), both the open and closed-loops transfer functions of the control system are deduced as:

O.L.T.F=G(s).H(s)=

$$\frac{(K.Kp.K_v.F_s.\lambda/UA).e^{-TS}[1-e^{-(a_1 s+a_o)\tau_1}]}{(1+\tau S)(1+\tau_v S)(1+\tau_d S)\left\{(\frac{C}{UA}S+1)(\alpha S^2+\beta S+1)-\frac{1}{2}[1-e^{-(a_1 S+a_o)\tau_1}]\right\}} \qquad (26)$$

$$C.L.T.F.=\frac{G(s)}{1+G(s)H(s)}=$$

$$=\frac{(K_p K_v F_s.\lambda/UA).(1+\tau_d S).[1-e^{-(a_1 s+a_o)\tau_1}]}{(1+\tau S)(1+\tau_v S)(1+\tau_d S)\left\{(\frac{C}{UA}S+1)(\alpha S^2+\beta S+1)-\frac{1}{2}[1-e^{-(a_1 S+a_o)\tau_1}]\right\}} \cdot \frac{1}{+\frac{KK_pK_vF_s\lambda}{UA}.e^{-TS}[1-e^{-(a_1 s+a_o)\tau_1}]} \qquad (27)$$

It is to be noted that the transportation lag is expanded by the padé second approximation, i.e.

$$e^{-\mu s} \simeq \frac{1-0.5\mu S+0.0833\mu^2 S^2}{1+0.5\mu S+0.0833\mu^2 S^2} \qquad (28)$$

where μ is any value denoting dead time.

After hard mathematical manipulations equations (26) is transformed to a quotient of fourth degree polynomial in S divided by tenth degree polynomial, while equation (27) is transformed to a quotient of fifth degree polynomial in S over tenth degree polynomial.

NUMERICAL DATA PROCESSED:

The invariant data is:

Boiler pressure 41 bar (615 psia)

Boiler saturation temperature = 251.5 °C (485 °F)

Superheated steam temperature = 454 °C (850 °F)

Dryness fraction of vapour in shell κ is assumed 0.88.

Vapour Volume in shell is computed as 0.40 m^3

ρ_w	= 8835	kg/m^3
ρ_{sh}	= 8005.3428	kg/m^3
c_w	= 0.3936	KJ/kg.K
c_f	= 4.187	KJ/kg.K
c_{sh}	= 0.4606	KJ/kg.K
C_w	= 28.6926	KJ/K
C_{sh}	= 138.6841	KJ/K
h_2	= 31677	KJ/m^2. hr. K
K_p	= 3.5	mm/K
K_v	= 0.05	(-)
K	= 0.4562	(-)
τ	= 0.3	s
τ_v	= 3	s
τ_d	= 6	s
T	= 1	s
a_1	= 1	(-)

Feed water temperature entering the boiler are 11 (240°F) in case of two feed water heaters and (320 °F) when three heaters are used. temperature rise per heater is assumed.

Condenser vacuum is assumed to be 0.0478 ba (28" 1/2 Hg vacuum) and 0.0647 bar abs. (2 Vacuum).

Bled steam saturation temperature is assumed 2.78°C (5 °F) higher than that of the cond outlet.

Normal boiler evaporation (100% load) is tak 27200 kg/hr (≃ 60000 Ib/hr) with variations re 90% and 110% of the normal load.

Table (1) indicates the remainder of the num data depending on the variations in feed temperature, condenser vacuum and boiler evapor

An illustration of the detailed calculatio demonstrated in the appendix applied to case (1)

RESULTS AND DISCUSSION

The transient response of only the plant wh surrounded by dotted lines in Figure (5) is displa Figure (6) for case (1). The temperature devia characterized by the influence of dead time, slow of response, very large steady state error reaching and no oscillations. Knowingly, the transfer func the plant is

$$\frac{\theta_f(S)}{X(S)}=\frac{(K_vF_s\lambda/UA)[1-e^{-(a_1 S+a_o)\tau_1}]}{(\frac{C}{UA}S+1)(\alpha S^2+\beta S+1)-\frac{1}{2}[1-e^{-(a_1 S}}$$

Despite the seemingly acceptable response o plant particularly after modifying the static erro existence of two external disturbances render automatic loop unavoidable.

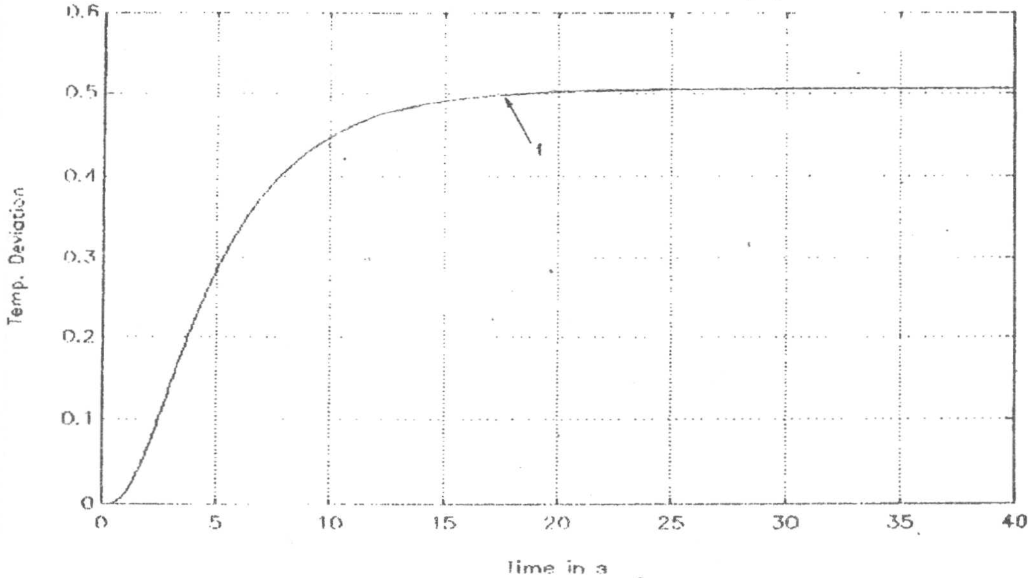


Figure 6. Transient response of the plant.

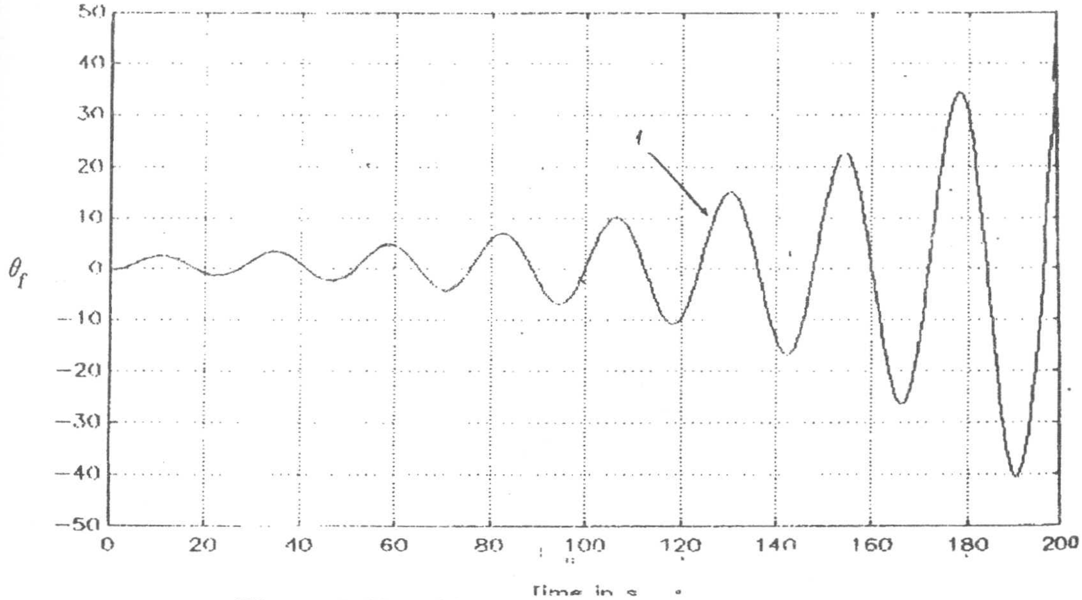


Figure 7. Unstable response of G.L (k_p. k_v = 0.4).

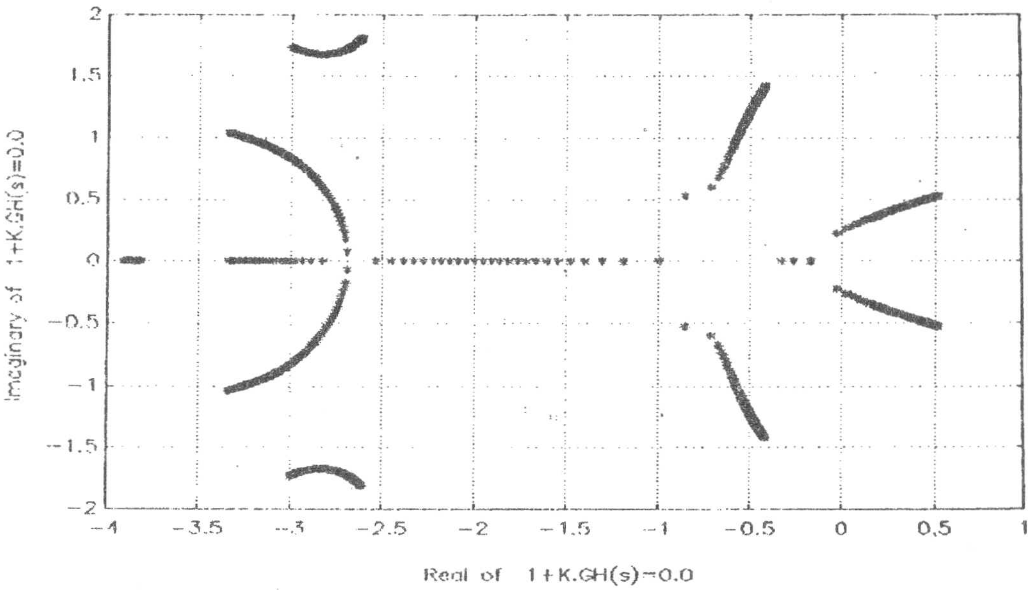


Figure 8. Root locus illustration (K=0 ---- 25, Case 1).

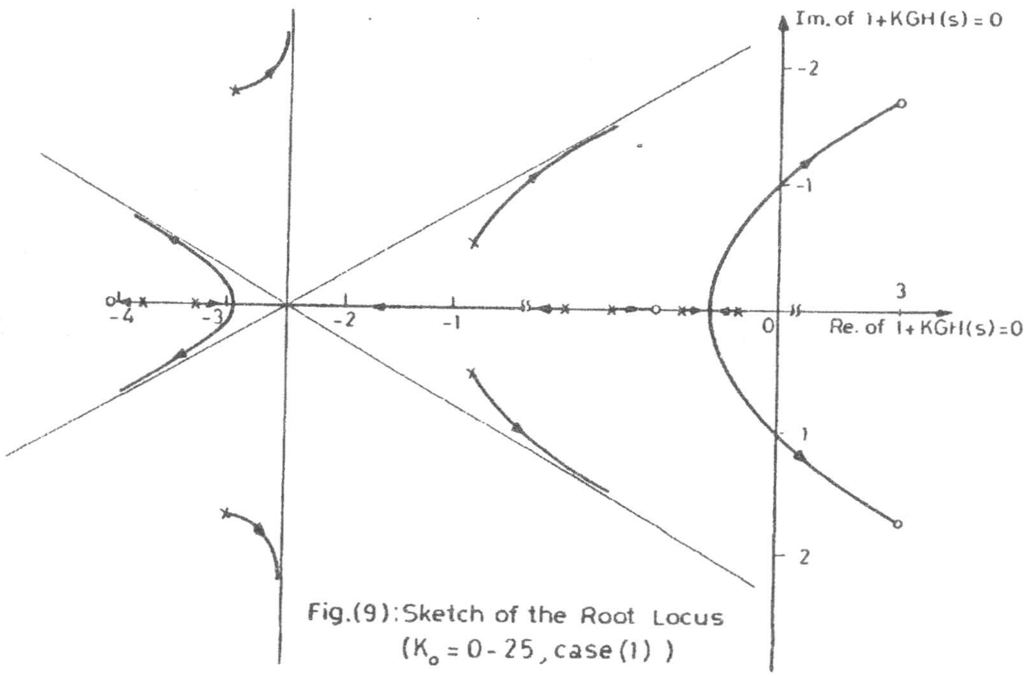


Figure 9. Sketch of the root locus (k_o=0 -- 25, Case (1)).

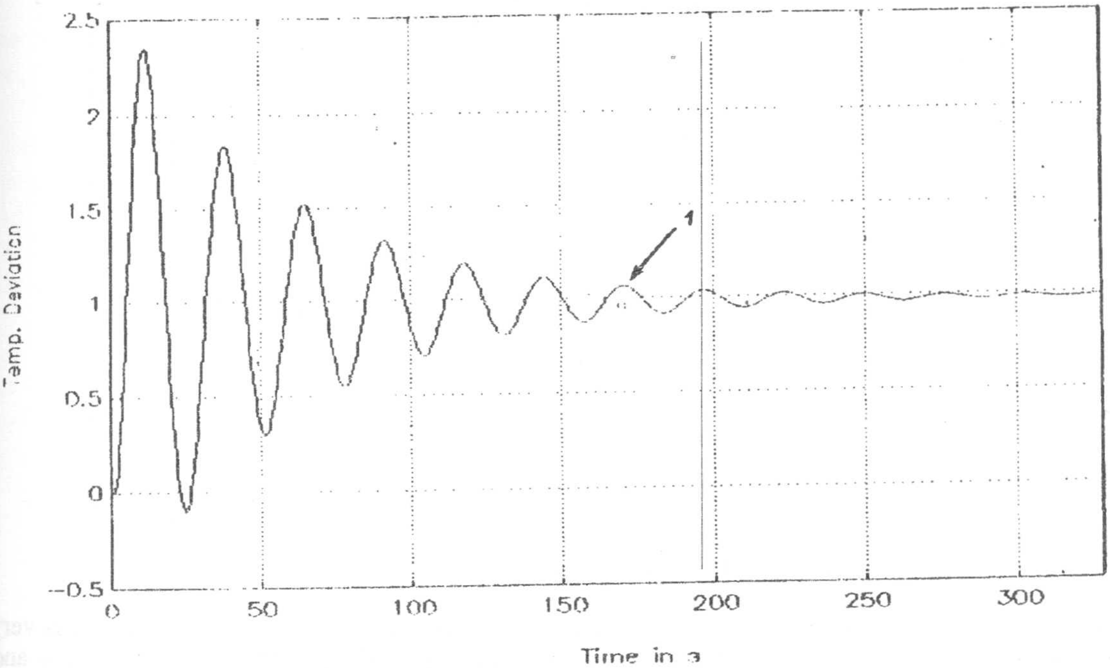


Figure 10. Transient response of C.L. (k_p k_v=0.3425).

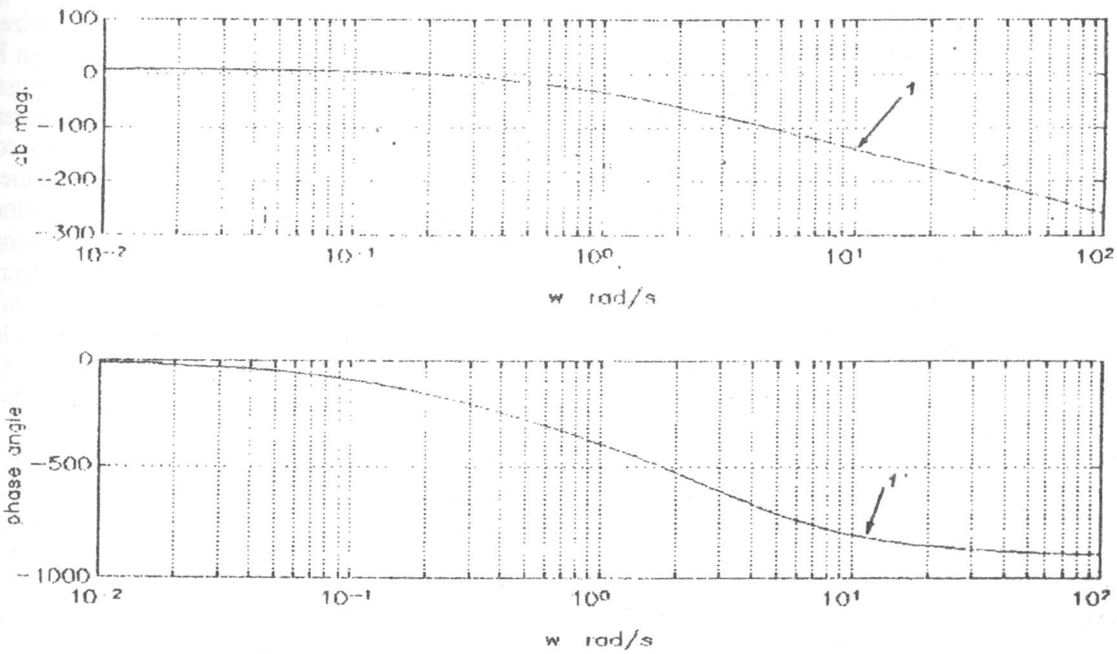


Figure 11-a. Bode plots of O.L. (k_p k_v=0.3425).

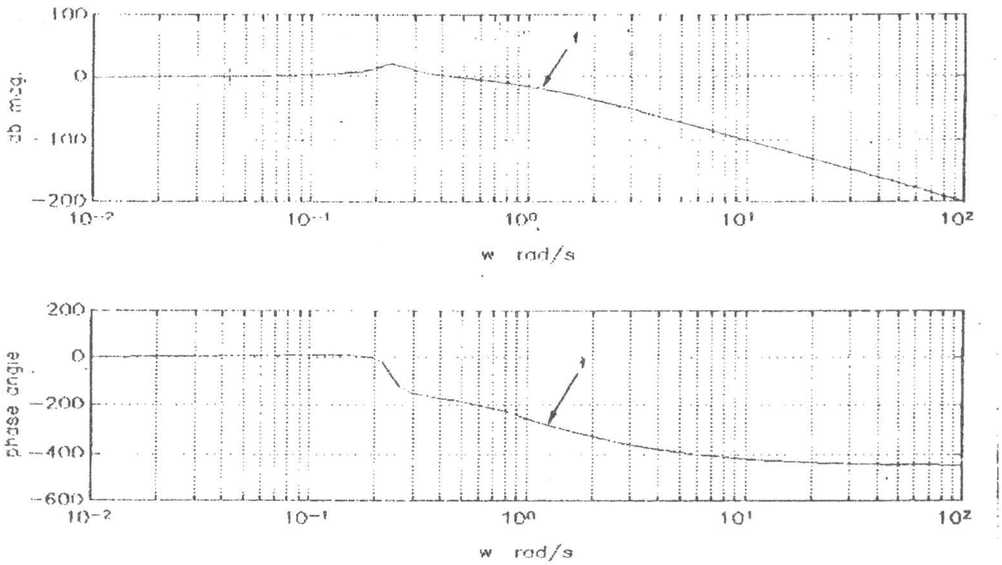


Figure 11-b. Bode plots of C.L. (k_p k_v=03425, k=0.73).

As preliminary exploration of the influence of the gains of the loop, the closed system proved to be unstable for case (1) when these values were selected: K_p K_v = 0.4 and K =1.

Two of the ten eigenvalues of the system are located in the right hand side of the complex plane. Detailed investigation of the stability of the closed loop is to establish the pole-zero mapping and plot the root locus of Evans by the aid of the open-loop transfer function K_o. G H (S) in terms of the overall gain K_o = K_p. K_v. K. The root locus configuration printed by the computer for case (1) and K_o ranging from 0 to 25 is shown in Figure (8). The Figure displays that the stability border of case (1) is close to K = 1 and K_p* K_v = 0.3. A more clarifying hand sketch of the root locus indicated in Figure (8) is presented in Figure (9). Extensive investigations of the stability problem of the closed loop by Routh criterion certifies that the border of stability for the twelve cases lies in the neighborhood of K_p K_v = 0.25 and K = 1. Should the value of K be reduced below 1 in order to get a reasonable steady state error approaching 0.02, while keeping the closed loop transfer function unchangeable, this implies dividing K_p K_v by K.

The influences of the gains K_p, K_v and K on both the transient response of the C.L to a unit step input and the frequency response of the O.L are displayed in Figures (10), (11-a,b), (12) and (13) for case (1).

Provided with the product K K_p K_v = 0.3425 and (K $= \frac{1}{1.37}$ = 0.73, K_p K_v = 0.25), Figure (10) illustrates the transient response of the C.L. The system posses a low damping factor with oscillatory overshoots; the maximum overshoot is very exce (135%) while the settling times for 2% and 4% s errors approaches 300 and 240 seconds respecti The open loop Bode plots of the preceding case shown in Figure (11-a); corresponding gain and p margins are 3.85 db & 27° respectively. Closed l Bode plots for this case, with characterized reso peak at ω = 0.23 rad/s are also shown in Figure b). When the product K.K_p.K_v was chosen as (with K = 0.5263, K_p K_v = 0.1 the transient respo obtained for case (1). Figure (12) shows a consider growth in the damping factor decreasing the numbe oscillations and reducing both the maximum oversh from 135% to 46% and the settling time-for 4% st error-from 240 to 70 seconds. A little decrease in speed of response is noticed too. In a similar man to Figure (11) , the O.L polar plot with the mentioned data is indicated in Figure (13) reveal gain and phase margins reaching 10.1db and 1 respectively.

This leads to the fact that reducing both K_p.K_v an improves the relative stability of this control syste

The final established selection of the gains is follows: K_p = 3.5, K_v = 0.05 and K = 0.4562. W the exception of the maximum overshoot percent which ranges from 30.5% to 48.5%, the chosen val of the gains K, K_p and K_v match with the requireme of absolute and relative stability, time and freque domain specifications. Since the objective of the pa is a control analysis and not a design problem, t parametric study will be carried on with the abo mentioned valves and the obtained results a demonstrated in Figures (14) through (28).

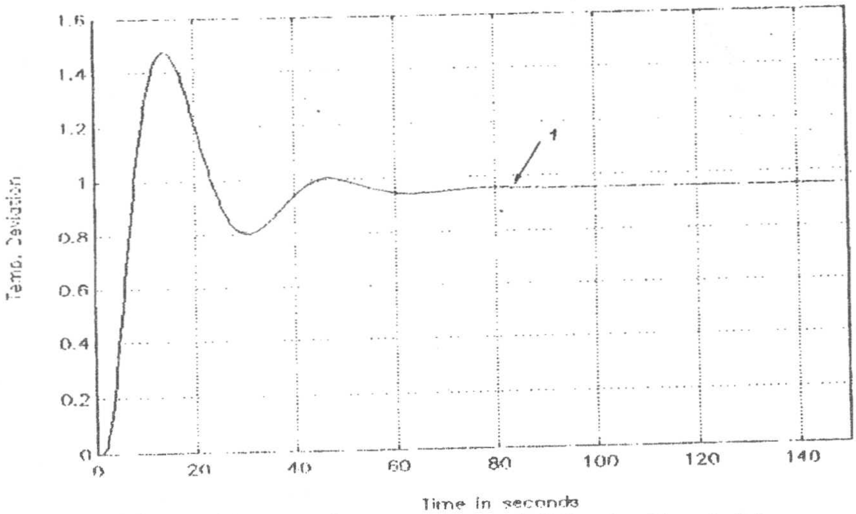


Figure 12. Transient response of C.L. (k_p K_v=0.19).

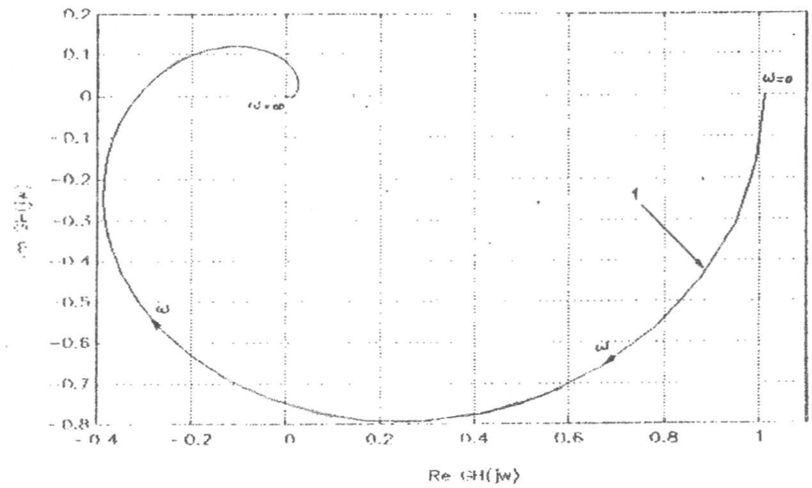


Figure 13. Polar plot (k_p k_v=0.19).

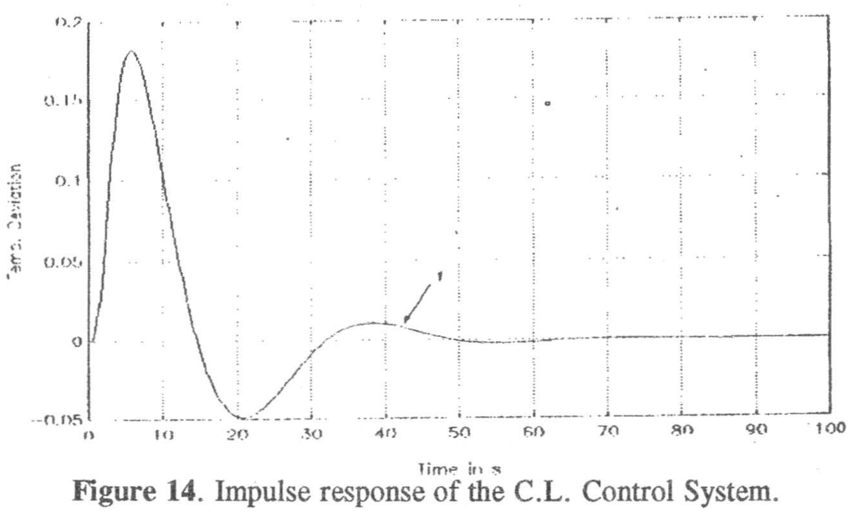


Figure 14. Impulse response of the C.L. Control System.

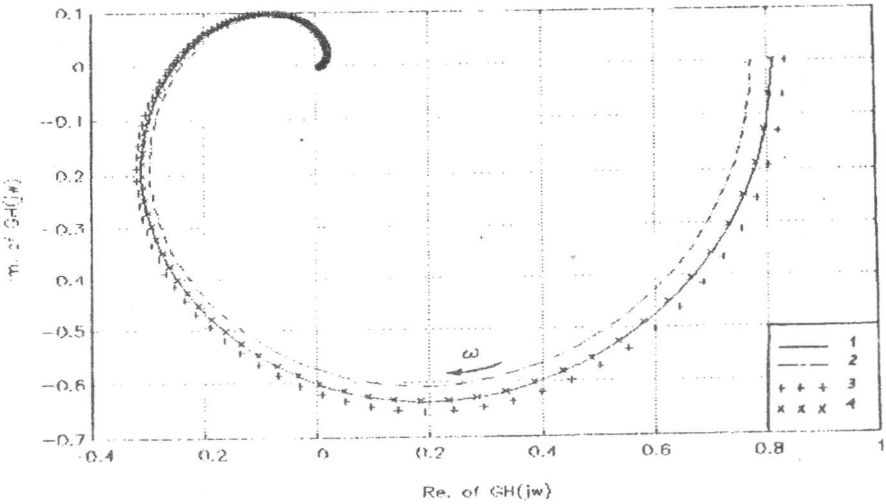


Figure 15. Effect of F.W.T. & Vaccum on polar plot.

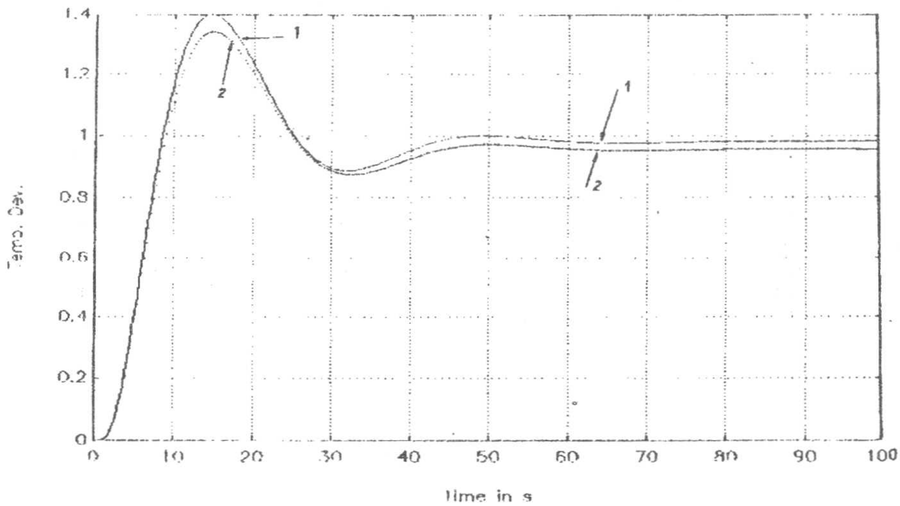


Figure 17. Effect of Vacuum variation.

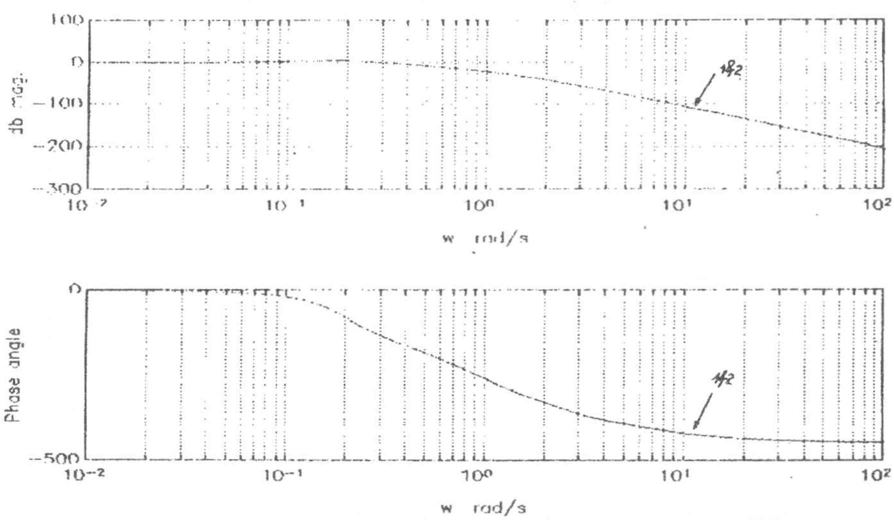


Figure 17. Bode plots of C.L. (k_p k_v=0.1754).

Figure (14) illustrates the impulse response of the C.L control system for case (1). The observed maximum overshoot is only 18% and the settling times for ± 5% and ± 2% of the final steady state value are 20 and 35 seconds respectively.

Before discussing the parametric analysis of the feed water temperature regulation, it is not surprising to find that the C.L transients of the twelve case studies are characterized by the narrow range (or even the constant value, e.g. the delay time) occupied by the delay, rise, peak and settling times beside the speed of response. To summarize these ranges:

Delay time is almost constant and equals 5.53 s

Rise time ranges from 4.9-5.75 s

Speed of response (after the dead time effect) ≃ tangent (80° - 81°) = 5.6713 - 6.3138

Peak time = 13.4 - 15.32 s

Settling time = 47 - 64.26 s (for 2% static error).

Limited variations in condenser vacuum and boiler load whereas dependence of F.W.T on the number of feed water heaters incorporated into the steam cycle give reason to the narrowness of the preceding ranges.

A relative stability comparison-displayed in polar plots between the first four case studies is shown in Figure (15).

A reduction of the vacuum from 0.0478 bar abs (28" 1/2 Hg vacuum) to 0.0647 bar abs (28" Hg vacuum) at 100% boiler load and invariant F.W.T = 115.5°C raises the gain margin from 3.9636 to 4.1536 and reduces the phase cross over frequency ω_π from 0.2507 to 0.2496 rad/s.

Adversely raising the F.W.T from 115.5°C to 160°C with the associated increase in number of heaters from 2 to 3 at 100% boiler evaporation and fixed vacuum of 0.0478 bar abs reduces the gain margin from 3.9636 to 3.8457 and increases ω_π from 0.2507 to 0.2517 rad/s.

The same phenomena hold good when comparing case studies 1 & 3 and 2 & 4. It is worthmentioning that through the whole range of the parametric analysis of the case studies, the phase margins (and gain cross over frequencies) are undefined due to the non-intersection of the unit circle with the polar plots.

The effect of vacuum variation at fixed load and feed water temperature in time domain is indicated in Figure (16). Decreasing the vacuum, decreases the maximum overshoot and increases the static error.

On the other hand, the effect of vacuum variation on the closed loop in the frequency domain is negligible, Figure (17). The system is characterized by a large bandwidth and no existence of resonant peak. Likewise, Figures (18) and (19) represent identical phenomena to Figures (16) and (17) respectively, but at different values of feed water temperatures.

It can be concluded that a change in condenser vacuum by 0.017 bar abs (≃ 1/2"Hg Vacuum) produces a change in the peak value ranging from 3.8% to 5.5% and a change in the settling time reaching from 1.3% to 2.2% respectively. In what concerns the effect of feed water temperature at constant boiler load and vacuum on the transients of the closed-loop, cases (1) & (3) and (2) & (4) are compared in Figures (20) and (21) respectively. Lowering the feed water temperature, lowers the maximum overshoot but increases the static error.

Similar changes approaching those values got from the change in condenser vacuum are obtained when varying the feed water temperature from 115.5 °C to 160°C (240°F to 320°F) while increasing the number of heaters from 2 to 3. The dependence of feed water temperature inlet to the boiler on the number of heaters gives reason to its limited effect on the condensate outlet temperature deviation.

An emphatic conclusion is that the closed loop frequency response in Bode forms are exactly typical of the Bode plots shown in Figure (19). Therefore, additional repeated graphs will not be attached to avoid monotony. The influenence of boiler load variation at fixed feed water temperature and vacuum on open-loop in db-magnitude versus phase angle plots are shown for cases (1), (5) and (9) in Figure (22). While the phase margins are undefined, minute changes in gain margins exist and will be displayed later.

Boiler part, normal and overloads effects on the transient response of the closed-loop are illustrated in Figure (23). Reducing the boiler load increases both the maximum overshoot and peak time beside decreasing the static error. A 10% increase or decrease in boiler load shifts the peak value from 3.7% to 5% and changes the settling time by 2.5% approximately. This effect when discussed on Bode plots of closed-loop Figure (24) shows intangible deviation on the db-magnitude plot and a slight variation on the phase angle plot.

Similar to closed-loop Bode plots of the twelve case studies, the plots have a distinguishing property of wide band width and no resonant peak.

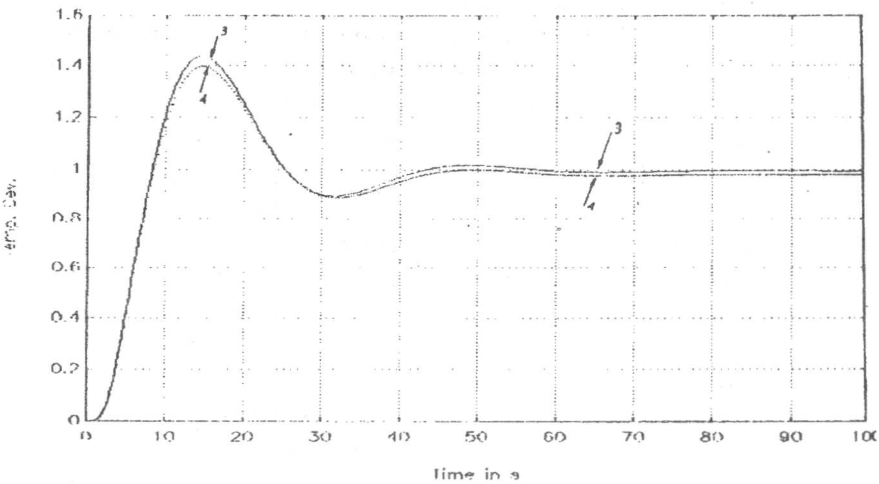


Figure 18. Effect of Vacuum variation.

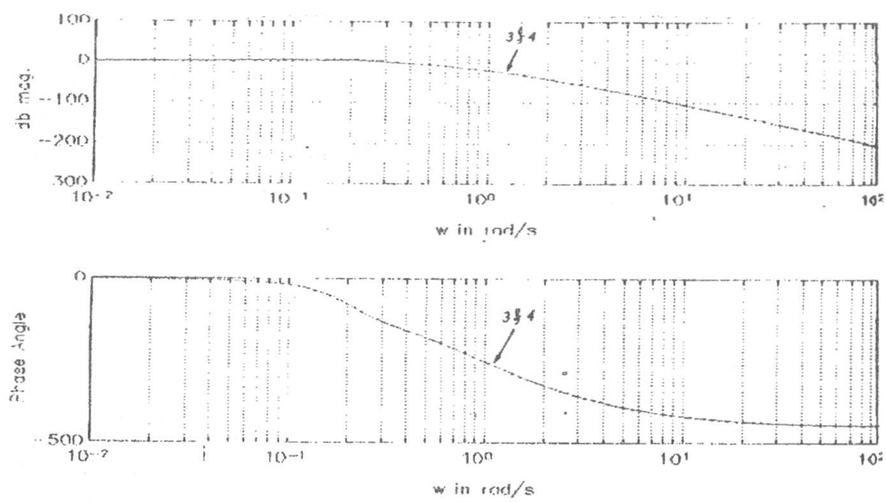


Figure 19. Effect of Vacuum variation on Bode plots of C.L.

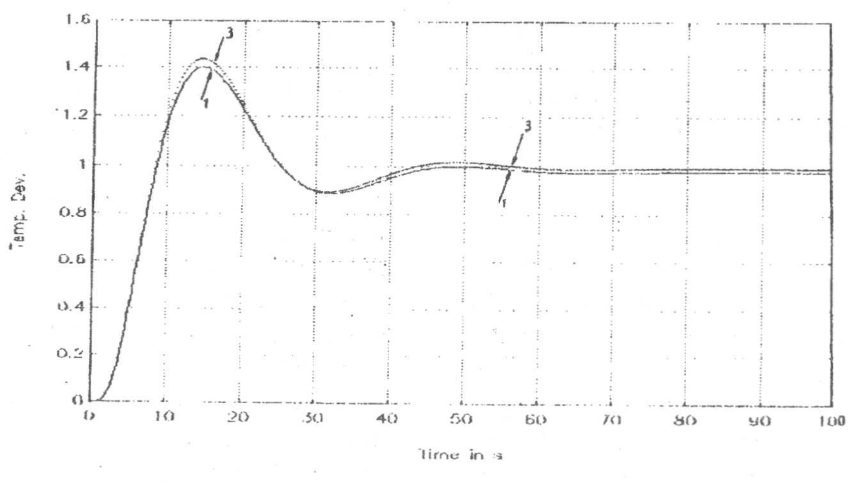


Figure 20. Effect of F.W.T.

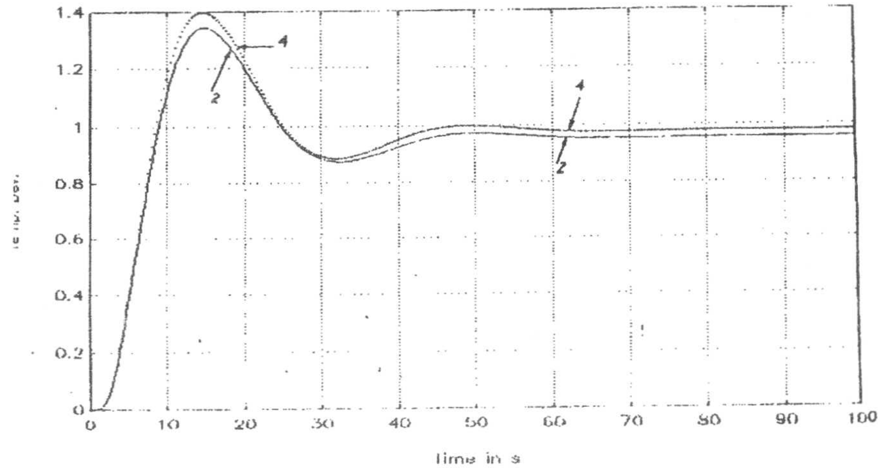
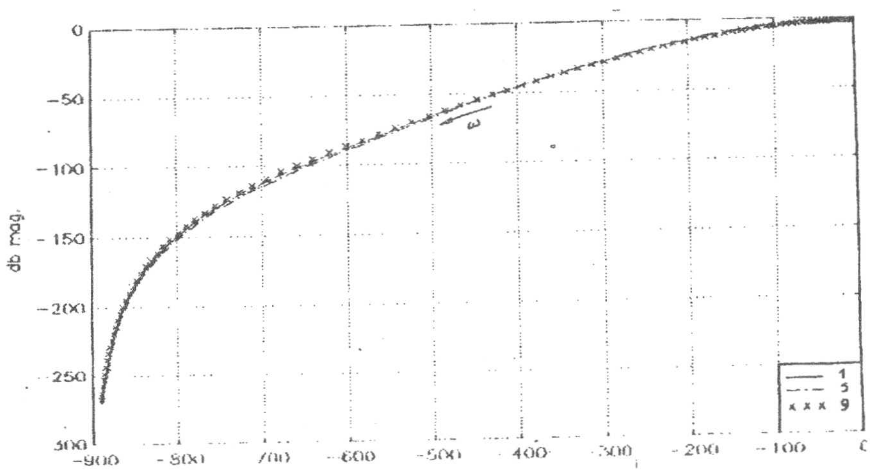


Figure 21. Effect of F.W.T.



Phase Angle

Figure 22. Effect of boiler load variation on db mag.-phase angle plot.

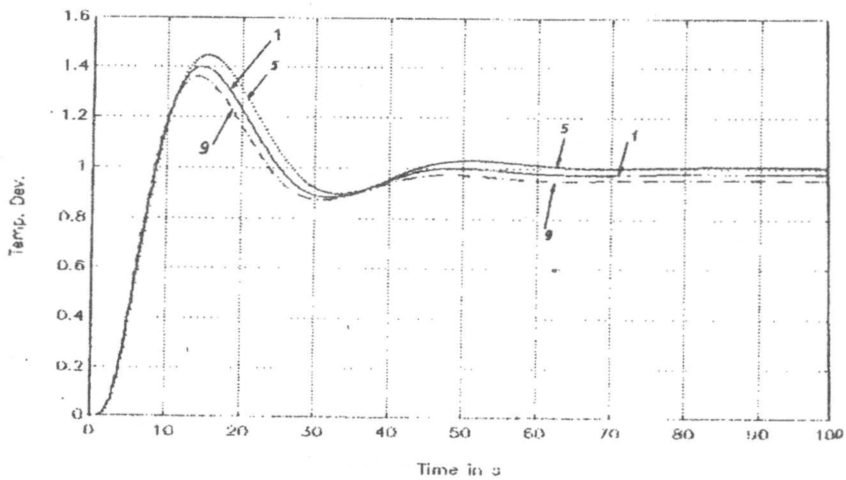


Figure 23. Effect of boiler load.

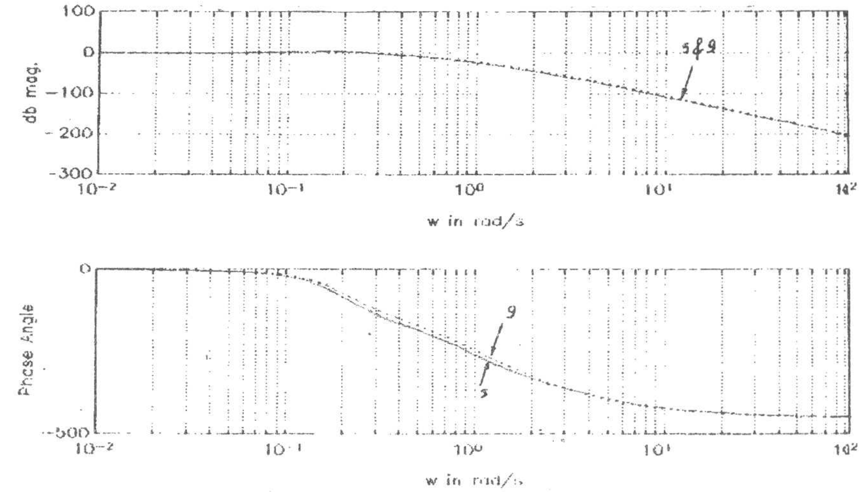


Figure 24. Effect of boiler load variation on Bode pots of G.L.

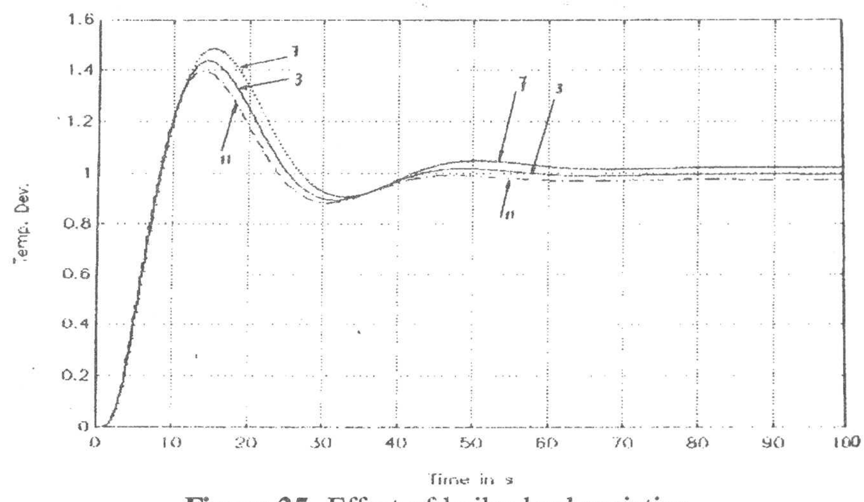


Figure 25. Effect of boiler load variation.

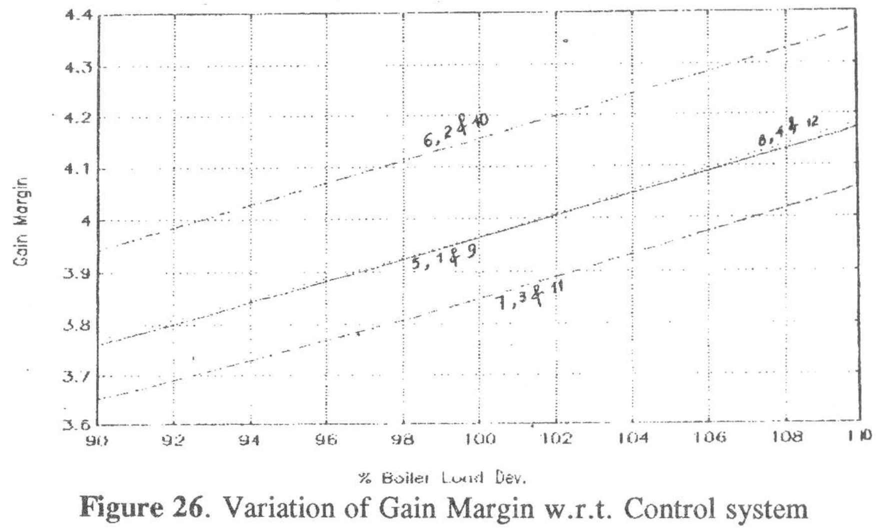


Figure 26. Variation of Gain Margin w.r.t. Control system parameters.

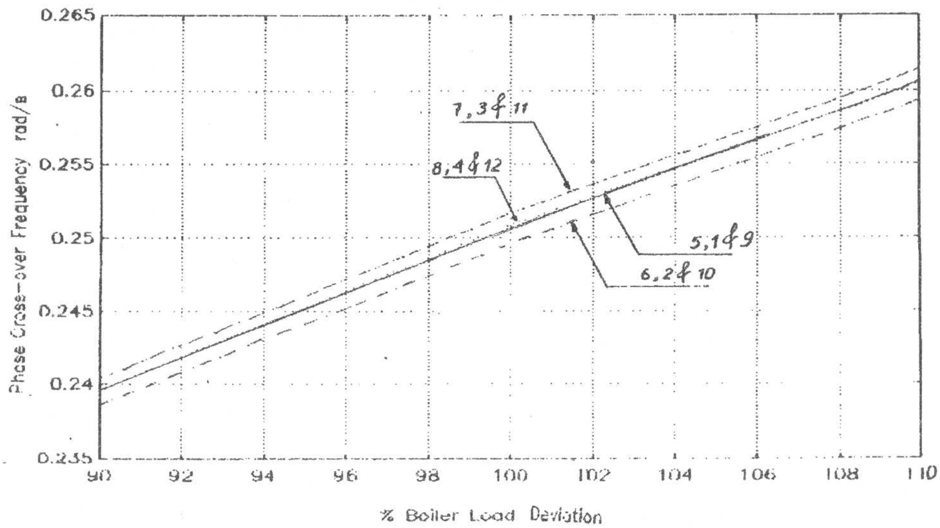


Figure 27. Variation of ω_π w.r.t. Control system parameters.

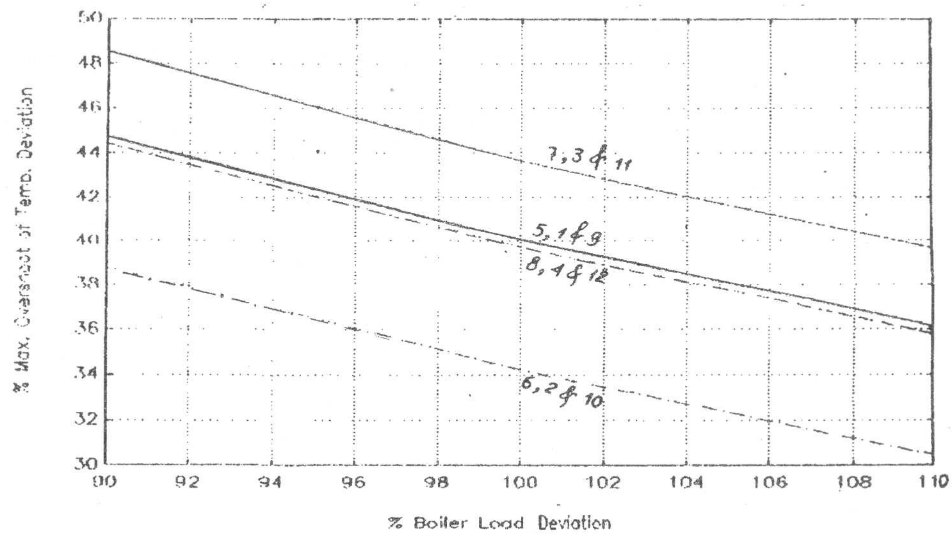


Figure 28. Variation of M_p w.r.t. Control system parameters.

Boiler load effect is discussed too in time domain for cases (3), (7) and (11) for feed water temperature equals 160°C and condenser vacuum equals 0.0478 bar abs. (28" 1/2 Hg vacuum). The concluded tendency for Figure (25) is identical to that shown in Figure (23).

The influence of control system parameters on gain margin, phase crossover frequency ω_π and maximum overshoot M_p are plotted in Figures (26), (27) and (28) respectively. Figure (26) reveals that the gain margin is increased by raising the boiler load, decreasing the condenser vacuum and lowering the feed water temperature. Extreme values of gain margin are 3.6531 and 4.372.

From Figure (27), it is evident that, while ω_π increases with the increase of boiler load, in contrast to gain margin, ω_π decreases by lowering the vacuum and reducing the feed water temperature. Maximum and minimum values of ω_π are 0.2614 and 0.2386 rad/s.

Lastly, the percentage maximum overshoot M_p which is displayed in Figure (28)- decreases with the increase of boiler load, the decrease of condenser vacuum and the reduction of feed water temperature. The lowest and highest observed values of M_p are 30.47% and 48.52%.

Programs for this study [28,29] had been executed in the Lloyd's computer laboratory at the department of

Marine Engineering and Naval Architecture, Alexandria University on the DELL-466/ME apparatus.

CONCLUSION

Dynamic models for feed water temperature regulation w.r.t. command and perturbation signals of fluctuating boiler load or condenser vacuum have been presented.

Computational verification of the first model has been executed in extensive parametric analysis of the control loop. Major attention was paid to various values of condenser vacuum, boiler evaporation and required feed water temperature inlet to the boiler which depends on the number of feed water heaters. Each parameter affects the complex dynamics of heat transfer with the associated transportation lag and time delay owing to the resulting variations in bled steam pressure, amount and heat content beside occurring changes in heat transfer coefficients and thermal capacities. The role played by the loop on absolute and relative stability has been scanned. The problem has been thoroughly analyzed in both time and frequency domains displaying which specifications are obtained and how far these are affected by the control system parameters.

The automatic loop dynamics proves insensitivity to slight deviations in system parameters.

REFERENCES

[1] Mc Adams, W.H., *Heat Transmission*, 3rd ed., McGraw-Hill Book Company, New York, 1954.

[2] Gould, L.A., *The Dynamic Behaviour and Control of Heat Transfer Processes*, Sc. D. Thesis, Massachussetts Institute of Technology, 1953.

[3] Cohen, W.C. and Johnson, E.F., *Dynamic Characteristics of Double-Pipe Heat Exchangers*, Ind. Eng. Chem., 48: 1031, 1956.

[4] Campbell, D.P. *Process Dynamics*, John Wiley & Sons. Inc., New York, 1958.

[5] Catheron, A.R., Goodhue S.H. and Hansen, P.D., *Control of Shell- and- Tube Heat Exchangers*, Trans. ASME Paper 59-IRD-14.

[6] De Bolt, R.R., *Dynamic Characteristics Steam-Water Heat Exchanger*, M.Sc. Th University of California, 1954.

[7] Less, S. and Hougen, J.O., *Pulse Testi Model Heat Exchange Process*, Ind. Chem, 48: 1064, 1956.

[8] Koppel, L.B., *Dynamics of a Flow-Fo Heat Exchanger*, I and EC Fundame 1:131, 1962.

[9] Fricke, L.H., Morris, H.J., Otto, R.E. Williams T. J., *Process Dynamics and An Computer Simulations of Shell-and-Tube Exchnagers*, Chem. Eng. Progr. Sympo Ser., 56 (31): 80, 1960.

[10] Oppelt, W. *Kleines Handbook techni Regelvorgänge*, Verlag Chemie, 1954.

[11] Quack, *Boilers with Load Regulation in National Economy*, Techn. Mitt, 5, pp. 193, 1957.

[12] Quack, *Automation in the Steam Power P* Techn. Mitt. Essen 50, 5, 1957.

[13] Profos, P. *Control of Steam Installat* Springer-Verlag, 1962.

[14] Profos, P. *Load-Dependent Adjustment o Controller Setting for Superheater Tempera Control*.

[15] Profos P. *The Dynamic Behaviour of Fo Flow Evaporators with Uneven Distribution*, Regelungstechnik 13, pp. 57 1965.

[16] Profos, P., *Consideration of Future Co problems in Steam Generating Power Stati* Mitt. VGB, 109, pp. 224-232, 1967.

[17] Doležal, R., *Some Constructional Steps fo Improvement of the Dynamic Characteristi Steam Generators*, Seminaire Internationa l' IBRA Bruxelles, 1966.

[18] Doležal, R. & Varcop, L., *Process Dyna* Chapter (19). *The Dynamic Characteristi the Economizer*, Elsevier Publishing Comp Ltd, Amsterdam, 1970.

[19] Varcop, L. *The Various Modes of Operati a Heat Exchanger*, Wärme, 71, pp. 121- 1965.

[20] Varcop, L. *The Dynamics of Forced F Evaporator Systems under Consideration Pressure Variations of the Evaporated Fl* Regelungstechnik, 15, pp. 404-412, 1967.

[21] Varcop, L., *Dynamics of the Evaporation Zone of an Evaporator with Unidimensional Flow,* Rozpravy Ceskoslovenske akademie ved, 77, 1967.

[22] Varcop, L. *On the Feedwater Supply and Fuel Flow Control in Once-Through Boilers,* IFAC Symposium On Multivariable Control Systems, 7/8 October Dusseldorf, 1968.

[23] Isermann, R., *The Determination of the Control Quality of Steam Temperature Control in Drum Boilers using an Analogue Computer Parts I and II,* Regelungstechik, 10, pp. 469-475 and 11, pp. 519-522. 1966.

[24] Masubuchi, M., *Dynamic Response and Control of Multipass Heat Exchangers,* Trans. ASME J. Basic Eng., 820:51, 1960.

[25] Yang, W. & Masubuchi, M., *Dynamics for Process and System Control,* Chapter (5), *Dynamic Behaviour of Distributed Systems,* Gordon & Breach Science Publishers, New York, 1970.

[26] Jackson, L. & Embleton, W., *Instrumentation and Control Systems,* Reed's Marine Engg. Series, Thomas Reed Publications, London, 2nd Edition, 1975.

[27] Harriott, P., *Process Control,* Chapter (11)- *The Dynamics and Control of Heat Exchangers* McGraw-Hill Publishing Company, New York, 1977.

[28] Melsa, J., *Computer Programs for Computational Assistance in the Study of Linear Control Theory,* McGraw-Hill Book Company, 1973.

[29] _______ *MATLAB, The Mathworks Incorporation,* Stanford, 1986.

[30] Mikheyev, M., *Fundamentals of Heat Transfer,* Peace Publishers, Moscow, 1965, Appendix, Table A-19, pp. 369.

Appendix

Brief Sample of Analysis of Procedure

Normal boiler load = 27200 kg/hr (≃ 60000 Ib/hr)
Feed water temperature 115.5 °C (240 °F)
Condenser vacuum 0.0478 bar abs. (28 1/2" Hg)
Dimensions of heat exchanger 50 cm (≃ 19.666")
Outer steel shell diameter with 9.5 mm (≃3/8") thickness.
Dimensions of tubes are:

d_i = 1.6916 cm (≃ 0.666")
d_o = 1.905 cm (≃ 0.75") x = 244 cm (≃ 8′)
Water velocity through the feed water heater (v) is selected as 0.6 m/s.
$\tau_1 = x/v = 4.07$ s
w is assumed to be approximetly 9.81 kN/m^3 (1000 kg$_f$ /m^3).

1) $$n = \frac{\bar{G}}{\frac{\pi}{4} d_i^2 v\, w * 3600} \simeq 56 \quad \text{tubes}$$

2) From steam tables:
Condenser satuation temperature 37.5 °C
$\Delta\theta_r$ = 39.03 °C (based on equal temperature rise per heater).
Saturation temperature of steam = 37.5 + 39.03+ 2.78 = 79.31 °C
which corresponds to a pressure equals 46060 Pa (0.4606 bar)
Steam pressure in shell (P) is assumed to be 3/8 of the bled steam pressure = 17270 Pa
Hence, from steam tables $\frac{\partial P}{\partial \theta_s}$ = 802 Pa/K at shell pressure.
Dryness fraction of bled steam (κ) in shell is assumed to be 0.88.
Specific volume of dry and saturated steam at shell pressure = 7.7303 m^3/kg, λ = 2365.3227 kJ/kg

3) $$\text{LMTD} = \frac{(79.31-37.5)-(79.31-76.53)}{\ln\frac{41.81}{2.78}} = 14.3945\,°\text{C}$$

4) $$UA = \frac{\bar{G} x 4.187 x \Delta\theta_r}{3600 x \text{ LMTD}} = 85.7727 \quad \text{kJ/K.s}$$

5) $$F_s = \frac{\bar{G} x 4.187 x \Delta\theta_r}{\lambda x 3600} \simeq 0.53 \quad \text{kg/s}$$

6) $$\text{Vapour volume in shell} = \frac{\pi}{4}\left[d_s^2 . n - d_o^2\right].(x) \simeq 0.4\ \text{m}^3$$

7) $$M_v = \frac{0.4}{0.88 x 7.7303} = 0.0592 \qquad \text{kg of steam}$$

8) $$C_v = \frac{M_v}{P}\lambda\left(\frac{\partial P}{\partial \theta_s}\right) = 6.5027\ \text{KJ/K}$$

9) The specific heat of wall c_w = 0.3936 KJ/ kg.K

$$\rho_w = 8835\ \text{kg/m}^3,\ C_w = \frac{\pi}{4}\left[d_o^2 - d_i^2\right].(x).n.c_w.\rho_w = 28.6926\ \text{kJ/K}$$

Table (1) Parameters Dependent Data.

Boiler Load	100% (27200 kg/hr)				90% (24480 kg/hr)				110% (29920 kg/hr)			
FWT	115.5 °C (2 heaters)		160 °C (3 heaters)		115.5 °C (2 heaters)		160 °C (3 heaters)		115.5 °C (2 heaters)		160 °C (3 heaters)	
Vacuum bar abs.	0,0478	0,0647	0,0478	0,0647	0,0478	0,0647	0,0478	0,0647	0,0478	0,0647	0,0478	0,064
Case No.	1	2	3	4	5	6	7	8	9	10	11	12
θ_b, °C	79.31	81.81	81.10	84.44	79.31	81.81	81.10	84.44	79.31	81.81	81.10	84.44
P_b, bar	0.461	0.510	0.495	0.566	0.461	0.510	0.495	0.566	0.461	0.510	0.495	0.566
$\Delta\theta_r$, °C	39.03	36.43	40.83	39.17	39.03	36.43	40.83	39.17	39.03	36.43	40.83	39.17
LMTD °C	14.40	13.78	14.83	14.43	14.40	13.78	14.83	14.43	14.40	13.78	14.83	14.43
v, m/s	0.60	0.60	0.60	0.60	0.54	0.54	0.54	0.54	0.66	0.66	0.66	0.66
τ_1, s	4.07	4.07	4.07	4.07	4.52	4.52	4.52	4.52	3.70	3.70	3.70	3.70
U.A kJ/K.s	85.77	83.63	87.12	85.88	77.20	75.27	78.41	77.29	94.35	92.00	95.83	94.47
F_s kg/s	0.53	0.50	0.55	0.53	0.48	0.45	0.50	0.48	0.58	0.54	0.61	0.59
λ, kJ/kg	2365.32	2360.05	2361.53	2354.51	2365.32	2360.05	2361.53	2354.51	2365.32	2360.05	2361.53	2354.51
$\partial p/\partial\theta_s$ Pa/K	802	905	869	980	802	905	869	980	802	905	869	980
ρ_s kg/m^3	0.129	0.142	0.138	0.157	0.129	0.142	0.138	0.157	0.129	0.142	0.138	0.157
M_v, kg	0.059	0.057	0.056	0.063	0.059	0.057	0.056	0.063	0.059	0.057	0.056	0.063
C_v, kJ/K	6.503	6.404	6.156	6.865	6.503	6.404	6.156	6.865	6.503	6.404	6.156	6.865
C kJ/K	173.88	173.78	173.53	174.24	173.88	173.78	173.53	174.24	173.88	173.78	173.53	174.24
F_m kg/m^2.s	600	600	600	600	540	540	540	540	660	660	660	660
h_f kJ/hr.m^2.K	17272.4	17272.4	17272.4	17272.4	15876.2	15876.2	15876.2	15876.2	18641.0	18641.0	18641.0	18641.0
T_1, s	3.7	3.7	3.7	3.7	4.02	4.02	4.02	4.02	3.423	3.423	3.423	3.423
T_2, s	0.4	0.4	0.4	0.4	0.398	0.398	0.398	0.398	0.4	0.4	0.4	0.4
T_{12}, s	0.82	0.82	0.82	0.82	0.896	0.896	0.896	0.896	0.763	0.763	0.763	0.763
a_o, s^{-1}	0.177	0.177	0.177	0.177	0.169	0.169	0.169	0.169	0.187	0.187	0.187	0.187

10) $\rho_{sh} = 8005.3428$ kg/m^3
$c_{sh} = 0.4606$ kJ/kg.K
$C_{sh} = \pi (d_{so} - d_s)(x) . \rho_{sh} . c_{sh}$
$= 138.6841$ kJ/K

11) $C = C_v + C_w + C_{sh} = 173.8794$ kJ/K

12) $h_1 = 0.0144\, c_f \dfrac{F_m^{0.8}}{d_i^{0.2}}$ (in F.P.S. System) [1]

where: F_m is the mass flow rate in (Ib/hr. ft^2)

Free area $= n \times \frac{\pi}{4} d_i^2 = 0.1355$ ft^2

$F_m = \dfrac{27200 \times 2.204}{0.1355} = 442361.2751$ Ib/hr. ft^2

and $c_f = 1$ Btu/Ib$_m$.°F

Then $h_1 = 0.0144 \times c_f \times \dfrac{(442361.2751)^{0.8}}{(0.666/12)^{0.2}}$

$= 843.5786$ Btu/hr. ft^2.°F

$= 17272.4332$ kJ/ hr. m^2.K

13) In order to determine h_2 [30]:

$h_2 = c_1\, b_1\, 4\sqrt{r_1}\, / 4\sqrt{L . \delta\theta}$ in M.K.S. system.
For horizontal tubes

$c_1 = 0.72$, $L = d_o = 0.01905$ m

From attached tabular form [30]:

at $\delta\theta = 80 - 20 = 60$ °C

Then; $4\sqrt{r_1} = 4.85$,

$b_1 = 2070$ and
$h_2 = 7565.5393$ Kcal/m^2.hr.K
$= 31677$ kJ/m^2.hr.K

14) $M_f = \frac{\pi}{4} d_i^2 . \rho_f = 0.2248$ kg/m length for one tube,
$A_1 = \pi . d_i = 0.0532$ m^2/m length for one tube,
$c_f = 4.187$ kJ/kg.K then:

$T_1 = \dfrac{M_f . c_f}{h_1 . A_1} \times 3600 = 3.7$ s

15) $M_w = \frac{\pi}{4}\left[d_o^2 - d_i^2\right] . \rho_w$
$= 0.5319$ kg/m length for one tube,
$A_2 = \pi\, d_o = 0.06$ m^2/m length of one tube,

then:

$T_2 = \dfrac{M_w . c_w}{h_2 . A_2} \times 3600 = 0.4$ s

16) $T_{12} = \dfrac{M_w . c_w}{h_1 . A_1} \times 3600 = 0.82$ s

17) $a = \dfrac{(T_1 S + 1)(T_{12} T_2 S + T_{12} + T_2) - T_2}{T_1 (T_{12} T_2 S + T_{12} + T_2)}$

$= \dfrac{(3.7 S + 1)(0.82 \times 0.4 S + 0.82 + 0.4) - 0.4}{3.7 (0.82 \times 0.4 S + 0.82 + 0.4)}$

$\simeq S + 0.1772$ 1/s

18) $e^{-a\tau_1} = e^{-(S+0.1772)\, 4.07} = 0.4861\, e^{-4.07S}$

19) $\dfrac{b}{a} = \dfrac{1}{T_1 T_2 S^2 + (T_1 + T_2 + T_1 T_2 / T_{12}) S + 1}$

$= \dfrac{1}{1.48 S^2 + 5.9049 S + 1}$

20) $K_v = 0.05$, $K_p = 3.5$ mm/K, , $K = 0.4562$, $\tau = 0.3$ s,
$\tau_v = 3$ s, $\tau_d = 6$ s
and $T = 1$ s

Then, $\Theta.L.T.F = \dfrac{\sum_{i=o}^{4} n_i S^i}{\sum_{j=o}^{10} m_j . S^j}$

where,

$$[n_o \; n_1 \; \dots \; n_4] = [51.5389 \;\; 277.528 \;\; -76.2319 \; -10.2971 \;\; 5.9246]$$

and

$$[m_o \;\; m_1 \;\; \dots \;\; m_{10}] = [63.7334 \;\; 1349.796 \;\; 11354.16 \;\; 48778.05 \;\; 115905.0 \;\; 157860.0 \;\; 127802.9 \;\; 60859.23 \;\; 16825.64 \;\; 2523.554 \;\; 159.8091]$$

$$\text{and} \quad C.L.T.F. = \frac{\sum_{i=o}^{5} p_i S^i}{\sum_{j=o}^{10} q_j S^j}$$

where,

$$[p_o \;\; p_1 \;\; \dots \;\; p_5] = [112.9681 \;\; 1399.0897 \;\; 4825.3955 \;\; 3119.5778 \;\; 813.0678 \;\; 77.\text{[illegible]}79]$$

and

$$[q_o \;\; q_1 \;\; \dots \;\; q_{10}] = [115.2723 \;\; 1627.324 \;\; 11277.93 \;\; 48767.75 \;\; 115910.9 \;\; 157860.0 \;\; 127802.9 \;\; 60859.23 \;\; 16825.64 \;\; 2523.554 \;\; 159.\text{[illegible]}]$$