DYNAMIC PERFORMANCE OF SHADED POLE MOTOR USING ANALOGUE SIMULATION

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ABSTRACT

An analogue simulation technique is described for the calculation of the transient performance of a shaded pole motor under dynamic starting conditions. The equations of performance are written in terms of two actual stator circuits and two equivalent rotor circuits for the squirrel cage. Comparison is made with the results obtained from another method used digital simulation.

1. INTRODUCTION

Some papers have been published on steady-state performance of shaded pole motors [1,2,3,4]. Chang [2] assumes 90 degrees displacement between the shaded and the unshaded portions of the pole which is quite reasonable when the pole arc is nearly equal to the pole pitch. The results are not very satisfactory when the pole arc is significantly different from the pole pitch. Butler and Wallace [4] consider harmonics up to the 15th; but where the pole arc is significantly different from the pole pitch, the results are not satisfactory. Sherer and Hertzog [3] have used Chang's method to study the performance with and without harmonics. Their results do vary from experiment when harmonics are taken care of.

Some other papers have been published on the dynamic performance of shaded pole motors [5,8,9]. The paper by Desai and Mathew [5] gives computer results with and without harmonics. It accounts for saliency and the actual displacement of the shading winding from the main winding by the determination of most of the motor constants experimentally. Lock [8] presents a method of analyzing the performance of the motor by using step-by-step numerical solution of basic coupled-circuit differential equations. The cage rotor is represented as a number of cascaded loops. each loop is represented by two adjacent bars and the two sections of the end rings joining the bars. Akpinar and Kaya [9] investigate he possibility of increasing the starting torque and efficiency, controlling the speed and reversing the direction of the rotor of the shaded pole motor. In addition the dynamic performance of the motor provided with this possibility is analyzed.

This paper presents a method of analyzing the dynamic behaviour of the motor by using analogue simulation. Harmonics are neglected because of the limited number of the available potentiometer on the analogue computer used. However, depending on the availability of the potentiometer, the presented method can be easily modified to include harmonics as explained in paragraph 3 of section 4.

The motor data of Desai and Mathew [5] is used. Therefore, saliency and the actual displacement of the main winding have been taken care of. The machine is represented by a "primitive machine" in which there are two actual circuits of the stator; one for the main winding and the other for the shading winding. The main winding is excited by the supply voltage, whereas the shading winding is closed onto itself. The rotor is, likewise, represented by two short-circuited coils in quadrature with each other; one of which has an axis coincident with the direct axis of the system, being that of the main winding. The approach is similar to that of Kron [1].

2. EQUATIONS OF PERFORMANCE

Figure (1) shows a schematic representation of the motor. Figure (2) is a representation of the motor as a "primitive machine" where the winding a_d and a_q represent the shaded winding resolved into two components along the direct and quadrature axes respectively.

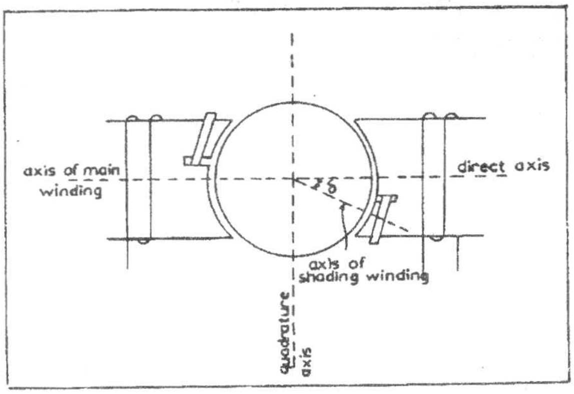

Figure 1. Schematic diagram of the shaded pole motor.

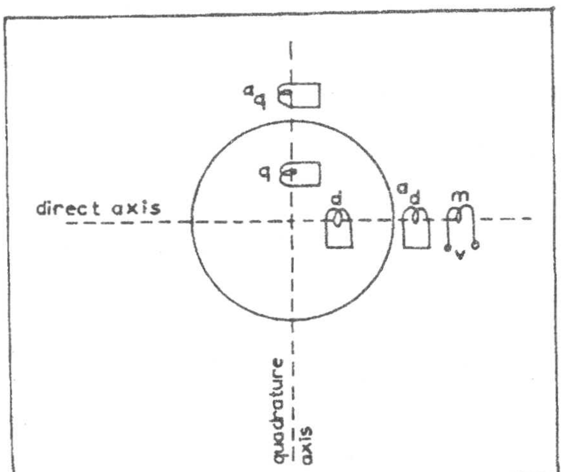

Figure 2. Two axis representation of shaded pole motor.

To simplify the problem, the following assumptions are introduced in deriving the equations of performance [6,7]:

1. Magnetic saturation is neglected, so that inductances are assumed independent of the magnitude of currents.
2. Harmonics are neglected so that mutual inductances between the stator and rotor circuits are obtained by considering the fundamental component of the flux only.
3. Effect of eddy currents and hysteresis is neglected.
4. The slotting effect is ignored.
5. The resistances are assumed independent of the magnitude and frequency of currents.

There are six equations. Four of them are for volt-ampere equations of the four electric circuits of t motor and are given in matrix form by equation The fifth equation given by equation (2) gives expression for the electromagnetic torque. The si equation given by equation (3) relates the electrom netic torque, the load and inertia torques.

Equation (1), equation (2), and equation (3) repres the nonlinear equations to be solved for the depend variables i_m, i_a, i_d, i_q and w_r. Different ini conditions can be assigned to study the differ conditions of operation.

$$\begin{bmatrix} V \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_m & 0 & 0 & 0 \\ 0 & R_a & 0 & 0 \\ 0 & pw_r L_{ar} \sin\delta & R_r & -pw_r L_{qq} \\ pw_r L_{md} & pw_r L_{ar} \cos\delta & pw_r L_{dd} & R_r \end{bmatrix} \cdot$$

$$+ \begin{bmatrix} L_{mm} & L_{ma} & L_{md} & 0 \\ L_{ma} & L_{aa} & L_{ar}\cos\delta & -L_{ar}\sin\delta \\ L_{md} & L_{ar}\cos\delta & L_{dd} & 0 \\ 0 & -L_{ar}\sin\delta & 0 & L_{qq} \end{bmatrix} \begin{bmatrix} di_m/dt \\ di_a/dt \\ di_d/dt \\ di_q/dt \end{bmatrix}$$

$$m_e = p\,[L_{md}\, i_m\, i_q + L_{ar}\, i_a\, i_d \sin\delta + L_{ar}\, i_a\, i_q \cos$$

$$+ (L_{dd} - L_{qq})\, i_q\, i_d]$$

$$m_e = J dw_r/dt + m_L$$

In the above equations, all the impedances related the shaded winding i.e. R_a, L_{ar}, L_{ma}, L_{aa} are referr to it. Thus i_a is the actual current in the shading rin On the other hand, all other impedances and curre are referred to the main winding.

3. NORMALIZED EQUATIONS

In order to solve the performance equations on analogue computer they have been normalized both time and magnitude. For time normalization t computer time τ is chosen 31.4 times the actual tim For magnitude normalization a base value is assign for each variable. Equal coefficient rule is applied estimate the base values. Six equations are written the normalized form in equations (4) through (6). The (*) denotes the derivative of the variable with respect to the computer time τ.

$$I^*_m = (V_m\, w/nI^*_{mm}X_{mm})V - (R_m w I_{mm}/X_{mm} n I^*_{mm})I_m - (X_{ma}I^*_{am}/X_{mm}I^*_{mm})I^*_a - (X_{md}I^*_{dm}/X_{mm}I^*_{mm})I^*_d$$

$$I^*_a = -(wR_a I^*_{am}/X_{aa}I^*_{am}n)I_a - (X_{ma}I^*_{mm}/X_{aa}I^*_{am})I^*_m + (X_{ar}\sin\delta\, I^*_{qm}/X_{aa}I^*_{am})I^*_q - (X_{ar}\cos\delta\, I^*_{dm}/X_{aa}I^*_{am})I^*_d$$

$$I^*_d = -(p\Omega_{rm}X_{ar}I_{am}\sin\delta\, /nX_{dd}I^*_{dm})I_a\Omega_r + (p\Omega_{rm}X_{qq}I_{qm}/nX_{dd}I^*_{dm})I_q\Omega_r - (wR_r I_{dm}/nX_{dd}I^*_{dm})I_d - (X_{md}I^*_{mm}/X_{dd}I^*_{dm})I^*_m - (X_{ar}I^*_{am}\cos\delta\, /X_{dd}I^*_{dm})I^*_a$$

$$I^*_q = -(p\Omega_{rm}X_{md}I_{mm}/nX_{qq}I^*qm)I_m\Omega_r - (p\Omega_{rm}X_{dd}I_{dm}/nX_{qq}I^*_{qm})I_d\Omega_r - (wR_r I_{qm}/nX_{qq}I^*_{qm})I_q + (X_{ar}I^*_{am}\sin\delta\, /X_{qq}I^*_{qm})I^*_a - (p\Omega_{rm}X_{ar}I_{am}\cos\delta\, /nX_{qq}I^*_{qm})I_a\Omega_r \quad (4)$$

$$M_e = (p/M_{em}w)(X_{md}I_{mm}I_{qm})I_mI_q + X_{ar}I_{am}I_{dm}\sin\delta)I_aI_d + (X_{ar}I_{am}I_{qm}\cos\delta)I_aI_q + (X_{dd} - X_{qq})(I_{qm}I_{dm})I_qI_d \quad (5)$$

$$\Omega^*_r = (M_{em}/Jn\Omega^*_{rm})M_e - (M_{em}/Jn\Omega^*_{rm})M_L \quad (6)$$

The base voltage is taken equal to the maximum value of the applied voltage. Thus the supply voltage in normalized form may be written as:

$$V = \sin 10\,\tau \quad (7)$$

The analogue computer setup is shown in Figure (3), Equations (4) are simulated in Figure (3-a), while figure 3(b) is a simulation of equations (5) and (6). Table II gives the potentiometer settings.

4. DYNAMIC PERFORMANCE

The normalized equations given above have been solved on an analogue computer to study the dynamic starting performance with the assumption of free acceleration.

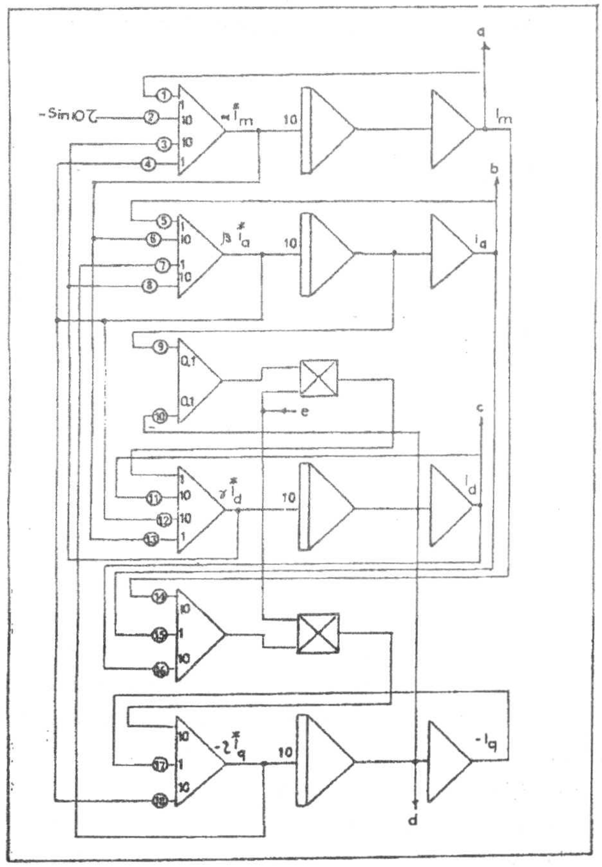

Figure 3-a. Analogue simulation of current equations.

The analogue computer used is made by EAI-Electronic Associates PTY ltd, Australia. It has twelve amplifiers, 24 potentiometer subjected to nonlinearity of 0.1%, 8 multipliers of accuracy 1% in all quadrants, 12 summers and 12 integrators. The latter may be used as summers. Each summer (or integrator) consists of a high performance operational amplifier and 5 computing resistors or (three resistors and two capacitors). Both the precision resistors and capacitors are matched to 0.25%. The computer is equipped with a 3 digits voltmeter which is used to monitor all computer variables.

24 potentiomters are available; all of them have been used. Due to the shortage in potentiometer higher harmonics could not be included. Sinusoidal voltage is applied to the main winding and a switching instant of 0 degree is assumed i.e. $V = V_m \sin wt$. However, if more potentiometer are available, it would have been

so easy and possible to include higher harmonics. Each additional harmonic requires two more potentiometer. The harmonics would have then been added up and fed to the top left summer of Figure (3-a) at its input where the function (- sin 10τ) is being fed at present.

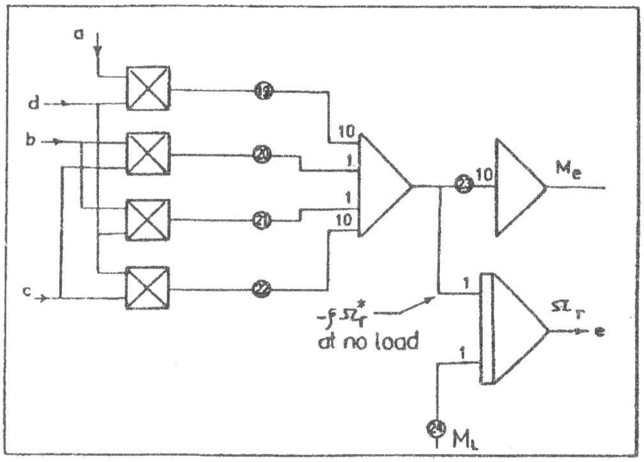

Figure 3-b. Analogue simulation of torque equation.

The motor is a 4-pole machine rated 50 watts, 220 volts. The motor constants as well as the base values are given in table I of the appendix. The potentiometer settings are given in table II of the appendix.

Torque/time and Speed/time curves are shown in Figures (4) and (5). The torque/time curve shows (i) fundamental frequency torque in the initial region, (ii) region of maximum torque and (iii) no load region with zero average torque with double frequency torque. This result is very much similar to that obtained by Desai and Mathew [5] without the consideration of harmonics. The distinct regions are also shown in the results of Akpinar and Kaya [9].

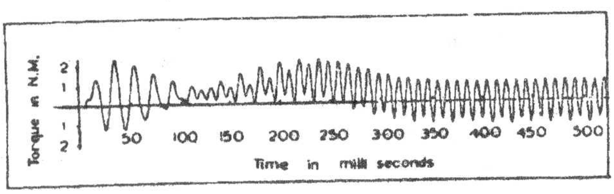

Figure 4. Torque/time curve.

When comparing the present results with that of reference [5] it is found that the maximum torque is the same in both results which is about 2.2 N.m. Also, the maximum torque occurs at about the same time in both results. However, there is a difference between the amplitude of the steady state no load torque; 1.2N in the analogue computer results compared to 1.7N in theirs.

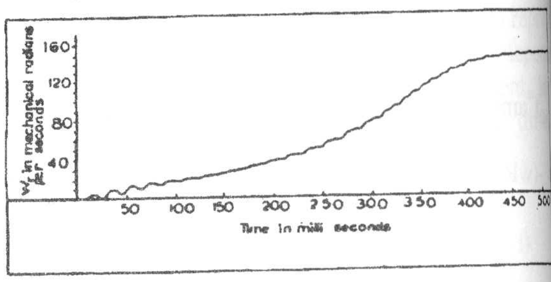

Figure 5. Speed/time curve.

The speed/time curve shows pulsations at the fundamental frequency in the initial starting reg Similar results are observed by Desai and mathew The steady state speed is reached at about 0.5 sec in the present results compared to 0.37 seconds in t results.

The main winding current/time is shown in Fig (6). It alternates at the rated frequency and has initial peak as the case of any inductive circuit. T results of reference [5] are similar to that of the pres results. However, the first peak is about 5 amperes the present paper compared to 4.6 amperes in referen [5].

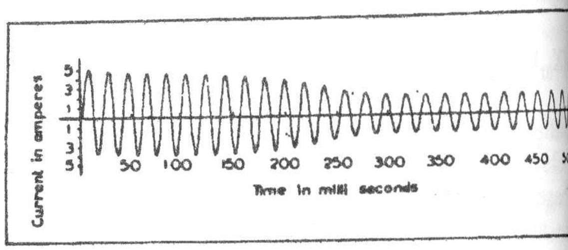

Figure 6. Main winding current/time curve.

Figure (7) shows the variation of the rotor d-a current, referred to the main winding, with time shows a first peak of about 3 amperes. The curves the main winding and the d-axis rotor currents sh that they remain substantially constant at the lo rotor values until the speed has increased substantia As for conventional machines the currents decreas their no load values.

The curve for the q-axis rotor current, referred to main winding, versus time is shown in figure 8. the interesting feature of this is that, it has a large d.c. component in the initial period which decays to zero after about 175 milliseconds, goes to negative values and becomes zero again at about 300 milliseconds. The steady state condition has zero d.c. component. Desai and Mathew [5] quoted that this current undergoes a phase reversal around 150 milliseconds. The steady state value is about 0.7 ampere.

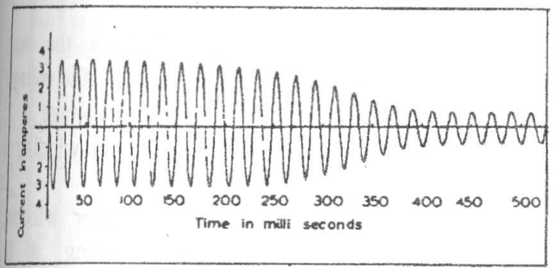

Figure 8. Rotor d-axis current/time curve.

The actual rotor current should be several times greater than the main winding current. However, it should be realized that the values of the components of the rotor current given in Figures (7) and (8) are referred to the main winding as stated earlier at the end of section 2.

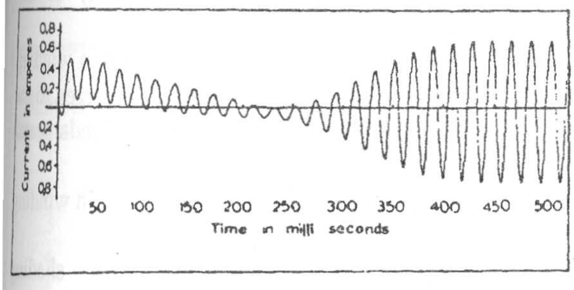

Figure 9. Rotor q-axis current/time curve.

The actual shading coil current/time curve is shown in Figure (9). It remains constant for a short period, shows some decrease around 200 milliseconds, and then recovers again. This confirms fairly well the esults of Desai and Mathew [5]. The initial, lowest nd no load amplitudes are estimated 233, 183, and 33 amperes respectively. The corresponding figures of eference [5] are 250, 200 and 300 amperes espectively.

The difference between the present results and their results are due to the errors expected in both analogue and digital computations. In comparing analogue and digital solutions, it should be realized that the accuracy of digital solution depends on the digital programmer himself as well as the equipment. It is his skill, the size and speed of the computer and the time allotted to obtain a solution which determine the accuracy of the final results.

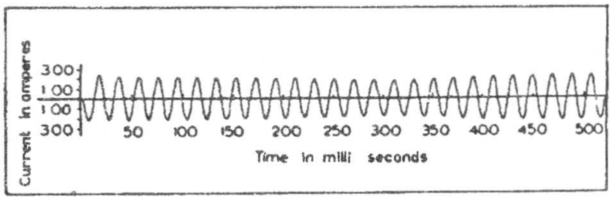

Figure 9. Shading winding current/time curve.

Some of the errors in analogue computation are due to Component errors, Mechanization errors and Readout errors. The first are due to the deviation of the component's output from its actual value based on constant (zero frequency) inputs e.g. amplifier drift. This is found null (up to the third decimal) during the period of computation. Mechanization errors have been reduced by: (1) reduction of the number of amplifiers. (2) optimization of potentiometer settings by rescaling. The readout error on a three decimal voltmeter is 1 in the third digit. While the setting error of potentiometer number 12, which represents the worst, could reach 7%, Other potentiometers have setting errors very much less than this value. However digital solutions are also subjected to readout errors.

Iterative procedures can be employed to improve the solution accuracy. The only draw back of this technique is the amount of computation and components required. Nevertheless, it is normal to use the analogue computer in order to determine the general characteristics of a physical system.

5. CONCLUSIONS

1. Transient performance of a shaded pole motor under dynamic starting conditions have been obtained by simulating the performance equations on an analogue computer.
2. A large fundamental frequency torque component is present which causes fundamental pulsation in the

rotor speed in the initial period of starting. This is confirmed experimentally by Desai and Mathew[5]. The present results, in general, agree to a great extent with the results of reference [5].

3. The advantage of using analogue simulation is that it can determine the general characteristics of a physical system promptly in the form of traces on an output device such as an x-y plotter.

6. NOMENCLATURE

I^*_{mm}, I^*_m	base value of the derivative of the main winding current with respect to τ and the normalized value $(di_m/dt)/nI^*_{mm}$ respectively.
I^*_{am}, I^*_a	base value of the derivative of the shaded coil current with respect to τ and the normalized value $(di_a/dt)/nI^*_{am}$ respectively.
I^*_{dm}, I^*_d	base value of the derivative of the d-axis rotor current, referred to the main winding, with respect to τ and the normalized value $(di_d/dt)/nI^*_{dm}$ respectively.
I^*_{qm}, I^*_q	base value of the derivative of the q-axis rotor current, referred to the main winding, with respect to τ and the normalized value $(di_q/dt)/nI^*_{qm}$ respectively
i_m, I_{mm}, I_m	main winding current; actual value, base value in ampere and normalized value i_m/I_{mm} respectively
i_a, I_{am}, I_a	shading winding current; actual value, base value in ampere and normalized value i_a/I_{am} respectively.
i_d, I_{dm}, I_d	rotor d-axis current, referred to the main winding; actual value, base value in ampere and normalized value i_d/I_{dm} respectively.
i_q, I_{qm}, I_q	rotor q-axis current, referred to the main winding; actual value, base value in ampere and normalized value i_q/I_{qm} respectively.
J	rotor moment of inertia.
m_e, M_{em}, M_e	electromagnetic torque; actual value, base value in N.m. and normalized value m_e/M_{em} respectively.
m_L, M_L	load torque; actual value in N.m normalized value m_L/M_{em} respective
n	ratio of computer time to real time t.
p	pole pairs.
R_m	resistance of the main winding.
R_a	resistance of the shading winding.
R_r	resistance of the rotor winding refer the main winding.
t	actual time.
v	instantaneous voltage to the main wi in volts = V_m sin wt.
V_m	maximum voltage applied to the winding; also the base value of the vol
w	angular frequency of applied voltage
X_l	leakage reactance of the main windi the fundamental frequency.
X_r	leakage reactance of the rotor win referred to the main winding, at fundamental frequency.
X_{mm}	self reactance of the main winding a fundamental frequency.
X_{aa}	self reactance of the shading winding a fundamental frequency.
X_{dd}	self reactance of the d-axis rotor win referred to the main winding, at fundamental frequency.
X_{qq}	self reactance of the q-axis rotor win referred to the main winding, at fundamental frequency.
X_{md}	mutual reactance between the main win and the d-axis rotor winding, referred to main winding, at the fundame frequency.
X_{ma}	mutual reactance between the main win and the shading winding.
X_{ar}	mutual reactance between the sha winding and the rotor winding.
τ	computer time.
w_r, Ω_{rm}, Ω_r	rotor angular velocity; actual value, b value in mechanical radians per seconds normalized value w_r/Ω_{rm}.
Ω^*_{rm}, Ω^*_r	base value of the angular acceleration w respect to τ and the normalized va $(dw_r/dt)/n\Omega^*_{rm}$ respectively.
δ	angle between the main winding and shading winding.
α	constant equals $I^*_{mm}/10I_{mm}$
β	constant equals $I^*_{am}/10I_{am}$

constant equals $I^*_{dm}/10I_{dm}$
constant equals $I^*_{qm}/10I_{qm}$
constant equals $\Omega^*_{rm}/\Omega_{rm}$

APPENDIX

Table I.

Motor constants				Simulation constants			
R_m	=	13.46	ohms	I_{mm}	=	5.51	A
R_a	=	11.9×10^{-4}	ohms	I_{am}	=	350	A
R_r	=	56.52	ohms	I_{dm}	=	5	A
X_{md}	=	333.0	ohms	I_{qm}	=	1.376	A
X_l	=	22.05	ohms	nI^*_{mm}	=	1500	$A.S^{-1}$
X_r	=	33.9	ohms	nI^*_{am}	=	61500	$A.S^{-1}$
X_{aa}	=	0.0033	ohms	nI^*_{dm}	=	1360	$A.S^{-1}$
X_{ma}	=	0.487	ohms	nI^*_{qm}	=	215	$A.S^{-1}$
X_{ar}	=	0.575	ohms	$n\Omega^*_{rm}$	=	1000	Nm
X_{qq}	=	115.76	ohms	M_{em}	=	1	
a	=	45	Degrees	n	=	31.4	
w	=	314	Radians/s	α	=	0.867	
J	=	0.0015	$Kg.m^2$	β	=	0.559	
$2p$	=	4		γ	=	0.866	
V_m	=	311	Volts	ζ	=	0.497	
Ω_{rm}	=	157	Radians/s	ξ	=	0.203	

Table II. Potentiometer Settings

pot. Number	Setting	Pot. Number	Setting
1	0.038	13	0.999
2	0.016	14	0.115
3	0.085	15	0.089
4	0.087	16	0.115
5	0.360	17	0.488
6	0.232	18	0.089
7	0.484	19	0.217
8	0.176	20	0.613
9	0.775	21	0.168
10	868	22	0.149
11	0.015	23	0.739
12	0.008	24	0.135

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