# ANALYTICAL PHOTOTRIANGULATION AND ITS PERSPECTIVES

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#### ABSTRACT

The main purpose of this paper is to establish aspects of analytical phototriangulation concepts and procedures, the backbone of which consists of various mathematical models. The modern analytical phototriangulation seems to be overwhelmed with the technique of adjustment computations. A rigorous solution, even by using the method of least squares, would not always justify the trouble expended. This is because the data, observed or calculated, do not depend merely on errors of observation but also have their origin in the inequalities of the terrain, atmospheric conditions, measuring instrument, camera, ...... etc. as well as the inadequacy of the mathematical models used for the purpose of processing, assessing and analyzing the data. Analytical phototriangulation is now on the verge of a great expansion in its applications. This is indicated by the developments of several new comparators and analytical plotters, which utilizes an on-line computer.

### 1. INTRODUCTION

Analytical phototriangulation is the most vivid example of intensive and successful development of photogrammetry.

It is based on the use of automatic precise devices and computers. For a comparatively short period of time-during two last decades a rigid theory of construction and adjustment of strip and block photogrammetric networks achieved very high level, highly precise comparators and monocular comparators with automatic recording of results of image measurements were created, technology of different methods of phototriangulation and methods of systematic errors effects were worked out in detail. These achievements enabled one to increase the accuracy and efficiency of analytical phototriangulation in comparison to another method.

Analytical phototriangulation received the general acknowledgement and broad application as the universal and efficient method for bridging.

### 2. THE STRIP PHOTOTRIANGULATION

Three types of the strip phototriangulation received the most spreading:

1. The partly dependent model method.

- 2. The independent model method.
- 3. The bundle method.

A single model created on stereopair is an elementary link in the first and second methods.

# 2.1. The partly dependent model method

It was worked out at the beginning of a development of the analytical phototriangulation.

The mathematical model of this method corresponds to phototriangulation carried out by means of universal stereoscopic plotters provided with arrangements to establish negative bases and to interchange an observation system and also carried out by stereoscopic plotters, which have no such arrangements but allow to determine orientation elements and to change photos in carriers.

In this method a common model of a strip is formed as a result of a successive construction of a single models in the universal coordinate system taken for the first model. Exterior elements of photos are found using absolute or arbitrary exterior elements of the photos and relative orientation elements of photos with arbitrary photobase. Therefore each successive model is transformed to the scale of the preceding one by tie points.

Coordinates of model points are calculated by formulae of the photographic intersection. The reduction of coordinates of common model points to ground coordinates of corresponding points of the terrain are carried out by control points.

The number of these points usually is enough to define elements of the exterior orientation of the model and eliminate residual systematic errors by means of polynomials of the second and third orders.

## 2.2. The independent model method

In this method the sequence of single model construction may be arbitrary. Every model is created in a local coordinate system. The axis X of the local system coincides with the photobase and the axis Z is established in the main base plane of the left photo.

The coordinates of the model points are obtained by formulae for stereopairs using the measured coordinates of image points, the elements of relative orientation and the arbitrary photobase.

One of the models, the first model, for example, is assumed as an original one. Others are connected to this model by means of tie points.

Seven elements of orientation, which are necessary for calculation of points coordinates, are defined in the system assumed for the original model. Thus a common model is obtained, exterior orientation of which is made by the same way as in the first method.

#### 2.3. The bundle method

The main principle of this method is based on the bundle of rays forming the photo. The aggregate of bundles belonging to the strip are simultaneously arranged so that the corresponding rays intersect and coordinates of control points are equal to their ground coordinates. If these conditions are met, the elements of exterior orientation of the photos and ground coordinates of pass points are defined.

The initial equation in this method is the interdependences between the coordinates of the image point and the coordinates of the corresponding point of the terrain.

The total number of such equation is two times more than the number of image point.

The number of unknowns are defined by:

$$N = 6m + 3h$$

where:

m -the number of the photographs h -the number of the pass points

Therefore, the large equation systems are obtained which contain the elements of exterior orientation of the photographs and coordinates of the pass points. These equations are solved by means of the least square method by iteration.

In principle, bundle adjustment requires the largest computer memory and includes the largest number of mathematical operations, while for polynomial adjustment these requirements are at a minimum.

The accuracy of the first and second methods is the same. Theoretically the bundle adjustment should give the highest accuracy especially when the self calibration technique is applied.

The disadvantage of strip phototriangulation is the necessity to provide each strip with ground control points. Therefore at present strip phototriangulations mainly used for prospecting and construction of linear objects.

### 3. BLOCK PHOTOTRIANGULATION

The wish to reduce the amount of field work as much as possible and to increase the accuracy of analytic bridging by using interstrip connections resulted in the development and application in practice of block phototriangulation which does not require ground control points for each strip.

There are three main methods of block phototriangulation

- 1. Strip network connection method
- 2. Independent models method
- 3. Bundles method

# 3.1. Strip network connection method

This method includes the construction of independent strip networks, their connection into a common net by tie points and exterior orientation by ground control points.

Deformation of strip nets and a common network

caused by residual systematic errors is taken into account by tie and control points by means of the second and third order polynomials.

The accuracy of this method is reduced considerably when the area of side over lap is less and there are no control points in this area.

# 3.2. Independent models method

It is based on the construction of single independent models, their connection into a block by tie points and exterior orientation of the common model by ground control points.

The accuracy of this method is higher than the accuracy of the first one.

### 3.3. Bundles method

The bundles method is the most rigid and accurate method of block phototriangulation.

The mathematical model of this method includes the equations of collinearity, formed for each image point of block net. Solving a set of such equations gives the exterior orientation elements of photographs and the coordinates of the pass points. The method makes it possible to perform simultaneous adjustment of photogrammetric, geodetic, and auxiliary measurements. Three sets of equations are founded and solved taking into account weights of measured quantities.

The necessity to solve large sets of equations is the main difficulty which prevents the method to be used widely. To reduce the order of normal equations the successive insertion of unknowns is used, the method of individual adjustment of exterior orientation elements and coordinates of pass points, multigroup block adjustment and quasiphotos. This enables one to use digital computer of medium memory for the method of bundles.

The method of quasiphotos is based on the division of a block into overlapping sub-blocks. An independent net is constructed for each sub-block and a quasiphotos is calculated (central projection of sub-block points). Then a block network by quasiphotos is made, the number of quasiphotos being several times less than the number of real ones.

Analytical phototriangulation allows to create control networks also on a celestial bodies, using the

information obtained from the spaceships. Therefore investigation of the geometrical characteristics of panoramic phototelevision and other special photographs is going on for them to be used for measurements.

# 4. MEASUREMENT DEVICES AND DIGITAL COMPUTERS

Automatic precise stereocomparators (single-carriage, double-carriage and three-carriage) and monocomparators are used for measuring the coordinates of points on photographs of 1-3 mic. accuracy.

Double-carriage stereocomparators are widely used. Three-carriage stereocomparators makes it possible to measure the coordinates of points simultaneously but the device is complex, large and expensive.

Single-carriage stereocomparators is simple but it requires photo outlines for all the points of the net. The observations by means of monocomparators require premarking of points on photograph.

One should take into account that the process of measuring the photographs is the most labour consuming in all office work in aero-phototriangulation.

### 5. CONCLUSION

The introduction analytical aero-phototriangulation in surveying allowed the 1.5 to 2.0 time increase in the accuracy of photogrammetric nets and the production of labour increased 3.0 to 4.0 times as compared to the triangulation on stereoplotters. The culture of production has increased, the manual labour has greatly decreased; all the computations are performed by computers.

The actual tasks of analytical phototriangulation result from the mentioned above:

- 1. The development of methods for determining elements of exterior orientation of photographs in flight and the methods to use these elements in constructing metric nets.
- 2. The development and introduction of effective methods of camera calibration and of taking into account systematic errors of photographs.

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- 2. The development and introduction of effective methods of camera calibration and of taking into account systematic errors of photographs.

- 3. The comparable estimation of measuring devices and their complete automation.
- 4. The choice of optimal methods for solving large sets of equations and the estimation of the aero-phototriangulation accuracy.
- 5. Detailed study of geometrical properties of space photographs and their application for control nets construction on planets as well as for solving different tasks in measurements.

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