EVALUATION OF THE MINIMUM LENGTH OF FILTER IN EARTH DAMS

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ABSTRACT

The present study in this paper aims to evaluate the minimum length of the filter downstream earth dams, based on an impervious base, by measuring experimentally the length of surface of seepage at the toe of the dam. The different parameters affecting on surface of seepage such as, the upstream retained water head, the base width of the dam and the change of the angle of inclination of the downstress face of the dam, are studied. A comparison is made between the different solutions which given by Schaffernak and Van Iterson, Casagrande, Pavlovsky, Gilbey and experimental results. Casagrande and Schaffernak formulas are modified to satisfy the experimental results and put in a new form.

NOTATION

A	minimum height of filter ($A = a \sin \alpha$),
a	surface of seepage,
$a_{(cas.)}$	Surface of seepage given by casagrande Eq. (2)
$a_{(sch.)}$	Surface of seepage given by Schafferank Eq. (1)
b	half inner distance between the two prespex plates,
d	the considered base width of the dam in equations,
d'	total base width of the dam, ($d' = d + 0.7\Delta$)
g	acceleration due to gravity,
h	the upstream retained water head,
h_o	tail water downstream,
K	hydraulic conductivity,
m	the upstream slope of the dam and equals "$\cot \beta$",
m_1	the downstream slope of the dam and equals $\cot \alpha$,
q	seepage discharge,
(A/h)	the minimum relative height of the downstream filter,
T	minimum height of the dam which equals the maximum upstream water depth,
ν	Kinematic viscosity of the oil used in the experiment,
α	angle of inclination of the downstream face of the dam,
β	angle of inclination of the upstream face of the dam which is constant and equals 45°, and
Δ	horizontal projection of the upstream slope which equals "$h \cot \beta$".

INTRODUCTION

Evaluation of the surface of seepage is studied experimentally in this paper so that minimum length of filter downstream earth dam, based on an impervious base, can be designed. The first approximate method that accounts for development of the surface of seepage as proposed independently in 1916 by Schaffernak [6] and Van Iterson [3], the length of the surface of seepage was given by the following equation:

$$a = \frac{d}{\cos \alpha} - \sqrt{\frac{d^2}{\cos^2 \alpha} - \frac{h^2}{\sin^2 \alpha}} \qquad (1)$$

Casagrande [1] analyzed the same problem and put the length of surface of seepage in the following form :

$$a = \sqrt{d^2 + h^2} - \sqrt{d^2 - h^2 \cot^2 \alpha} \qquad (2)$$

A modified form was obtained by Gilbey [2] for determining surface of seepage and depends upon quantity of seepage as follow:

$$a = \frac{1}{\sin^2 \alpha} \frac{q}{K} \qquad (3)$$

Pavlovsky [4] established the following equation considering tail water downstream "h_o"

$$\frac{q}{K} = \frac{a_o}{m_1} \left[1 + \ln \frac{a_o + h_o}{a_o} \right] \qquad (4)$$

substituting $h_o = 0$, $a_o = a \sin \alpha$ and $m_1 = \cot \alpha$. To satisfy the problem studied in this paper, equation (4) can be written as follow:

$$a = \frac{\cos\alpha}{\sin^2\alpha}\,\frac{q}{K} \qquad (5)$$

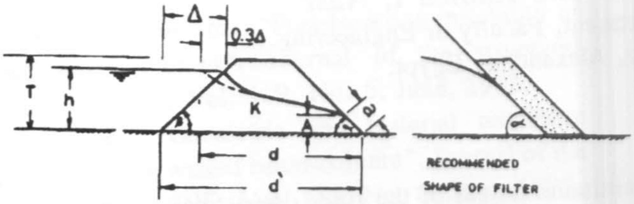

Figure 1. Geological section.

EXPERIMENTAL MODEL

The shown model in Figure (2) is used to measure experimentally the length of surface of seepage, from which the minimum relative height of filter downstream the dam can be evaluated. The model is consists of two vertical prespex plates (1). The spacing between the two plates is kept constant by using Klingarite washers (2) 1.5 mm thick. The angle of inclination of the downstream face of the dam is changed from 90 to 80, 70,60,50,40 and 30° by cutting triangular sector each experiment. The upstream face of the dam (3) has an inclination of 45° to the horizontal and is fed from a tank (4) having an overflow tube (5) to control the retained head upstream. The main supply tank (6) is connected to the feeder tank (4) by a tube (7) and the flowing oil (supper 7500-20 w/50) is controlled by a valve (8). A vertical channel (9) is used to collect the seepage discharge at the toe of the dam. A graduated tube (10) is used to measure the collected oil in a certain time by a stop watch. A tank (11) receives the excess oil passing through tube (5), this oil is lifted again to the main supply tank (6) by a small centrifugal pump (12) through the pipe (13). Temperature is recorded in each experiment and the corresponding hydraulic conductivity of the soil (K) is calculated

$$K = \frac{b^2 \cdot g}{3\nu}$$

* *Procedure of experimental work*

The experimental procedure is carried out as follow:

1. For a constant values of $d' = 72$ cm and $\alpha = 90^\circ$, the upstream retained head "h" is changed to 4,6,8,10 and 12 cm and the corresponding measured values of the length of surface of seepage and seepage discharge are recorded. The phreatic surface are also recorded in such case.
2. For the same value of $d' = 72$ cm, "α" is changed to 80,70,60,50,40 and 30° . The step No. 1 is repeated for different values of the angle of inclination "α".
3. The same experimental procedure in steps 1 and 2 are repeated for different values of $d' = 60$ and 48 cm.
4. A total of (105) experimental runs were carried out.

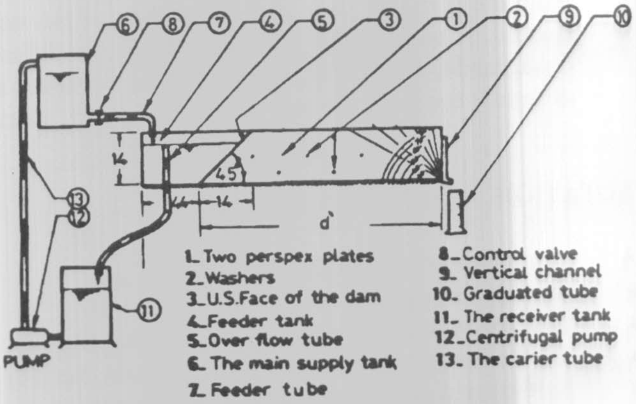

Figure 2. An experimental model DIME. IN CM.

ANALYSIS OF RESULTS

Evaluation surface of seepage at the toe of earth dam important to make a design for the minimum length filter which may be constructed downstream. Th parameters affecting on the surface of seepage (a) are the retained water height upstream the dam (h), the widt of the dam (d') at base and the angle of inclination of th downstream face of the dam (α) . In the prese experimental study, the angle (α) is considered 30, 4 50, 60, 70, 80 and 90° for a constant value of the ba width of the dam and angle of inclination of the upstrea face "β". The base width (d') is changed into 48, 60 a 72 cm. Surface of seepage is recorded in ea corresponding value of (h). Analysis of results shows th surface of seepage (a) decreases with the increase of bo the angle (α) and the base width of the dam (d') , whi increases with increasing the retained upstream wat head (h). A design charts are plotted between t minimum relative height of filter (A/h) and the relati upstream retained water head (h/d), Figures (3-a, b a c). Seepage discharge (q/kh) is also represented vers (h/d) as shown in the same figures.

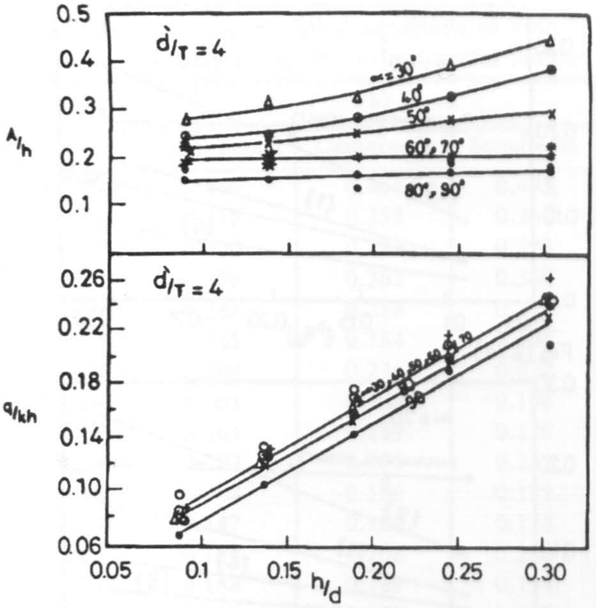

Figure 3-a. (h/d) versus both of (A/h) and (q/kh).

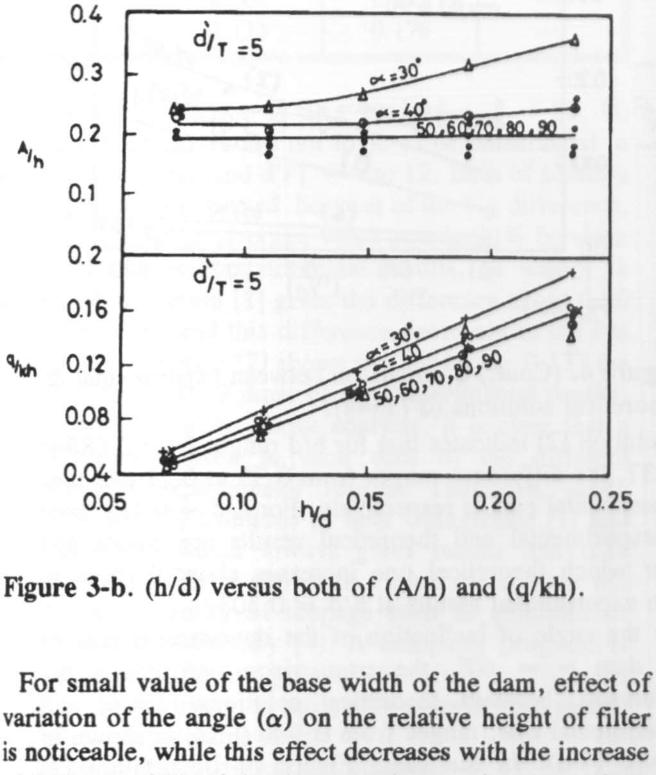

Figure 3-b. (h/d) versus both of (A/h) and (q/kh).

For small value of the base width of the dam, effect of variation of the angle (α) on the relative height of filter is noticeable, while this effect decreases with the increase of the base width of the dam, much more increase of the base width makes this effect almost constant, as shown from Table (1).

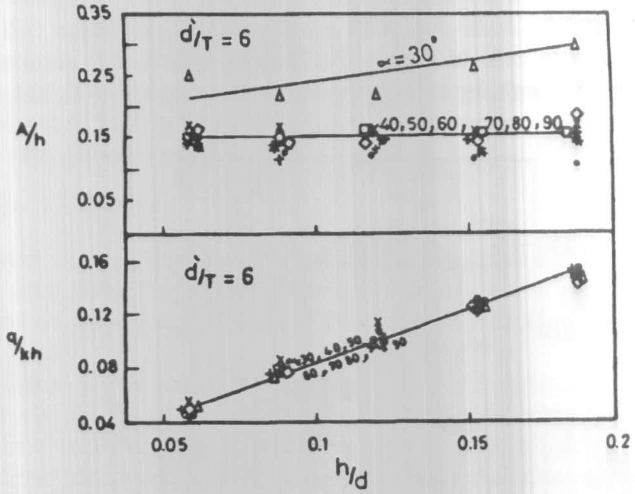

Figure 3-c. (h/d) versus both of (A/h) and (q/kh).

Table 1.

h/d	α	A/h		
		d'/T = 4 & d'= 48	d'/T = 5 & d' = 60	d'/T = 6 & d' = 72
0.15	30	0.32	0.28	0.26
	40	0.26	0.22	0.16
	50	0.23	0.22	0.16
	60	0.20	0.22	0.16
	70	0.20	0.22	0.16
	80	0.17	0.20	0.16
	90	0.17	0.20	0.16

The minimum relative height of filter is analyzed for the angle of the downstream face of the dam, $\alpha = 30^\circ$, and can be taken (A/h) = 0.32 for $(h/d) \leq 0.15$. For $0.15 < (h/d) \leq 0.3$ the minimum relative height of the filter may be considered 0.45.

For $\alpha = 40, 50, 60$ and 70° and for both of $(h/d) \leq 0.15$ and $0.15 < (h/d) \leq 0.3$, the minimum relative height of the filter can be taken 0.20 and 0.40 respectively. For $\alpha = 80$ and 90° and for $(h/d) \leq 0.3$, the minimum relative height of the filter may be evaluated 0.2. It is clear from the above results that the length of filter decreases with increase the angle of the downstream face of the dam.

A comparison is made between the experimental and the different theoretical solutions given by [1,2,4 and 6]. It is evident that, for $\alpha = 30^\circ$ and d' = 48 cm (d'/T = 4 to 12), both of the solutions[6] and [1] have a difference ranges from 0.36 to 0.72 and from 0.28 to 0.68 less than experimental results for (h/d) ranges from 0.088 to 0.303 respectively, as shown in Figure (4-a).

Solution [4] gives a difference equals 0.44 less than experimental results for h/d = 0.088, this difference reaches to 0.28 for h/d = 0.137 and decreases till reaches to zero at h/d = 0.275 after which still almost coincides with the experimental results until h/d = 0.303.

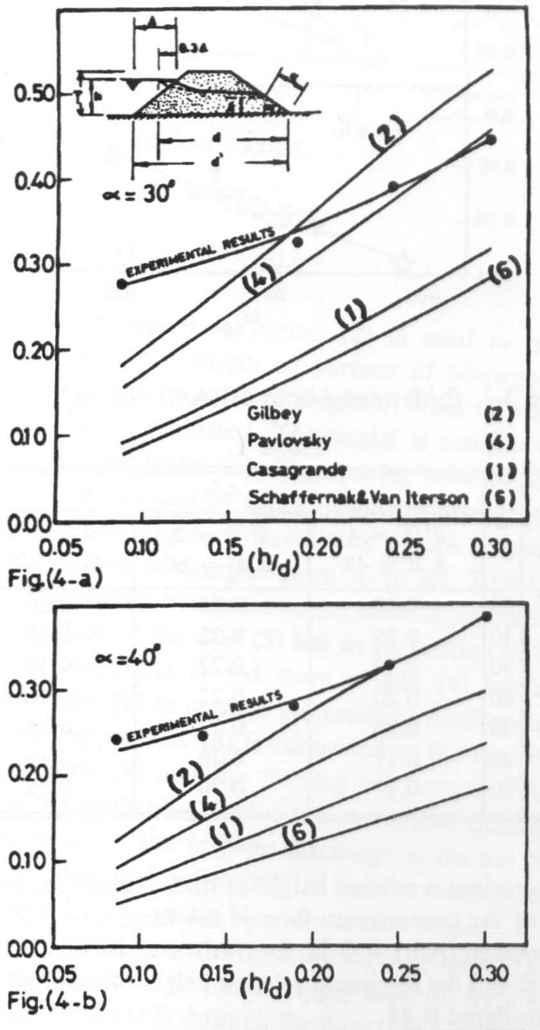

Figure 4. Comparison between experimental and theoretical solutions (d'/T = 4).

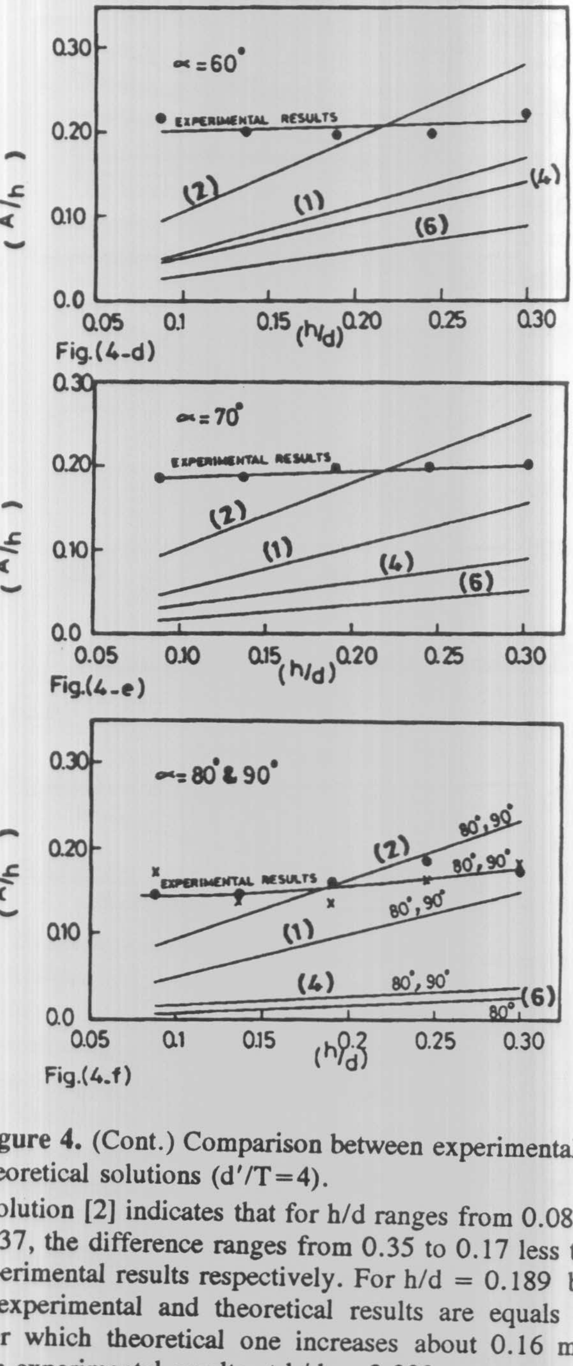

Figure 4. (Cont.) Comparison between experimental & theoretical solutions (d'/T=4).

Solution [2] indicates that for h/d ranges from 0.088 to 0.137, the difference ranges from 0.35 to 0.17 less than experimental results respectively. For h/d = 0.189 both of experimental and theoretical results are equals and after which theoretical one increases about 0.16 more than experimental results at h/d = 0.303.

For the angle of inclination of the downstream face of the dam α = 60°, the comparison indicates a big difference, between theoretical solutions [1,4,6] and experimental one, ranges from 0.3 to 0.88, as shown in Figure (4-d). In the same Figure (4-d), solution [2] indicates that for h/d = 0.137 difference about 0.30 less than experimental results and reaches to zero of h/d = 0.215 after which increases , this difference increases to 0.58 for small value of h/d = 0.088.

Table 2. Sample of the minimum relative height of the filter according to the modified equations of both Casagrande and Schaffernak, and experimental results.

α	h	h/d	(A/h)		
			Experimental	Casagrande	Schafferak
30	12	0.303	0.446	0.464	0.475
	12	0.235	0.358	0.358	0.367
	12	0.188	0.300	0.289	0.298
40	12	0.303	0.386	0.362	0.340
	12	0.235	0.246	0.289	0.273
	8	0.120	0.145	0.184	0.194
50	8	0.189	0.249	0.234	0.222
	8	0.148	0.201	0.199	0.196
	10	0.153	0.161	0.189	0.179
60	6	0.137	0.202	0.208	0.212
	6	0.108	0.188	0.186	0.199
	10	0.153	0.147	0.180	0.178
70	6	0.137	0.188	0.208	0.202
	6	0.108	0.188	0.189	0.194
	8	0.120	0.153	0.169	0.157
80	4	0.088	0.172	0.233	0.270
	4	0.071	0.222	0.221	0.268
	8	0.120	0.135	0.169	0.145
90	10	0.244	0.190	0.233	----
	4	0.071	0.200	0.223	----
	6	0.088	0.133	0.176	----

The same comparison is also made for $\alpha = 80, 90$ (although these angles are not logic to be constructed in earth embankment), and $d'/T = 4$ to 12. Both of solution [6] and [4] can not be used because of the big difference, which reaches to an average value equals 90% between these solutions and experimental results, as shown in Figure (4-f). Solution [1] gives the difference about 0.30 for $h/d \geq 0.189$ and this difference decreases to 0.17 at $h/d = 0.303$. Solution [2] shows that for $h/d \geq 0.137$ the difference equals 0.18 more or less experimental results, Figure (4-f). From the above analysis, it is clear that a big difference caused in such case of the different solutions and experimental results. Therefore, study directed to modify solutions of both Casagronde [1] and Schaffernak [6] which always gives results less than experimental results as shown in Figure (4) and are not depend upon quantity of seepage such as solutions of Gilbey [2] and Pavlovsky [4]. A computer program is used; to fit each solution [1] and [6], given by formulas (2) and (1) respectively, to satisfy the experimental results and put in the following form:

Casagrande formula after modification:

$$a=0.7+1.36a_{(cas)} \text{ for } h>0.0 \text{ [Corr. Coeff. 0.96].} \quad (6)$$

Schaffernak formula after modification:

$$a=1.05+1.52a_{(sch.)} \text{ for } h>0.0 \text{ [Corr. Coeff. 0.96]} \quad (7)$$

where $a_{(cas.)}$ and $a_{(sch.)}$ are denoted by the formulas (2) and (1) respectively. Dimensions of equations (6) and (7) in meters.

Table (2) shows some results of the new shape of casagrande and Schaffernak formulas if compared with the experimental results.

CONCLUSIONS

From analysis of the experimental study for determining the minimum length of filter downstream earth dam, based on an impervious base, the following conclusions are made:

- * The required length of filter increases with decreasing both of the angle of inclination of the downstream face and the base width of the dam, also length of filter increases with increasing the retained upstream water head.
- * The minimum relative required length of filter may be taken, for $\beta = 45^o$, as follow:

α^o	(A/h)	
	$h/d \leq 0.15$	$0.15 < h/d \leq 0.30$
30	0.32	0.45
40,50,60 & 70	0.20	0.40
80 & 90	0.20	0.20

- * Based on the experimental results, modified formulas of both Casagrande and Schaffernak are put in a new form, equations (6) and (7) respectively.
- * A design charts are plotted between both of (A/h) and (q/Kh) versus (h/d), for values $d'/T=4,5$ and 6.

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