BEHAVIOUR OF BEAM-COLUMNS SUBJECTED TO NORMAL AND BIAXIAL BENDING

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ABSTRACT

A numerical procedure to determine the load-deformation response of H-columns under biaxial bending is presented in this study. An incremental-iterative procedure based on the finite difference technique is employed in this study to simulate the elastic and inelastic behaviour of prismatic beam-column subjected to increased loads. The influence of residual stresses and initial deformations are also included in the study. In the adopted procedure the flexural stiffness about x and y axes, and the warping stiffness changes are included according to the distribution of yielded portions of cross section during each stage of loading. Correlation with existing experimental results shows excellent agreement.

INTRODUCTION

A review of the literature pertaining to biaxially loaded columns presented by Chen [1]. Wagner [2] was the first to investigate flexural-torsional buckling. Goodier [3] formulated governing differential equations pertaining to beam-columns subjected to biaxial bending under the conditions of small displacement and rotation. Equations were also derived by Timoshenko [4], and Vlasov [5] under similar assumptions. Goodier's equations which pertain to members loaded identically at each end, were solved exactly by Culver [6] and approximately by Thurlimann [7]. Birnstil, Harstead and Leu [8] have indicated in their investigation the inelastic response of H-columns under biaxial bending. Birnstial [9] also conducted experiments on isolated steel H-columns, which were tested under biaxial eccentric loading. Inelastic behaviour of rotationally restrained columns under biaxial bending and torsion using finite difference method have been reported by Vinnakota [10].

Nakashima [11] studied buckling and post buckling behaviour of steel beam - columns. Meek [12] studied the effect of plastic area deformation after local yielding.

The aim of this study is to develop a numerical, simplified procedure for the solution of a generalized problem for biaxially loaded columns with different eccentricities at each end. The influence of residual stresses and initial deformation are included in this study. The finite difference method is employed to solve the general differential equations of equilibrium. A computer program is developed in order to determine the load-deformation's response. The numerical solutions are compared with both experimental and analytical investigations, to verify the capabilities of the proposed model.

DERIVATION OF MATHEMATICAL MODEL

The following assumptions are made in the analysis:

1. The stress-strain diagram of the material is ideally elastic-perfectly plastic.
2. Strain hardening is neglected.
3. The cross section retains its original shape during deformations.
4. The deformations are considered small .
5. Yielding is governed by normal stress only.

The normal strain $\epsilon_{(\eta,\xi,\omega)}$ at a point (η,ξ,ω) of a cross section situated at a distance z from the origin, and the deformations (u, v, θ) of the pole of this cross section are related by the following formula

$$\epsilon_{(\eta,\xi,\omega)} = \epsilon_o + \nu'' \eta - u'' \xi - \theta'' \omega + \epsilon_r \quad (1)$$

where ϵ_r is the residual strain.

If the strain $\epsilon_{(\eta,\xi,\omega)}$ is inferior to the yield strain of the material, the fiber is still elastic, and thus

$$\sigma_{(\eta,\xi,\omega)} = E_{(\eta,\xi,\omega)} * [\epsilon_o + v''\eta - u'' \xi - \theta'' \omega] + \sigma_{r(\eta,\xi,\omega)} \quad (2)$$

where $\sigma_{(\eta,\xi,\omega)}$ is the normal stress at a point (η,ζ,ω) of the cross section situated at a distance z from the origin $E_{(\eta,\xi,\omega)}$ is Young's modulus at point (η,ξ,ω), $\sigma_{r(\eta,\xi,\omega)}$ is the residual stress at point (η,ξ,ω).

When the strain $\epsilon_{(\eta,\xi,\omega)}$ is equal to or superior to the yield strain $\sigma_{(\eta,\xi,\omega)} = \pm \sigma_y$

where σ_y is the yield stress

The internal forces are:

$$\text{Axial force } P = \int \sigma_{(\eta,\xi,\omega)} \, dA \tag{3}$$

The internal moments about the axes (ξ, η and ζ) are

$$M_{\xi\ int} = \int \sigma_{(\eta,\xi,\omega)} \, \eta dA \tag{4}$$

$$M_{\eta\ int} = - \int \sigma_{(\eta,\xi,\eta)} \, \xi dA \tag{5}$$

$$M_{\zeta\ int} = Gk_T\theta' + \theta' \int \sigma_{(\eta,\xi,\omega)}(\xi^2+\eta^2)$$

$$dA - \theta''' \int E_{(\eta,\xi,\omega)} \, \omega^2 \, dA \tag{6}$$

In order to examine the external forces consider a column in a deflected configuration at an arbitrary section $z = z_c$ as shown in Figure (1). The deflection of the centroid c of the local coordinate ξ and η are u and v. The location of c is defined by C:(u,v,z). Taking another local coordinates (X,Y,Z) which is parallel to the global coordinates (x,y,z). The bending moment M_x and M_y at a section z produced by the end moments

$$M_x = M_{xo} + \frac{z}{L}(M_{xl} - M_{xo}) \tag{7}$$

$$M_y = M_{yo} + \frac{z}{L}(M_{yl} - M_{yo}) \tag{8}$$

where $M_{xo} = - P.e_{yo}$ $\quad M_{yo} = P.e_{xo}$ at $z = 0$

$M_{x1} = - Pe_{yl}$ $\quad M_{yl} = Pe_{xl}$ at $z = L$

$$M_{X(ext)} = M_x + P.v \tag{9}$$

$$M_{Y(ext)} = M_y - P.u \tag{10}$$

$$M_{Z(ext)} = - \frac{u}{L}(M_{xl} - M_{xo}) - \frac{v}{L}(M_{yl} - M_{yo}) \tag{11}$$

In order to equate the internal forces M_ξ & M_η and M_ζ in equations (4), (5) and (6) with the external forces M_x & M_y and M_z a relation between the two coordinate axes (ξ,η,ζ) and (x,y,z) is needed. They are related through the rotation matrix [R].

$$[R] = \begin{bmatrix} 1 & \theta & -\dot{u} \\ -\theta & 1 & \dot{v} \\ \dot{u} & \dot{v} & 1 \end{bmatrix}, \text{ and thus:} \tag{12}$$

$$\begin{bmatrix} M_\xi \\ M_\eta \\ M_\zeta \end{bmatrix}_{ext} = [R] \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}_{ext} \tag{13}$$

$$M_{\xi ext} = M_x + Pv + \theta M_y - Pu\theta + \frac{u}{L}(M_{xL} - M_{xo})u' + \frac{v}{L}(M_{yL} - M_{yo})u'$$

$$M_{\eta ext} = -M_x\theta - Pv\theta + M_y - P.u + \frac{u}{L}(M_{xL} - M_{xo})v' - \frac{v}{L}(M_{yl} - M_{yo})v' \tag{14}$$

$$M_{\zeta ext} = M_x\dot{u} + Pvu + M_y\dot{v} - Puv - \frac{u}{L}(M_{xl} - M_{xo}) - \frac{v}{L}(M_{yl} - M_{yo})$$

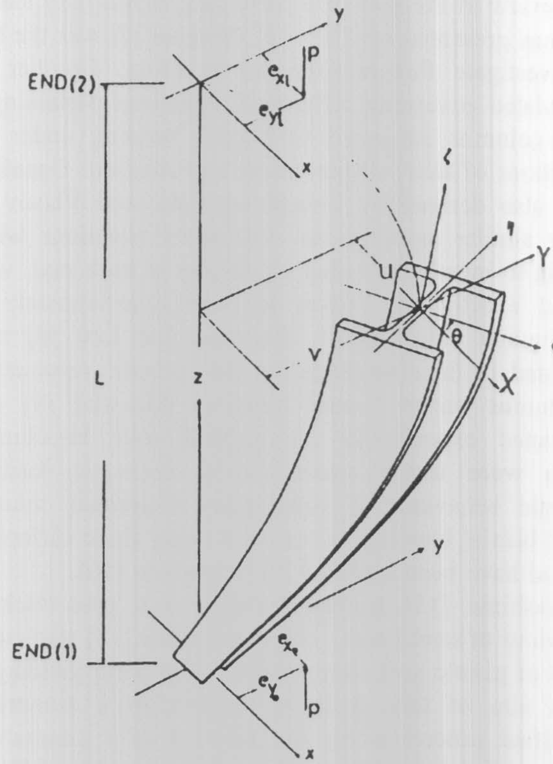

Figure 1. Column in space.

The differential equations are obtained by equating the internal moments to the external moments in the deformed configuration. However for doubly symmetric section such as H-shape,it may be assumed that :

$$\int E_{(\eta,\xi,\omega)}\eta dA = \int E_{(\eta,\xi,\omega)}\xi\eta dA = \int E_{(\eta,\xi,\omega)}\omega\eta dA = \int E_{(\eta,\xi,\omega)}\xi dA$$

$$\int E_{(\eta,\xi,\omega)}\omega dA = \int E_{(\eta,\xi,\omega)}\omega\xi dA = 0.0$$

$$\int E_{(\eta,\xi,\omega)}\eta^2 dA = EI_{x(i)}$$

$$\int E_{(\eta,\xi,\omega)}\xi^2 dA = EI_{y(i)}$$

$$\int E_{(\eta,\xi,\omega)}\omega^2 dA = EI_{\omega\,(i)} \qquad (15)$$

The flexural stiffness about x and y axes, and the warping stiffness are not constant but change according to the distribution of the yielded portion of cross section. If all the nonlinear terms of displacements are neglected and the effect of residual stresses and initial geometrical imperfections are considered then:

$$EI_{y(i)}\, u'' + P(u_o + u) + M_x\,(\theta_o + \theta) = M_y (16)$$

$$EI_{x(i)}\, v'' + P(v_o + v) + M_y\,(\theta_o + \theta) = -\, M_x (17)$$

$$EI_{\omega(i)}\theta''' - (Gk_T + k'')\,\theta' + M_x(u'_o + u') +$$

$$M_y(\dot{v} + \dot{v}_o) - \frac{v + v_o}{L}(M_{yL} - M_{yo}) - (\frac{u - u_o}{L})(M_{xL} - M_{xo}) = \dot{k}\dot{\theta}_o \qquad (18)$$

where u_o, v_o and θ_o are the initial displacement

and $k'' = \int (\sigma + \sigma_r)(\eta^2 + \xi^2)\, dA$
$k' = \int \sigma(\eta^2 + \xi^2)\, dA$

Eqs (16,17 and 18) are three nonlinear and nonhomogeneous differential equations of equilibrium for a column with three unknowns u,ν and θ.

SOLUTION TECHNIQUE

The determination of the load - deformation response of the column under a given load is reduced for seeking a solution of Equations (16,17 and 18)

Considering the appropriate boundary conditions at each end, these equations are coupled in u,v, and θ. As the problem is not amenable to closed-form solution, a finite difference procedure is therefore employed. The column is divided into m segments of equal length. The derivatives in equations (16,17 and 18) are replaced by central differences at the pivotal points.

This results in a matrix equation of the form

$$[A]\,[C] = [W]$$

In which [C] = a vector of the unknown deformations (u.v and θ); [A] = a square matrix of size three (m-1); and [W] = a vector of representing the right hand side of the equilibrium equations.

The deformations corresponding to a given increment of forces are computed by an iterative procedure. In the first iteration, an increment of the deformations is computed by using the flexural and warping stiffness corresponding to the previous known deformations. In the next iteration, another increment of deformation is computed by using the flexural and warping stiffness of the updated deformations. The unbalanced forces can be computed by comparing the internal forces at the updated deformations and the external applied forces for each iteration. Iterations for eliminating the unbalanced forces are continued until the unbalanced forces are negligible.

The flexural and warping stiffness are calculated numerically by dividing the cross section into finite elements as shown in Figure (2). The strain and stress in each element were computed as the average values at its centroid.

$$EI_{\xi(i)} = \frac{-\int \sigma\eta dA}{v''} \qquad (20)$$

$$EI_{\eta(i)} = \frac{-\int \sigma\xi dA}{u''} \qquad (21)$$

$$EI_{\omega(i)} = \frac{-\int \sigma\omega dA}{\theta'''} \qquad (22)$$

The ultimate load is given by the condition that instability is imminent where large increases in deformation results from small increments of the load .

NUMERICAL VERIFICATIONS

In order to verify the simplicity, accuracy, and efficiency of the proposed model for predicting the deformational response of columns a number of numerical examples have been selected for the inelastic postbuckling behaviour of isolated steel H-column. These examples take into account the initial geometric imperfection, residual stresses, and load eccentricities . The results are compared with experimental results of Birnstiel [9] and Aribert and Abdel Aziz [13].

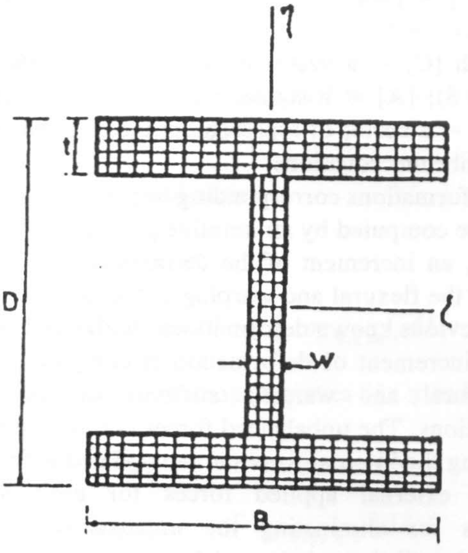

Figure 2. H-shape.

The cross section properties length, yield stress, initial geometric imperfection and the eccentricities of loading at the two ends for the four column specimens are given in Tables (1),(2) and (3). The boundary conditions at each end of the column were:

(1) lateral displacement is prevented ($u'' = v'' = 0$)
(2) Twisting is prevented ($\theta' = 0$)
(3) The cross sections at the ends are permitted to rotate about any axis in xy plane (v'' and u'' known),
(4) Warping of the end cross section is prevented ($\theta' = 0$).

Table 1. Dimension of column specimens in m_m.

Example	Length	Nominal size	Cross section properties			
			Flange width	Flange thickness	Depth	web thickness
1	2438.4	5x6 H	127.3	12.2	159.7	8.4
2	3048	4WF13	101.9	8.9	104.6	7.62
3	1000	HEA100	100	8	100	6
4	1000	HEA100	100	8	100	6

Table 2. Eccentricities of the loading and yield stresses.

Example	Eccentricities of loading (mm)						Nominal yield stress
	Top		Bottom		Average		
	e_x	e_y	e_x	e_y	e_x	e_y	MP_a
1	-23.4	70.6	-21.6	72.9	-22.6	71.6	248
2	12.7	67.8	8.6	70.4	10.7	69.1	448
3	7	--	--	--	--	---	286
4	-	--	--	--	---	---	286
	-						

Table 3. Initial Displacement of Column Specimens.

Example	Initial Displacements		
	Sweap maximum bow in x direction	camber maximum bow in y direction	Twist (Radian)
1	0.0	0.762	0.001
2	1.016	-1.016	0.007
3	32.1	---	---
4	17.0	---	---

The results of the proposed technique show a good agreement with those of the experimental investigations as shown in Figures (3) through (6).

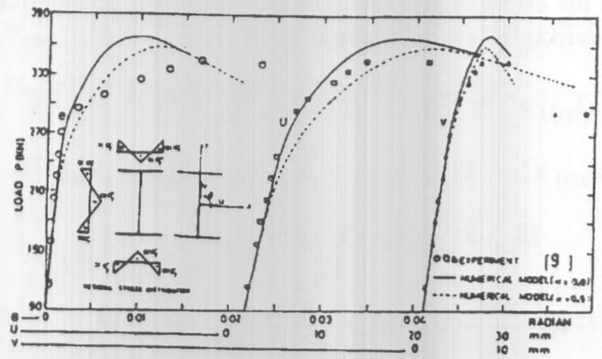

Figure 3. Load versus midheight-deformation curves example (test no.7 of Ref. 9).

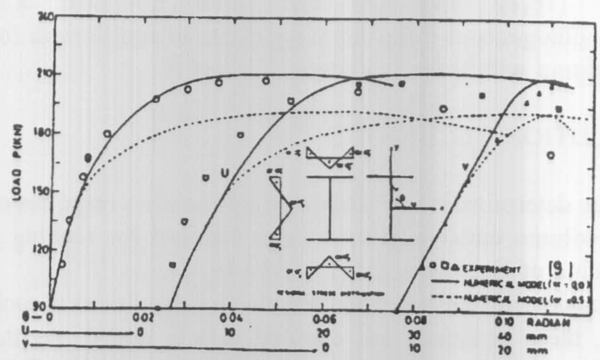

Figure 4. Load versus midheight deformation curves: example 2 (test no. 13 of Ref 9).

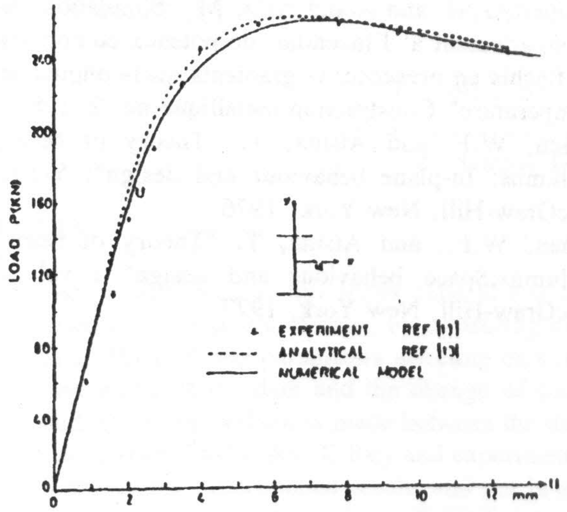

Figure 5. Load versus midheight displacement curve: example 3.

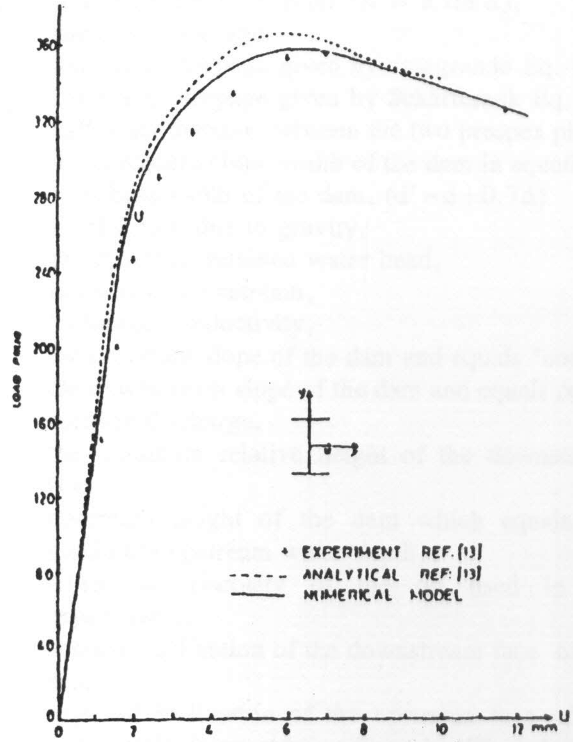

Figure 6. Load versus midheight displacement curve: example 4.

CONCLUSIONS

The proposed numerical model presented in this study simulates the behaviour of eccentrically loaded columns not only in the precritical zone but also in the post critical one. The influence of residual stresses and initial imperfections are also considered. The flexural stiffnesses about x and y axes, and the warping stiffness changes according to the distribution of the yielded portion of cross section during each stage of loading. The finite difference technique is employed to solve the general differential equations of equilibrium. The predicted ultimate loads and load-deformation responses show satisfactory agreement with the available tests and other analytical approaches. In spite of being a solution for elementary problems, the presented model can be implemented in solving complicated structural problems.

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