APPROXIMATE SOLUTION FOR FLEXURAL-TORSIONAL BUCKLING OF AXIALLY LOADED COLUMNS

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ABSTRACT

Singly symmetrical open section columns under axial load are only considered herein. In this study an approximate formula for determining the torsional-flexural buckling load of thin-walled open sections is developed. In order to evaluate the validity of this formula, the results are compared with the exact solution as well as with some experimental results. The proposed approximate formula can be used for determining the ultimate load of columns accurately.

INTRODUCTION

Thin walled cold-formed open sections are widely used as load-carrying structural members. For these shapes, because of their low torsional rigidity and open configuration, torsional-flexural buckling may be a critical mode of failure. There are three possible modes in which axially loaded columns may buckle. These columns can either bend in the plane of one the principal axes, or twist about the shear center axis or bend and twist simultaneously.

For any given member, depending on its length and the geometry of its cross section, one of these three modes will be critical. For hot-rolled structural steel sections, the load at which flexural buckling can occur is always less than the buckling loads corresponding to the other possible modes of failure. The Euler theory is therefore used to determine the carrying capacity of most of the sections. This, however, is not the case for thin-walled open sections.

The basic theory of torsional-flexural buckling is adequately documented by Timoshenko and Gere [1], Vlassov [2] and Bleich [3].

THE BASIC THEORY OF TORSIONAL-FLEXURAL BUCKLING

Consider a column of constant open section composed of (n) thin flat plates as shows in Figure (1). The plates may be of nonuniform thickness but are sufficiently thin where their lateral stiffness is negligible in comparison with the stiffness in their own plane. The length of the column is large as compared with the dimensions of the cross section.

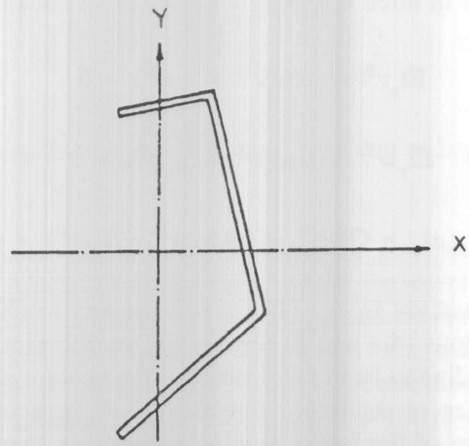

Figure 1. Thin walled open section.

The coordinates Y,Z are the principal centroidal axes of the section and X is the longitudinal center line. Under the action of external loads the column will deform and it is assumed that the cross section of the column do not distort.

The basic differential equation of bending and twisting can be derived from the theorem of stationary potential energy. The potential energy U consists of the strain energy of the deformed column V and the potential energy of the external loads U_w. Where

$$V = \frac{1}{2} \int (EI_z v''^2 + EI_y \phi''^2 + EI_\omega \phi''^2 + GJ\phi'^2)\, dx \quad (1)$$

$$U_\omega = \frac{1}{2}\int_0^L [-\sigma A(v^{I^2} + w^{I^2}) - 2\sigma A z_o v^I \phi^I + 2\sigma A y_o w^I \phi^I - \sigma I_o \phi^{I^2}]\, dx \tag{2}$$

where I_o is the polar moment of inertia of the cross section referred to the shear center.

Finally the complete expression for the potential energy (U) is the sum of Eqs. (1 an 2).

$$U = \frac{1}{2}\int_0^L [EI_z v^{II^2} + EI_y w^{II^2} + EI_\omega \phi^{II^2} + GJ\phi^{I^2} - \sigma A(V^{I^2} + w^{I^2}) - 2\sigma A z_o v^I \phi^I + 2\sigma A y_o w^I \phi^I - \sigma I_o \phi^{I^2}]\, dx \tag{3}$$

By using the theorem of stationary potential energy, eq. (3) results in three differential equations of buckling

$$EI_z v^{Iv} + \sigma A(v^{II} + z_o \phi^{II}) = 0 \tag{4.a}$$

$$EI_y w^{Iv} + \sigma A(w^{II} + y_o \phi^{II}) = 0 \tag{4.b}$$

$$EI_\omega \phi^{IV} + (\sigma I_o - GJ)\phi^{II} + \sigma A(z_o v^{II} - y_o w^{II}) = 0 \tag{4.c}$$

All derivatives in eq. (4) are with respect to (X) axis, and equations (4.a and b) express the equilibrium of the forces tending to bend an element of the column about Z and Y axes respectively. Equation (4.c) expresses the equilibrium of the forces tending to twist an element of the member about the shear center.

If the cross section of the column has one axis of symmetry such as channel sections, where ($Z_o = 0$ and $y_o \neq 0$), Eqs. (4) can be written in this form:

$$EI_z v^{IV} + Pv^{II} = 0 \tag{5.a}$$

$$EI_y w^{Iv} + P(w^{II} - y_o \phi^{II}) = 0 \tag{5.b}$$

$$EI_\omega \phi^{IV} + (r_o^2 P - GJ)\phi^{II} - Py_o w^{II}) = 0 \tag{5.c}$$

The solution of Eq. (5), for monosymmetric cross section column which has simple end supports, so that the ends are free to warp and to rotate about the Y and Z axes but cannot rotate about X axis or deflect in the Y and Z directions, is made according to these assumptions:

$$V = A_1 \sin\frac{\pi}{L}x$$

$$W = A_2 \sin\frac{\pi}{L}x$$

$$\phi = A_3 \sin\frac{\pi}{L}x$$

By equating the determinate of Eq. (5) to zero

$$(P_{cr} - P_z)[r_o^2(P_{cr} - P_y)(P_{cr} - P_\phi) - P_{cr}^2 y_o^2]] = 0 \tag{6}$$

where

P_z The Euler load about Z axis
P_y The Euler load about Y axis
P_ϕ The buckling load for pure torsion

$$P_z = \pi^2 E\frac{I_z}{L^2}$$

$$P_y = \pi^2 E\frac{I_y}{L^2}$$

$$P_\phi = \frac{1}{r_o^2}(\frac{\pi^2}{L^2}EI_\omega + GJ)$$

Eq. (6) can be derived as:

$$(P_{cr} - P_z)[\beta P_{cr}^2 - (P_y + P_\phi)p_{cr} + p_y P_\phi] = 0 \tag{7}$$

Eq. (7) has three solutions, the first one, which represents the critical flexural buckling load, is

$$p_{cr_1} = P_z \tag{8}$$

The other two solutions can be obtained by solving the following quadratic equation

$$\beta P_{cr}^2 - (P_y + P_\phi)P_{cr} + P_y P_{cr} = 0$$

where $\beta = r_c^2 / r_o^2$
rc Radius of gyration about the centroid
ro Radius of gyration about the shear center

$$P_{cr} = \frac{1}{2\beta}[(p_y + P_\phi) - \sqrt{(P_y + P_\phi)^2 - 4\beta P_y P_\phi}] \tag{9}$$

Eq. (9) leads to the critical torsional-Flexural buckling load, which is always smaller than P_y and P_ϕ, but it may be either smaller or larger than P_z.

APPROXIMATE THEORETICAL MODEL

As a consequence of the flexural-torsional buckling behaviour described before and referring to eq. (5), the flexural-torsional buckling results from the interaction of two effects; bending about major axis (Y), and twisting about the shear center.

The problem of obtaining the critical flexural-torsional buckling load can be simplified, if it is restricted to singly-symmetrical cross sections. Fortunately, this grouping includes most of the cold formed shapes commonly used for compression members.

For singly symmetrical cross sections, (Y axis is the axis of symmetry), which may buckle in the flexural-torsional mode, the critical buckling load is given by the interaction of Eqs. (5.b and c). These equations are referring to the displacement (ω) and the angle of rotation (ϕ).

For determining a simple formula for the flexural-torsional buckling load, it is assumed that the relationship between these two parameters is

$$\omega = \frac{m}{2}\phi \qquad (10)$$

where m the distance between shear center and the web plate

This relation is assumed according to the shape of the cross section after buckling, as shown in Figure (2).

Eqs. (5.b, c and 10) give two values for the flexural-torsional buckling load, the first one is

$$P_{cr_1} = \frac{m}{m - 2y_o} P_y \qquad (11)$$

The second value is

$$P_{cr_2} = \frac{2}{2r_o^2 - y_o m}\left(\frac{\pi^2}{L^2}EI_\omega + GJ\right) \qquad (12)$$

A comparison, between the results obtained from Eq. (11) and Eq. (12), is made for different shapes and cross section dimensions. This comparison shows that P_{cr2} is always less than P_{cr1}. Therefore P_{cr2} can be considered as the approximate critical flexural-torsional buckling load for two hinged-axially loaded cold-formed column. Thus,

$$P_{cr_{P.T}} \quad \frac{2}{2r_o^2 - y_o m}\left(\frac{\pi^2}{L^2} EI_\omega + GJ\right) \qquad (13)$$

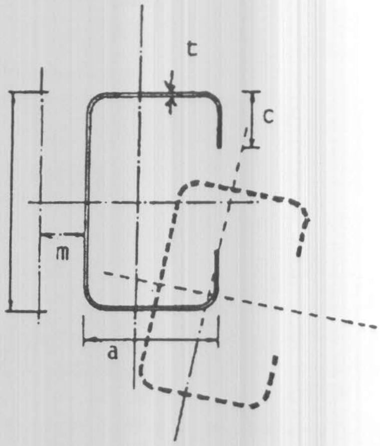

Figure 2. Flexural-Torsional buckling of lipped channel.

Figures (3) through (9) show the comparison between the results from Eq. (9), which represent the critical flexural-torsional buckling loads, and the results from the approximate equation Eq. (13), for different cross sectional geometries. This comparison shows that the approximate equation always gives values less than the exact equation with a maximum difference less than 8 percent. This means that the approximate equation results in conservative values and can be used for determining the critical axial flexural-torsional buckling load for different cross sections which has one axis of symmetry accurately.

EXPERIMENTAL VERIFICATION

In order to verify the results obtained by the approximate equation, a comparison with pervious test results is carried out. In this study the ultimate loads are calculated by using the approximate solution and compared with the experimental ultimate load conducted by Thomasson [4], Loughlan [5], Mulligan [6]. The results show a very good agreement between the critical load calculated by the approximate equation and the exact equation. The ultimate loads are computed according to AISI (7) as shown in Tables (1,2 and 3).

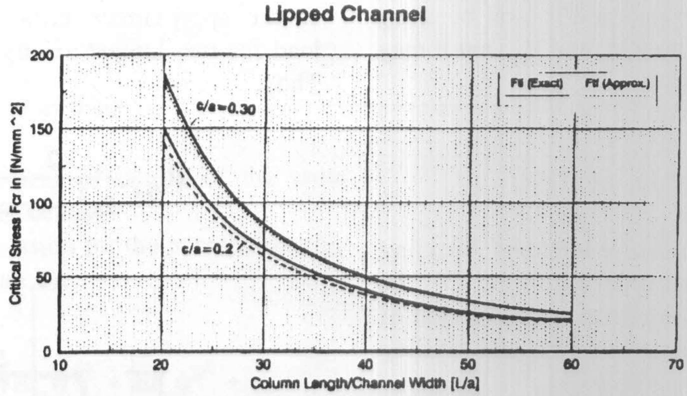

Figure 3. Column curves for a/t = 60, b/a =1.0.

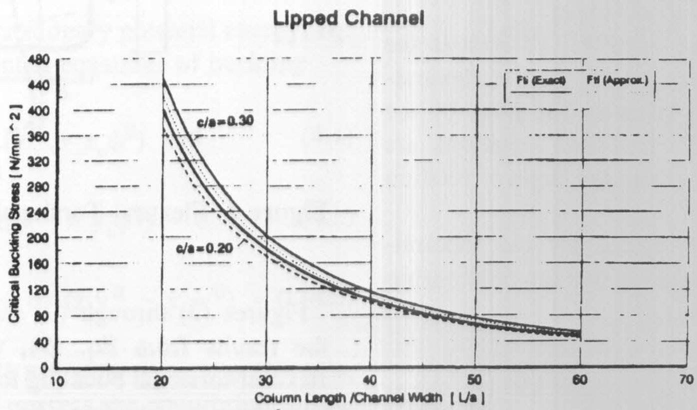

Figure 4. Column curves for a/t = 60, b/a = 2.0.

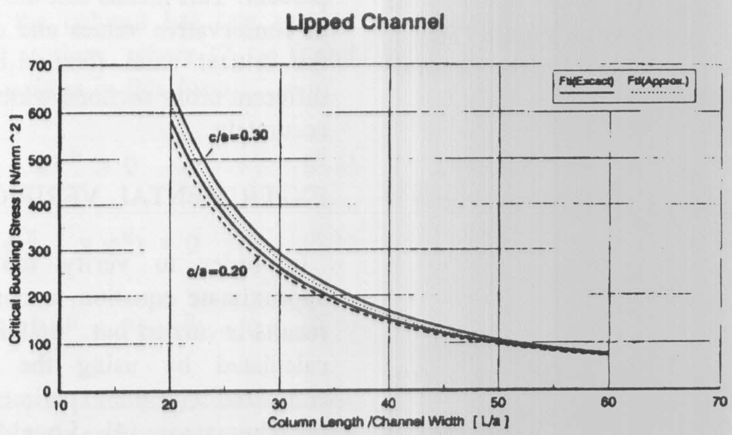

Figure 5. Column curves for a/t = 60, b/a = 3.0.

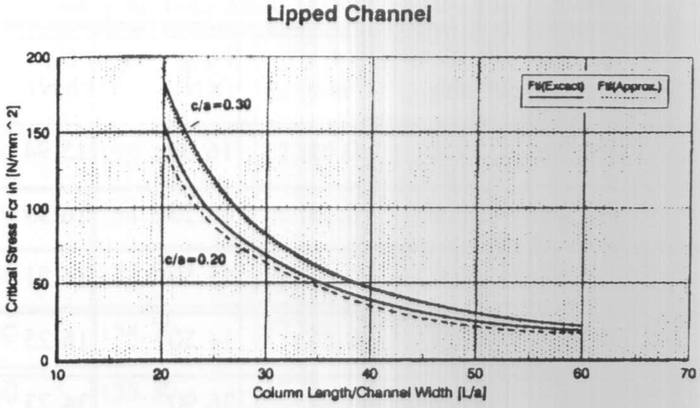

Figure 6. Column Curves for a/t = 100, b/a = 1.0.

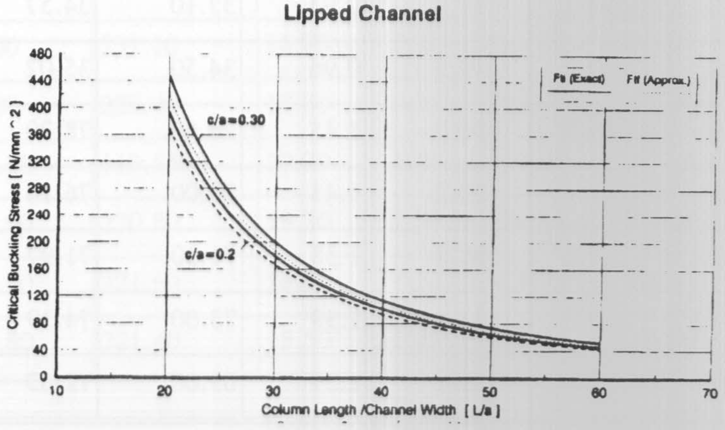

Figure 7. Column Curves for a/t = 100, b/a = 2.0.

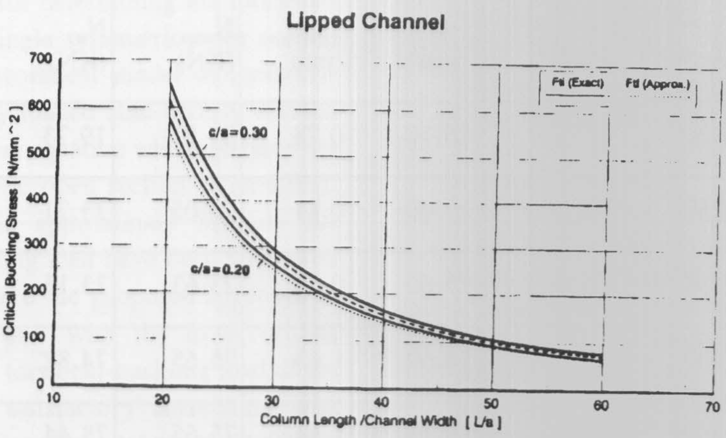

Figure 8. Column Curves for a/t = 100, b/a = 3.0.

Table 1. Thomasson's [1978] Long Column Tests.

spec No	L mm	a mm	b mm	c mm	t mm	N_{exp} (KN)	N_u (KN)	N_{exp}/N_u	P_{cr}/P_{crt}
A 71	2690	100.3	299.2	19.6	0.63	16.00	15.94	1.004	0.975
A 74	2690	100.5	299.2	20.6	0.64	16.20	16.59	0.976	0.975
A 75	2690	100.5	299.2	20.0	0.64	15.50	16.51	0.940	0.975
A 76	2690	100.2	300.0	20.2	0.65	14.50	16.25	0.900	0.975
A101	2690	100.5	299.8	20.2	0.94	36.90	34.73	1.062	0.976
A102	2690	100.5	299.7	20.0	0.94	35.00	34.68	1.009	0.976
A103	2690	100.6	299.3	19.6	0.94	37.10	34.57	1.073	0.976
A104	2690	99.6	299.3	19.5	0.96	34.50	35.02	0.985	0.977
A151	2690	100.0	299.3	20.3	1.45	76.60	78.20	0.980	0.976
A152	2690	100.0	300.1	20.2	1.43	70.00	76.16	0.920	0.977
A153	2690	99.8	299.8	20,8	1.38	71.30	71.97	0.993	0.976
A154	2690	100.4	300.6	23.4	1.39	73.00	74.39	0.981	0.972
A156	2690	99.8	299.4	21.1	1.39	69.00	72.63	0.950	0.975

Table 2. Loughlan's [1979] Long Column Tests.

spec No	L mm	a mm	b mm	c mm	t mm	N_{exp} (KN)	N_u (KN)	N_{exp}/N_u	P_{cr}/P_{crt}
L 15	1905	62.80	153.90	25.30	0.78	21.81	19.73	1.105	0.878
L 16	1600	62.90	153.90	25.60	0.81	23.05	23.21	0.993	0.876
L 17	1295	62.90	152.00	25.60	0.78	23.63	23.13	1.022	0.878
L 31	1905	61.80	178.20	24.60	1.63	75.65	74.82	1.011	0.930
L 32	1600	61.60	177.80	24.70	1.63	75.65	78.44	0.964	0.930
L 33	1295	62.00	176.70	24.90	1.63	80.10	85.44	0.937	0.927

Table 3. Mulligan's [1983] Long Column Tests.

spec No	L mm	a mm	b mm	c mm	t mm	N_{exp} (KN)	N_u (KN)	N_{exp}/N_u	P_{cr}/P_{crt}
GM1	1600	79.90	155.40	17.60	1.14	43.61	42.47	1.027	0.887
GM2	1905	79.90	154.80	16.50	1,14	46.28	38.64	1.198	0.891
GM3	3076	79.90	155.80	16.50	1.17	36.49	39.45	0.925	0.891
GM4	3073	80.00	154.20	17.10	1.14	37.38	38.31	0.976	0.886
GM5	1905	79.30	155.30	17.70	1.22	52.51	44.21	1.187	0.887
GM6	1829	80.10	230.40	16.70	1.14	42.72	43.90	0.973	0.969
GM7	2416	79.90	230.80	16.90	1.14	38.94	38.04	1.014	0.969
GM8	2997	80.00	231.10	16.80	1.12	33.82	31.05	1.089	0.969
GM9	2413	80.00	229.40	17.80	1.22	48.06	44.16	1.088	0.966
GM10	2519	112.60	113.550	19.20	1.22	48.95	45.74	1.070	0.810
GM11	1908	112.90	220.80	19.30	1.22	54.74	52.34	1.046	0.905
GM12	2517	112.50	221.60	18.80	1.22	53.85	49.71	1.105	0.899
GM13	2519	112.80	221.40	18.50	1.22	52.51	48.02	1.093	0.907

CONCLUSION

An approximate formula for determining the torsional-flexural buckling load of single symmetric open section is developed. The basic theoretical model of torsional-flexural buckling for axially loaded cold-formed column is adopted and the differential equation which governs the elastic buckling of symmetric open section is modified. The results of the simple approximate formula for estimating the critical buckling load have been compared with rigorous solutions where the proposed approach is shown to compare very well with the exact critical buckling loads. The critical torsional-buckling load of the approximate model gives satisfactory correlation with some previously conducted experimental work. Therefore the proposed approximate formula can be used practically for determining the critical torsional flexural buckling load for axially loaded columns.

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