TRANSIENT UNCONFINED SEEPAGE TO A SLOT WITH VERTICAL FACES

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ABSTRACT

A new analytical solution is developed to trace the movement of the seepage surface along the face of a vertical slot, fully penetrates an unconfined aquifer. A new expression is obtained to determine the time at which the steady state is reached. It will also enable to determine the height of the seepage surface at any time during the fall of the free surface. Another new expression is also obtained to determine the height of the seepage surface at the steady state and it is compared with other previous solutions and very good agreement is obtained. The two expressions obtained are presented graphically to simplify their uses

NOTATION

A	area perpendicular to the flow
g	gradient of the free surface at the slot
g_m	maximum gradient of the free surface at the slot
h	height of the free surface at a distance x from the slot
h_s	height of the free surface at the slot
h^*_s	h_s/H_1
H_1	depth of the original water table
H_2	depth of water in the slot
k	permeability coefficient
l_s	length of the slot
L	width of the block
m	height of the seepage surface
Q	flow into the slot
S	specific yield
t	time
t_c	time at which the steady state is reached
T^*	non-dimensional time
∀	volume

INTRODUCTION

The analysis of unconfined seepage through porous medium plays an important role in the solution of many geotechnical problems, like those concerning the stability of downstream faces of earth dams due to rapid drawdowns. The problem may be also arises due to pumping from a slot with vertical faces and fully penetrates an unconfined aquifer. Figure (1) shows a homogeneous and isotropic aquifer bounded at the left by a fully penetrating vertical slot and situated above an impervious base. The aquifer is fully saturated up to a constant level, $z = H_1$. There is no recharge by rainfall and consequently the steady situation shows a horizontal water table. Once the pumping starts in the slot the water level in the slot is rapidly dropped to a lower elevation ($z=H_2<H_1$) resulting in a lowering of the water table which will propagate inland with time. Since water velocity in the porous medium is less than that in the slot, the free surface intersects slot face at some distance above the water level in the slot and a surface of seepage of height m is developed as shown in Figure (1).

Seepage surfaces may cause an instability or failure to excavations due to excessive leakage at their faces. Therefore the determination of the seepage surface height is urgently needed to geotechnical and designer engineers.

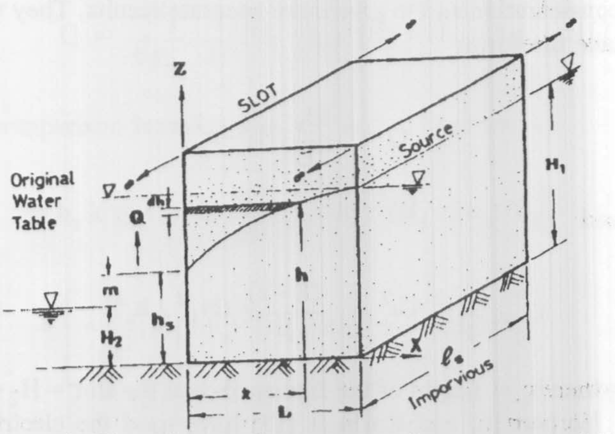

Figure 1. Flow from a line source of seepage to a fully penetrating slot of infinite length.

The analysis of seepage in unconfined aquifers is complicated because the position of the free surface is initially unknown. Dupuit [9] developed an analytical solution to determine the flow and the position of the free surface through a homogeneous block in the steady state (see Figure (1)):

$$Q = \frac{k l_s}{2L}(H_1^2 - H_2^2) \qquad (1)$$

and

$$H_1^2 - h^2 = \frac{2Q}{k l_s}(L-x) = (\frac{L-x}{L})(H_1^2 - H_2^2) \qquad (2)$$

where

Q	=	flow into the slot
k	=	permeability coefficient
l_s	=	length of the slot
H_1	=	depth of the original water table
H_2	=	depth of water in the slot
h	=	height of the free surface at a distance x from the slot
L	=	width of the block

Although Dupuit ignored the seepage surface in his analysis Eq. (1) computes the rate of flow satisfactorily. Muskat [12], Kochina [8], Harr [5] and Chapman [2] used the conformal mapping method to solve the problem in the steady state. They presented groups of dimensionless charts to determine the seepage surface height. Chapman [2] suggested that the Dupuit equations (Eqs. (1) and (2)) may be modified by substituting of h_s instead of H_2 to take the effect of the seepage surface into consideration and to give more accurate results. They will take the form;

$$Q = \frac{k l_s}{2L}(H_1^2 - h_s^2) \qquad (3)$$

and

$$H_1^2 - h^2 = \frac{(L-x)}{L}(H_1^2 - h_s^2) \qquad (4)$$

where h_s = height of the free surface at the slot $= H_2 + m$

Herbert [6] and Rushton [15] have used the electrical analogue to solve the problem of seepage through an earthen block. However, such analogues are generally expensive to construct and accurate measurements are difficult. Sand model is also used to solve groundwater flow problems. Although it gives quick results but accuracy is subject to the fact that the local variations in permeability have a proportionality greater effect in the model than in the prototype. A further difficulty is the height of the capillary fringe is usually a much larger proportion of the depth of flow in the model than in the prototype.

Several numerical attempts have been made to determine the seepage surface height as well as the position of the free surface in the transient and the steady states. Jeppson [7] and Rushton [15] have used the finite difference method, while Taylor et al [16], Neuman et al [13], France [4] and Cividini et al [3] have used the finite element method to solve this problem. Liggett [10] has used the boundary element method to analyses the same problem. Although the numerical methods are used to solve many complex problems of groundwater a lot of efforts is needed to prepare the input data. Moreover, the most tedious aspects of the use of the numerical methods are the basic processes of subdividing the domain and of generating error-free input data for the computer. Errors in the input data may go undetected and the erroneous results obtained therefrom may appear acceptable.

According to the Author knowledge, no closed forms are obtained to determine the time at which the steady state is reached. The main objective of this research is to develop a new analytical solution to trace and determine the height of the seepage surface during the fall of the free surface from its initial to the final position, due to pumping from a vertical slot fully penetrates an unconfined homogeneous aquifer.

THEORY

Figure (2) shows different stages of falling of the free surface. During the first stage (first minutes of pumping) water originates from the confined storage in the porous medium. In the second stage t_1, $t_2 > 0$ (after few hours of pumping) the free surface, close to the slot, is considered as a source of water and its gradient g starts to increase quickly towards the slot and reaches a maximum value of g_m at the slot. The increase of drawdown in the slot causes a lowering of the free surface which will propagate inland with time. This will continue until the steady state is reached at $t=t_c$ at which the drawdown in the free surface becomes insignificant. It should be noted that the free surface has the some maximum gradient value g_m at the slot during different stages of pumping. This is attributed to the fact that the free surface must approach the face of the slot or the well tangentially with a vertical velocity of k during the pumping (Boulton [1], Zee et al [17], Raudkivi et al [14] and Moghazi [11]). Thus during this time, $t \leq t_c$, the flow into the slot will be

$$Q = A\,k\,g_m = l_s\,h_s\,k\,g_m \tag{5}$$

where

A = area perpendicular to the seepage flow
g_m = maximum gradient of the free surface at the slot
h_s = height of the free surface at the slot

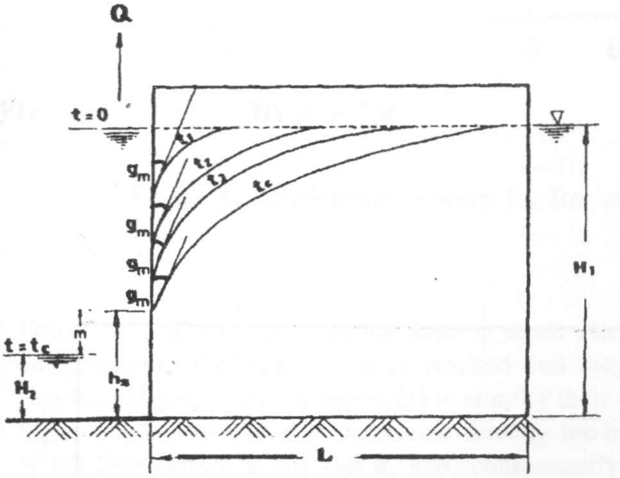

Figure 2. Different stages of pumping from a slot.

This flow has come from the space above the free surface and it has a volume $\forall$ (see Figure (2)):

$$\forall = \int_{h_s}^{H_1} l_s\, x\, dh \tag{6}$$

where dh = thickness of strip has a width of x and a length of l_s

This volume $\forall$ contained water of volume $S\forall$, where S is the specific yield of the aquifer. Assuming the aquifer has a constant value of S, thus

$$S\forall = \int_0^{t_c} Q\,dt \tag{7}$$

or

$$Q = S\,\frac{d\forall}{dt} \tag{8}$$

where t_c = time at which the steady state is reached. According to Eq. (4)

$$\frac{H_1^2-h^2}{H_1^2-h_s^2} = 1-\frac{x}{L} \quad \text{or}\quad x = L\left[1-\frac{H_1^2-h^2}{H_1^2-h_s^2}\right] \tag{9}$$

Substituting of Eq. (9) in Eq. (6), obtains

$$\forall = l_s\int_{h_s}^{H_1} L\left[1-\frac{(H_1^2-h^2)}{(H_1^2-h_s^2)}\right]dh$$

$$= \frac{l_s L}{H_1^2-h_s^2}\left[\frac{h^3}{3}-h_s^2 h\right]_{h_s}^{H_1} = \frac{l_s L}{H_1^2-h_s^2}\left[\frac{H_1^3}{3}-H_1 h_s^2+\frac{2}{3}h_s^3\right]$$

$$= \frac{l_s L}{(H_1-h_s)(H_1+h_s)}(H_1-h_s)^2\left(\frac{H_1}{3}+\frac{2}{3}h_s\right)$$

$$= \frac{l_s L}{3}\,\frac{(H_1-h_s)(H_1+2h_s)}{(H_1+h_s)} \tag{10}$$

By comparing Eqs. (3) and (5), yields

$$L = (H_1^2 - h_s^2)\,/\,2\,h_s\,g_m \tag{11}$$

Substituting of Eqs. (10) and (11) in Eq. (8)

$$Q = \frac{S l_s}{6 g_m}\,\frac{d}{dt}\left[(H_1-h_s)^2(H_1+2h_s)/h_s\right] \tag{12}$$

A comparison between Eqs. (9) and (12) gives

$$6\,h_s\,k\,g_m^2 = S\,\frac{d}{dt}\left[(H_1-h_s)^2(H_1+2h_s)/h_s\right]$$

$$= \left[-\frac{1}{h_s^2}(H_1-h_s)^2(H_1+2h_s)+\frac{2}{h_s}(H_1-h_s)^2\right.$$

$$\left.-\frac{2}{h_s}(H_1-h_s)(H_1+2h_s)\right]\frac{dh_s}{dt} \tag{13}$$

or

$$6\frac{K}{S} g_m^2 dt = \frac{1}{h_s^3}\Big[-(H_1-h_s)^2(H_1+2h_s) + 2(H_1-h_s)^2 h_s$$

$$-2h_s(H_1-h_s)(H_1+2h_s)\Big]\ dh_s$$

$$= \frac{H_1-h_s}{h_s^3}\left[-H_1^2 - H_1 h_s - 4h_s^2\right]\ dh_s$$

$$= -\left[\frac{H_1^3}{h_s^3} + 3\frac{H_1}{h_s} - 4\right] dh_s \qquad (14)$$

An integration of Eq. (14) yields

$$-6\frac{K}{S} g_m^2 [t]_o^{t_c} = \left[-\frac{H_1^3}{2h_s^2} + 3H_1 \ \ln h_s - 4h_s \right]_{H_1}^{h_s} \qquad (15)$$

or

$$6\frac{K}{S}\frac{t_c}{H_1} g_m^2 = 3\ln\frac{H_1}{h_s} + \frac{H_1^2}{2h_s^2} + 4\frac{h_s}{H_1} - 4.5 \qquad (16)$$

Since the free surface must approach the slot tangentially with a vertical velocity of k the value of g_m is taken equal to unity. Then. Eq. (16) is simplified to the form

$$T^* = 3\ \ln\ (1/h_s^*) + 1/2(h_s^{*2}) + 4h_s^* - 4.5 \qquad (17)$$

where $T^* =$ a non-dimensional time $= 6\dfrac{kt_c}{SH_1}$ (18)

and

$$h_s^* = h_s/H_1 \qquad (19)$$

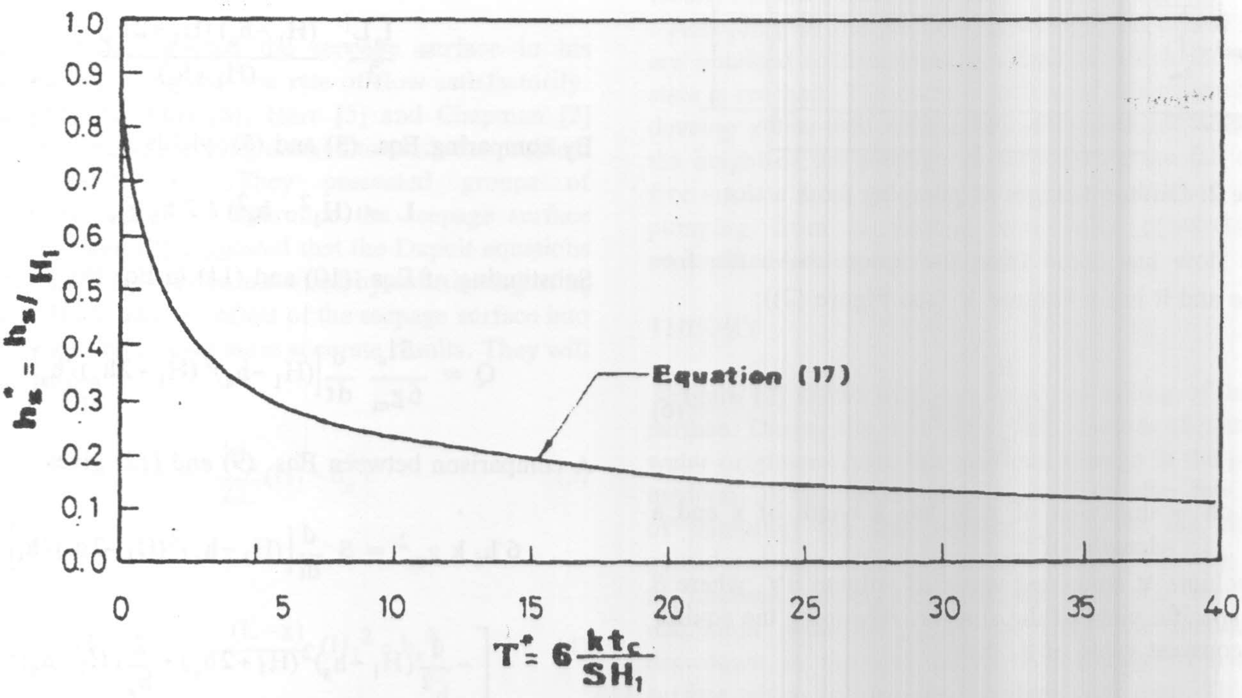

Figure 3. Relationship between the free surface height at the slot and the time.

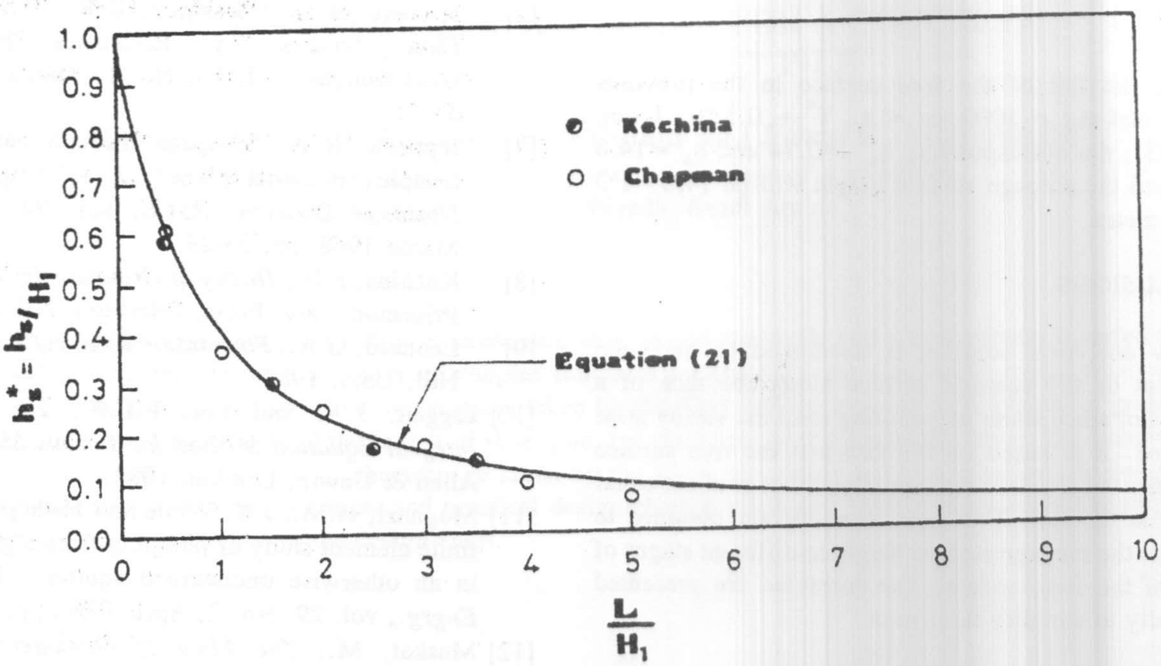

Figure 4. Relationship between the free surface height at the slot and the geometry of the aquifer.

Equations (17) to (19) give the time t_c since starts of pumping until the steady state is reached and they are represented graphically in figure (3) to simplfy their uses. Figure (3) is also useful to determine directly the height of the free surface at the slot h_s and, consequently, the seepage surface height m at any time ($t<t_c$) during the fall of the free surface by substituting of t instead of t_c. At the steady state the value of h_s^* can be obtained by rearranging Eq. (11) in the form

$$\frac{L}{H_1} = \frac{1}{2}\left(\frac{1}{h_s^*} - h_s^*\right) \qquad (20)$$

or

$$h_s^* = \sqrt{\left(\frac{L}{H_1}\right)^2 + 1} - \left(\frac{L}{H_1}\right) \qquad (21)$$

Equation (21) is represented graphically in Figure (4) in terms of the geometry of the aquifer. A comparison between h_s^* calculated by Eq. (21) and the corresponding values obtained, at the steady state, by Kochina and Chapman charts are also shown in Figure (4) and an excellent agreement is obtained between all results.

EXAMPLES

Some examples are analysed herein in order to show the simplicity of the use of Eqs. (17) and (21) to determine the seepage surface height m and the corresponding time.

Example 1

For a rectangular block of porous medium of a width of 25 metre the permeability coefficient and specific yield are 0.001 m/sec and 0.886 respectively. The upstream and downstream water levels are maintained at 20 and 1.3 metres respectively above the horizontal impervious base. Referring to Figure (1);

$L/H_1 = 1.25$ and $H_2/H_1 = 0.065$. Using Eq. (21) yields $h_s^* = h_s/H_1 = 0.35$

and $m/H_1 = 0.35\text{-}0.065 = 0.2858$ and the seepage surface height m = 5.72 metre. Applying Eq. (17) and for $h_s^* = 0.35$ the non-dimensional time $T^* = 4.13$. Using Eq. (18), then, $t_c = 3.388$ hours at which the steady state is reached.

Example 2

During the fall of the free surface in the previous example and at t = 1000 sec < t_c, $T^* = 0.3386$. Using Figure (3), the corresponding $h_s^* = 0.74$ and $h_s = 14.8$ metre, and the seepage surface height will be 14.8 - 1.3 = 13.5 metre.

CONCLUSIONS

A new analytical solution is developed to trace the movement of the seepage surface along the face of a vertical slot since starts of pumping until the steady state is reached. It is based on the fact that the free surface approaches the slot face tangentially with a gradient equal to unity at the slot. Two new formulae are obtained to determine the seepage surface height at different stages of falling of the free surface. The formulae are presented graphically to simplify their uses.

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