DESIGN OF DOWNSTREAM FILTER FOR A CONCRETE DAM WITH AN END SHEET PILE

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ABSTRACT

The case of confined seepage beneath a concrete dam with an end sheet pile and a downstream inverted filter is investigated in this paper. The theory of complex functions and the Schwartz-Christoffel transformation are used to establish equations by which the seepage taken by the filter may be estimated. A system of graphs is presented to study the effects of the length of sheet pile, the thickness of the permeable layer, the base width of dam and the length of filter on the seepage taken by the filter. The study led to a criterion for designing the optimal length of filter. A simple and practical design chart is presented. Finally a numerical example is provided to explain the use of the design chart.

INTRODUCTION

The problem of seepage beneath a concrete dam resting on a permeable soil of finite depth has been investigated by many researchers adopting different approaches (Muskat 1937, Pavlovsky 1956, Harr 1962 and Kochina 1962). According to Forchheimer's trial and error method the problem was graphically solved (Casagrande 1935). An approximate solution for the problem accounting for the existence of a downstream filter (Hathoot 1980) was presented. Another case in which the filter was installed partially beneath the dam floor and extending in the downstream direction was also investigated (Hathoot 1986). The case of seepage beneath a dam with sheet pile was solved independently by (Muskat 1937 and Pavlovsky 1956). They also provided graphical solutions for the case of symmetrically placed pilings (Harr 1962). In their solutions they did not consider the existence of a downstream filter. Hence the objective in this paper is to investigate the proper length of a downstream filter for a dam with an end sheet pile considering flow through a permeable soil of limited depth, Figure (1).

MATHEMATICAL MODEL

The differential equation that governs the two dimensional flow in porous media is that of Laplace:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad (1)$$

where ϕ is the velocity potential given by:

$$\Phi = k(\frac{P}{pg} - y) + C \qquad (2)$$

in which k is the hydraulic conductivity of soil, p is the pressure, ρ is the density of water, g is the acceleration due to gravity, C is a real constant and y is the vertical coordinate of a point in the z-plane, Figure (2-a).

In the mathematical treatment the following assumptions are considered: (1) The soil underneath the dam is homogeneous and isotropic; (2) the upstream channel bed and the filter are equipotentials, whereas the base of the dam, the sheet pile and the impervious stratum are streamlines; and (3) water follows the path of least resistance and is being taken mostly by the filter. Thus channel bed downstream from the filter is considered to be a streamline (Hathoot 1986).

The correspondence between points in the z and t-planes is shown in Figures (2-a) and (2-b). From the Schwartz-Christoffel transformation between the z and t-planes (Harr 1962) we have:

$$z = M\int_0^t \frac{tdt}{(1 - t^2)(\delta^2 - t^2)^{1/2}} + N \qquad (3)$$

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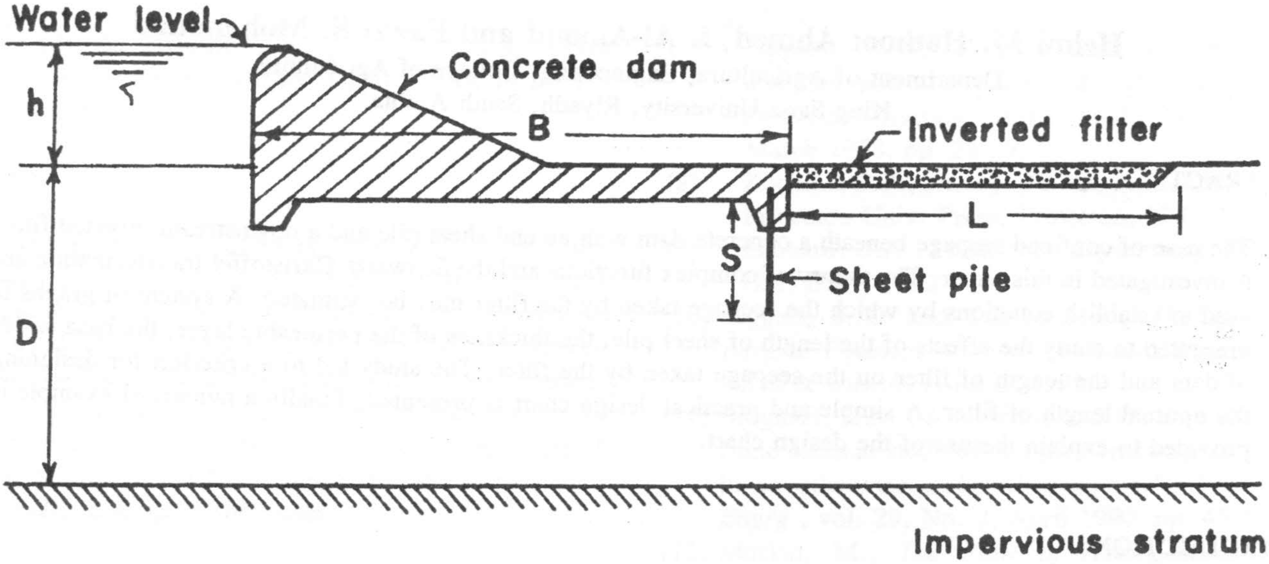

Figure 1. Geological section.

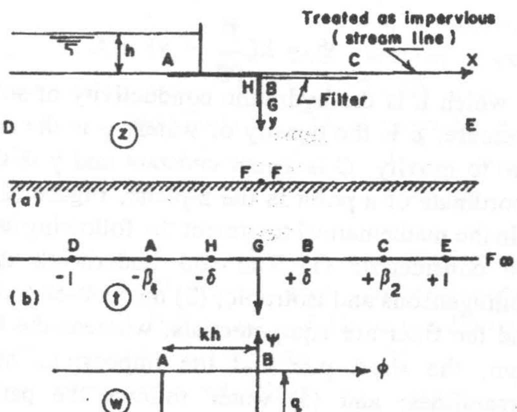

Figure 2. z,t and w planes.

in which M and N are complex constants.

Making the substitutions $\delta^2 - t^2 = \tau^2$, $t dt = -\tau d\tau$ and $1 - t^2 = \tau^2 + \delta_1^2$ where $\delta_1^2 = 1 - \delta^2$, we obtain

$$z = -\frac{M}{\delta_1}\left\{\arctan\left[\frac{(\delta^2 - t^2)^{1/2}}{\delta_1}\right] - \arctan\left(\frac{\delta}{\delta_1}\right)\right\} + N \qquad (4)$$

At points G, z = i S and t = 0; we find from Eq. 3 that:

$$N = iS \qquad (4')$$

in which S is the length of sheet pile.

At points B, z = 0 and t = δ; we have from Eq. 4:

$$\frac{M}{\delta_1}\arctan\left(\frac{\delta}{\delta_1}\right) + iS = 0 \qquad (5)$$

and hence Eq. 4 reduces to:

$$z = -\frac{M}{\delta_1}\arctan\left[\frac{(\delta^2 - t^2)^{1/2}}{\delta_1}\right] \qquad (6)$$

Considering points F, z = iD and t = ∞; and substituting into Eq. 6 we get:

$$M = \frac{-2iD\delta_1}{\pi} \qquad (7)$$

in which D is the thickness of the permeable soil, and hence

$$t = \pm\left(\delta^2 + \delta_1^2 \tanh^2\frac{\pi z}{2D}\right)^{1/2} \qquad (8)$$

Substituting from Eq. 7 into Eq. 5 we get:

$$\frac{2iD}{\pi}\arctan\left(\frac{\delta}{\delta_1}\right) = iS$$

from which

$$\delta = \sin\frac{\pi S}{2D} \text{ and } \delta_1 = \cos\frac{\pi S}{2D} \qquad (9)$$

therefore

$$t = \pm\cos\frac{\pi S}{2D}\left[\tan^2\frac{\pi S}{2D} + \tanh^2\frac{\pi Z}{2D}\right)^{1/2} \qquad (10)$$

Equation 10 is the required transformation between the z and t-planes.

Substituting points A, z = - B and t = - β_1 into Eq. 10:

$$\beta_1 = \cos\frac{\pi S}{2D}\left(\tan^2\frac{\pi S}{2D} + \tanh^2\frac{\pi B}{2D}\right)^{1/2} \qquad (11)$$

in which B is the base width of dam.

Following the same for points C, z = L and t = β_2:

$$\beta_2 = \cos\frac{\pi S}{2D}\left[\tan^2\frac{\pi S}{2D} + \tanh^2\frac{\pi L}{2D}\right]^{1/2} \qquad (12)$$

Figure (3) provides graphical evaluation of both β_1 and β_2, (Harr 1962).

Now let us consider the mapping of the w-plane onto the t-plane, Figures (2-b) and (2-c), from the Schwartz-Christoffel transformation we have:

$$W = M_1\int_0^t \frac{dt}{[(t-\delta)(t-\beta_2)(t+1)(t+\beta_1)]^{1/2}} + N_1 \qquad (13)$$

Performing the integration of Eq. 13 (Spiegel 1963 and Tuma 1979) we get:

$$W = M_1\left[F(m,\theta) - F(m,\theta_1)\right] + N_1$$

in which M'_1 and N_1 are complex constants, $F(m,\theta)$ and $F(m,\theta_1)$ are elliptic integrals of the first kind, m,θ and θ_1 are given as follow:

$$m = \left[\frac{(1+\beta_2)(\delta+\beta_1)}{(\beta_1+\beta_2)(\delta+1)}\right]^{1/2} \qquad (15)$$

$$\theta = \arccos\left[\frac{(\delta-\beta_2)(1+t)}{(1+\beta_2)(\delta+t)}\right]^{1/2} \qquad (16)$$

and

$$\theta_1 = \arccos\left[\frac{(\delta-\beta_2)}{\delta(1+\beta_2)}\right]^{1/2} \qquad (17)$$

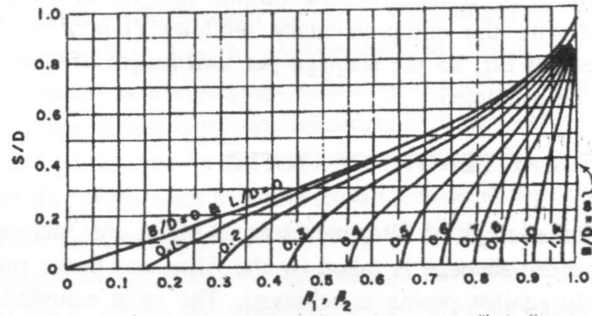

Figure 3. β_1, and β_2 versus S/D, According to Eqs. 11 & 12.

Considering points C, w=-iq and t=+β_2 and substituting into Eq. 14:

$$-iq = -M'_1\,F(m,\theta_1) + N_1 \qquad (18)$$

in which q is the seepage per unit length of dam. Similarly for points D, w=-kh-iq, t=-1 we get:

$$-kh - iq = M'_1\,[K - F(m,\theta_1)] + N_1 \qquad (19)$$

in which k is the hydraulic conductivity of soil, h is the net head acting on the dam, Figure (1), and K is the complete elliptic integral of th first kind.

From Eqs. 18 and 19 we get:

$$M_1 = \frac{-kh}{K} \qquad (20)$$

and

$$N_1 = -iq - \frac{kh}{K}F(m,\theta_1) \qquad (21)$$

Substituting from Eqs. 20 and 21 into Eq. 14

$$W = -\left[\frac{kh}{K}F(m,\theta) + iq\right] \qquad (22)$$

Equation 22 provides the required transformation between the w and t-planes.

Now considering points B, w=0 and $t=+\delta$ and substituting into Eq. 22 we get (Abramowitz and Stegun 1970):

$$\frac{q}{Kh} = \frac{k'}{K} \tag{23}$$

in which K' is the complete elliptic integral of the first kind with the complementary modules $(1-m^2)^{1/2}$.. By means of Eq. 23 the seepage per unit length of dam, q, can be evaluated.

OPTIMAL LENGTH OF FILTER

As the length of filter increases its initial cost increases but more seepage is taken by the filter and hence more safety against piping is achieved. The most economical condition is that in which a sufficiently high seepage percentage is taken by a limited length of filter. For convenience Figures (4) through (7) are plotted to illustrate the variation of the seepage ratio, q/kh with the filter length ratio L/D for various sheet pile length ratios S/D. Each of the above mentioned figures corresponds to a specific B/D value. In all the figures the rate of increase of the seepage ratio rapidly decreases at high filter length ratios and at seepage ratios close to their maximum values.

Table 1. Increase in Filter Length Corresponding to 1% increase in seepage.

Increase in Seepage Taken by Filter	Corresponding Average increase in Filter Length
95 % - 96 %	10.4 %
96 % - 97 %	13.0 %
97 % - 98 %	15.5 %
98 % - 99%	25.9 %

The average percentage increases in the seepage ratios corresponding to one percent increase in the filter length ratio for seepage percentages between 95 % and 99 % are given in Table 1. From this table it is evident that the percentage increase in filter length is abnormally high for the one percent increase in seepage between 98% and 99%. This indicates that the 98% seepage percentage might be taken as the design criterion for the optimal length of filter.

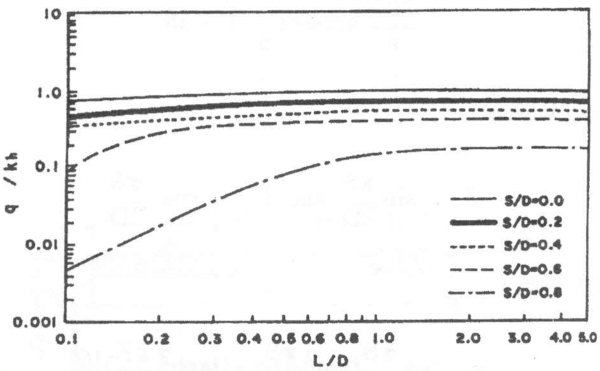

Figure 4. Seepage ratio versus filter length ratio (B/D =0.2).

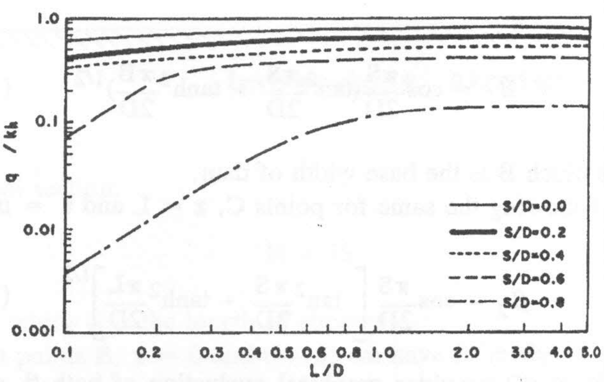

Figure 5. Seepage ratio versus filter length ratio (B/D = 0.4).

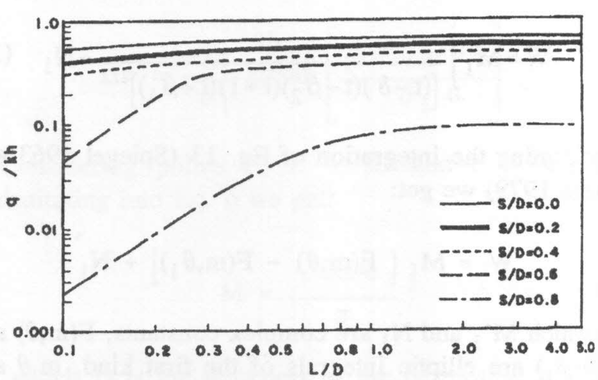

Figure 6. Seepage ratio versus filter length ratio (B/D = 0.6).

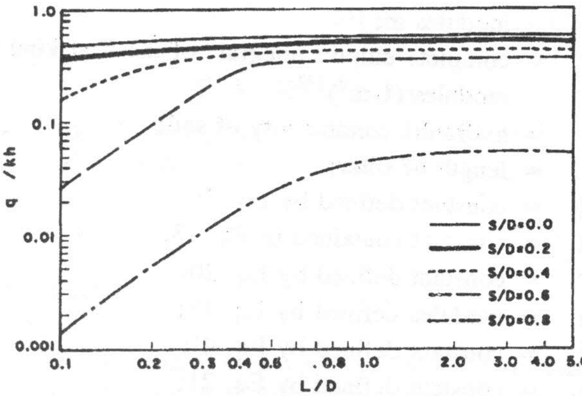

Figure 7. Seepage ratio versus filter length ratio (B/D = 0.8).

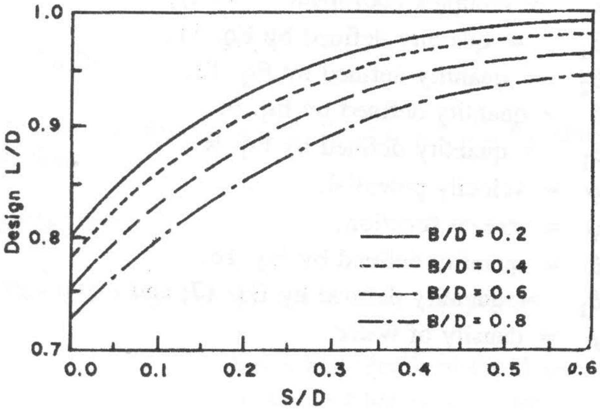

Figure 8. Design filter length ratio versus sheet pile length ratio.

Figure (8) was constructed on the basis of 98% seepage criteria and hence can be used for design purposes.

EXAMPLE

It is required to find the length of the downstream filter of a dam for the following conditions: D = 22.0m, B = 12.0m and S = 1.5m.

Solution

$$\frac{S}{D} = \frac{1.5}{22.0} = 0.068;$$

$$\frac{B}{D} = \frac{12.0}{22.0} = 0.545$$

from Figure (8), considering the above values we have:

L/D = 0.815, from which:

L = 0.815 (22.0)

= 17.93 m (taken L = 18.0 m).

CONCLUSIONS

A study on the optimal length of downstream filter indicates that the length increases with an increase in the length of an end sheet pile and the thickness of the permeable soil beneath the dam. Uneconomical increase in the filter length corresponds to relatively small increases in the seepage taken by the filter when high seepage percentages are considered. Specifically, a one percent increase in the seepage percentage (from 98% to 99%) corresponds to about 26% increase in the length of filter. Accordingly the 98% seepage is taken as the filter length design criterion since though only 2% of seepage is not taken by the filter yet, saving in filter length that may exceed 50% are attained. A chart for designing the proper length of filter is provided and the solution of a numerical example shows that the chart is simple and practical.

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APPENDIX I.

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APPENDIX II. NOTATIONS

The following symbols are used in this paper:

B = base width of dam;
C = constant contained in Eq. 2;
D = depth of permeable soil beneath dam;
g = acceleration due to gravity;
h = effective head acting on dam;
i = $(-1)^{1/2}$
K = complete elliptic integral of the first kind with modules m;
K' = complete elliptic integral of the first kind with modules $(1-m^2)^{1/2}$;
k = hydraulic conductivity of soil;
L = length of filter;
M = constant defined by Eq. 7;
M_1 = constant contained in Eq. 13;
M_1 = constant defined by Eq. 20;
m = modules defined by Eq. 15;
N = constant defined by Eq. 4';
N_1 = constant defined by Eq. 21;
p = pressure at a point;
S = length of sheet pile;
w = complex potential = $\phi + i_\psi$;
z = complex coordinate = x+iy;
β_1 = quantity defined by Eq. 11;
β_2 = quantity defined by Eq. 12;
δ = quantity defined by Eq. 9;
δ_1 = quantity defined by Eq. 9;
ϕ = velocity potential;
ψ = stream function;
θ = quantity defined by Eq. 16;
θ_1 = quantity defined by Eq. 17; and
ρ = density of water.