A TRANSIENT METHOD FOR MEASURING THERMAL PROPERTIES OF HUMID PARTICULATE MATERIALS WITH A CONSTANT HEAT FLUX ANNULAR CYLINDRICAL HEATER

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ABSTRACT

A transient method for measuring the thermal properties of humid particulate materials is introduced. A cylindrical copper heater, energized by a constant electric power input, develops a thermal field in the particulate material. The mechanism of heat transfer in the humid material is assumed to be solely pure conduction. The mathematical model, formulated to simulate the transient heat flow in the particulate material and the heater body, was solved using a finite difference implicit scheme. In- situ and laboratory experiments were conducted to measure the thermal properties of soil materials at the South Cairo Electricity Generating Plant. The thermal properties were calculated from the mathematical model by adjusting the theoretical temperature- time history at the heater surface to that measured experimentally.

NOMENCLATURE

c	Specific heat of soil material, J/Kg K.
c_1	Specific heat of heater material, J/Kg K.
Fo	Fourier number, equation (3).
I	Electric current, amp.
k	Thermal conductivity of soil material, W/m K.
k_1	Thermal conductivity of heater material, W/m K.
L	Dimensionless calculating domain length.
l	Calculating domain length, m.
l_1	Heater length, m.
q_o	Heat generated per unit volume, W/m^3.
R	Dimensionless radial coordinate, equation (3).
r	Radial coordinate, m.
r_1	Heater radius, m.
r_2	Calculating domain radius, m.
T	Temperature, ^{o}C.
T_o	Soil initial temperature, ^{o}C.
V	Electric voltage, volt.
X	Dimensionless axial coordinate, equation (3).
x	Axial coordinate, m.
α	Thermal diffusivity of soil material, m^2/s.
α_1	Thermal diffusivity of heater material, m^2/s.
Δ	Dimensionless heater thickness.
δ_1	Heater thickness, m.
θ	Dimensionless temperature, equation (3).
ρ	Density of soil material, Kg/m^3.
ρ_1	Density of heater material, Kg/m^3.
τ	Time, s.
Ω	Thermal resistivity, m K/W.

INTRODUCTION

In order to measure a thermal property of a material, as for example the thermal conductivity or the thermal diffusivity, a thermal field has to be established experimentally in a specimen of the material. The temperature field may be steady or unsteady with respect to the time. Then temperatures or heat fluxes, at certain locations and times, have to be measured. Next, a mathematical solution, describing the temperature field established in the specimen, has to be derived. The solution may be either analytic or numerical. The analytic solution can be a closed form expression or in the form of series. The solution, whether analytic or numerical, contains the thermal properties of the material. If these properties are varied until the theoretical solution yields values for the temperatures and heat fluxes as those measured at the respective locations and times, in the experiment; then we will know the thermal properties.

Several methods were devised to determine the thermal properties of the particulate materials. Kersten [1] built a steady equipment for measuring the thermal conductivity. The specimen is contained between two concentric cylinders. The inner cylinder is at a higher constant temperature. The internal line heat source method was first developed by Schleiermacher [2], and later modified in [3] through [6]. Beck [7] used the numerical theoretical solutions with the experimental measurements to evaluate the thermal properties. Abdel-Wahed et al. [8] developed a closed form analytic solution for the quasi-steady transient heating method in an annular cylindrical specimen. The inner side is heated by a constant heat flux. This approach was later modified to two sides heating in [9]. The transient field established by a cylindrical heater, having a finite length and embedded in an infinitely large loose powdered material, was employed by Abdel- Wahed et al. [10] to determine the thermal properties of the loose materials.

The present work lies under the frame of direct cooperation between the University and the Society. A gas and steam combined cycle extension is proposed to increase the generating capacity of the South Cairo Electricity Generating Plant. The new extension will be electrically connected to the existing power house by a 66 KV underground cable. This connection happens to be the first high tension cable in Egypt that runs in the ground. In order to properly size the cable, the thermal conductivity of the ground material needs to be known. In the present work, a thermal field will be created using a long cylindrical heater poked in the soil material to be tested. The heater receives constant heat flux. The temperature at the heater outside surface and in the soil rises monotonically with the time. The temperature field is a transient two dimensional one. If the temperature-time history on the outside surface of the heater is measured, and compared with histories, computed from the mathematical solution and parameterized by the material thermal properties, then the thermal conductivity and diffusivity of the soil will be determined.

THEORETICAL ANALYSIS

Figure (1) shows a graphical statement of the problem. The heater is heated by an electric current circulated in the coil embedded in the heater cylindrical annular body. In view of the low thermal capacity of the air gap compared to that of either the heater or soil materials, the transient heat transfer inside the air will be neglected. The conduction transfer of heat in the constant physical properties heater body and soil material is governed by the two dimensional transient heat conduction equation:

$$\rho c \frac{\partial T}{\partial \tau} = k \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) + k \frac{\partial^2 T}{\partial x^2} + q_o \qquad (1)$$

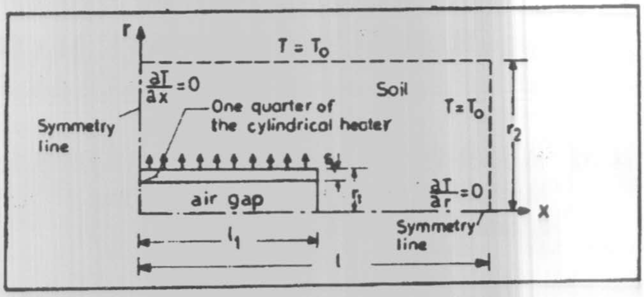

Figure 1. Graphical statement of the mathematica problem.

where q_o = (I * V)/heater volume

I and V are the current and the applied voltage of th electric energy input to the heater. The boundar conditions of the above equation are:

Outside boundaries:

$$T = T_o \text{ at } r = r_2 \text{ and at } x = l \qquad \text{(2-a}$$

Symmetry lines:

$$\partial T/\partial r=0 \text{ at } r=0, \text{ and} \partial T/\partial x=0 \text{ at } x= 0 \quad \text{(2-b}$$

The physical properties of the heater material and soi are respectively: α_1 , k_1 , ρ_1 and c_1 for the heater. and α , k , ρ and c for the soil.

Introducing the following dimensionless variables:-

$$\Theta = \frac{T-T_o}{q_o r_1^2/k_1}, Fo = \frac{\alpha_1 \tau}{r_1^2}, X = \frac{x}{r_1}, R = \frac{r}{r_1} \qquad \text{(3}$$

where T_o is the soil initial temperature. Inserting th above set of variables into equation (1) and its boundar conditions (2), we get:

In the soil:

$$\frac{\partial \Theta}{\partial Fo} = \frac{\alpha}{\alpha_1} \frac{1}{R} \frac{\partial}{\partial R}\left(R \frac{\partial \Theta}{\partial R}\right) + \frac{\alpha}{\alpha_1} \frac{\partial^2 \Theta}{\partial X^2} \qquad \text{(4-a}$$

In the heater:

$$\frac{\partial \Theta}{\partial Fo} = \frac{1}{R} \frac{\partial}{\partial R}\left(R \frac{\partial \Theta}{\partial R}\right) + \frac{\partial^2 \Theta}{\partial X^2} + 1 \qquad \text{(4-b}$$

and the boundary conditions (2) transform to:

Outside boundaries:

$$\Theta = 0 \text{ at } R=R_2 \text{ and } X = L \qquad \text{(5-a}$$

Symmetry lines:

$$\partial\Theta/\partial R=0 \text{ at } R=0, \text{ and } \partial\Theta/\partial X=0 \text{ at } X=0 \qquad (5\text{-b})$$

Equations (4-a) and (4-b), in essence, have the same form. They differ only in the value of the dimensionless material property and the magnitude of the generation term. The material property equals one in the heater and α/α_1 in the soil, while the generation term is one in the heater and zero in the soil. It is not possible to solve this mathematical build up analytically. The alternative is to follow a numerical approach. The implicit scheme was used to solve the problem. The result of the solution depends exclusively on the value of the diffusivity ratio α/α_1.

The following is the procedure followed to determine the thermal properties:

1- Select a value for α/α_1 and solve equations (4-a) and (4-b) with the boundary conditions (5-a) and (5-b).

2- Plot the Θ vs. Fo relation for the point on the heater surface at $R = 0$ and $X = 0$.

3- From the experimental measurements of T vs. τ and the dimensionless variables introduced in equation (3), compute the Θ vs. Fo experimental relation.

4- Compare the Θ vs. Fo experimental and theoretical relations at various values of α/α_1 until they coincide at the right dimensionless diffusivity ratio α/α_1.

5- Knowing the thermal diffusivity of the heater material, the soil thermal diffusivity α can be determined.

6- Performing a heat balance on the heater yields:

$$IV = \rho_1 c_1 . 2\pi r_1 l_1 \delta_1 \frac{\partial T}{\partial \tau} + (-k\frac{\partial T}{\partial r})2\pi r_1 l_1 = q_o 2\pi r_1 l_1 \delta_1 \qquad (6)$$

Inserting the dimensionless variables from equation (3), transforms equation (6) after arrangement to:

$$\frac{k}{k_1} = \Delta[(\frac{\partial\Theta}{\partial Fo} - 1)/(\frac{\partial\Theta}{\partial R})_{R=1}] \qquad (7)$$

where Δ is the dimensionless heater thickness δ_1/r_1. The gradients $(\partial\Theta/\partial Fo)$ and $(\partial\Theta/\partial R)_{R=1}$ can be computed from the numerical results.

7- The soil thermal conductivity can then be computed with the aid of equation(7) if the heater thermal conductivity is known.

EXPERIMENTAL APPARATUS

The details of the experimental set-up used in measuring the in-situ soil properties at the South Cairo Electricity Generating Plant is shown in Figure(2). The probe was manufactured from 10 mm outside diameter electrolytic copper tube. The tube wall thickness is 1 mm. A screw thread 1 turn/mm is machined on the tube outside surface. The heating element, which is an 0.2 mm diameter copper wire, is embedded in the thread groove. The wire is coated with varnish. The electric heating power is delivered to the heating wire from a DC power supply, having a capacity of 75 Watt. It has analog read-out for the current and voltage. Two 30 AWG copper-constantan thermocouples are attached to the probe surface. The output of one thermocouple is delivered through an electronic cold junction to a DC chart recorder having a full scale of 0.001 volt. The other thermocouple is wired to a 0.1 °C resolution temperature digital read-out. The idea of using the second measuring system is to cross check the performance of the chart recorder in view of the existence of high tension electric cables in the site of the experiment. The test procedure for the in situ experiments was as follows:

1- The probe is poked in the soil at the required location and depth.

2- It is left there for 10 to 15 minutes to attain the soil temperature.

3- The electric heater is energized and at the same moment the chart recorder sheet is advanced.

4- By the end of the heating, the temperature on the digital read-out is logged and the total time of the experiment was read using an electronic stop watch.

5- Knowing the current and voltage inputs to the heater together with the physical properties of copper, it is possible to recast the temperature vs. time history obtained by the chart recorder into the dimensionless form Θ vs. Fo.

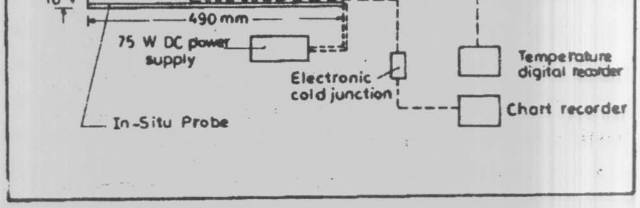

Figure 2. Experimental setup.

The experimental arrangement employed to measure the properties of the soil in the laboratory is similar to the in-situ one. The outside diameter of the shelby soil specimens was 10 cm. Therefore, a smaller probe was designed to suit the size of the shelby specimen. It was manufactured from a copper tube, having an outside diameter and length of 3.2 and 75 mm respectively. The tube wall thickness is 0.6 mm. The heating element consists of a closely spaced 0.2 mm varnish coated copper wire wound around the copper tube. The test procedure for the laboratory experiments to find the variation of the soil thermal resistivity with its moisture content was as follows:

1- A 10 cm long sample is taken out of the shelby tube and weighed.
2- The probe is poked inside the sample, and the total weight of the sample with the probe inside it is determined.
3- Next, the specimen is left in air to loose some of its moisture. After a sensible change in the moisture content, a second thermal resistivity test is done. The time period for an appreciable moisture variation is observed to be 24 hours at the beginning of the experiment, which increases as the specimen dries out. In order to reduce the length of the drying time period, at the end of an experiment, the specimen is placed in an oven held at 40°C. The sample is removed from the oven, left to cool naturally in air to the ambient temperature, reweighed, and its resistivity is measured.
4- At the end of the experiment, the specimen is dried out completely in an oven at 110°C for 24 hours to determine the initial moisture content.

Experimental Program

The material of the ground at the power plant site is highly irregular in nature. The bottom layer consists of clay sediments on the rock bed. Strata of backfillings, with different material structures and depths, were heaped on the clay layer to restore the ground elevation at the new extension to the old plant yard level. Six locations, enroute of the high tension cable from the power houses to the transformer area, were selected to perform thermal resistivity measurements at them, Figure(3). At each location, both in-situ thermal properties measurements as well as shelby samples for laboratory thermal testing were taken at three levels. The classification of the soil material and the elevation of the thermal measurements at the six respective locations are displayed in Figure(4). The experimental program is two folds. The first comprises an in-situ measurement of the thermal resistivity at the eighteen points shown in Figure(4). The second, consists of determining the relation between the thermal resistivity and the moisture content of eight samples. Six out of them are the shelbies obtained from the plant site, while the remaining two are for proposed backfilling materials.

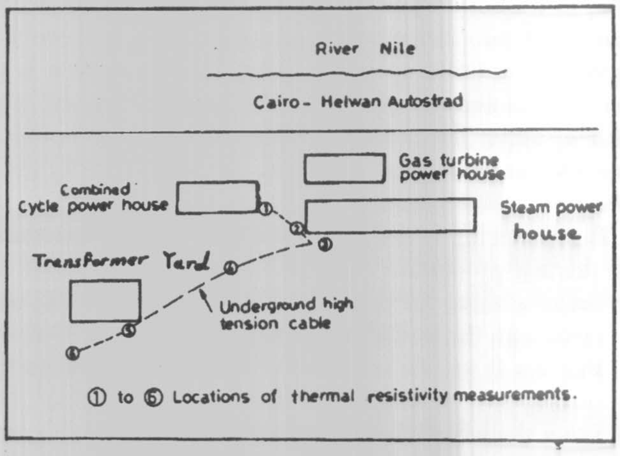

Figure 3. Route of the underground high tension cable.

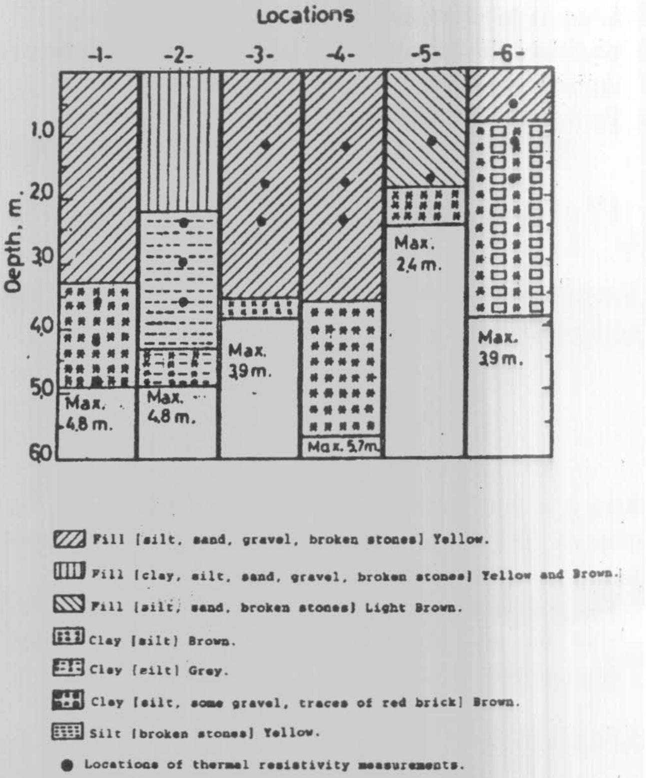

Figure 4. Classification of the soil material at the six locations.

RESULTS AND DISCUSSION

In-Situ Measurements:

Samples of the in-situ measurements of the Θ vs Fo history are displayed on Figure (5). The bottom part of the figure is for an experiment at location 3 and depth 2.4 m, while the upper parts are both for location 2 at two different depths, 2.4 and 3 m. Inspection of the result of location 3 shows that the theory predicts in an excellent way the temperature-time history. However, there is some departure between the theoretical prediction and the measured histories at location 2. The reason for this departure will be discussed in commenting on the laboratory results. Table(1) includes the in-situ thermal properties of the materials en route of the high tension cable. The reference thermal properties in the table k_1, α_1, and $\rho_1 c_1$ are for pure copper. Some locations, show high irregularity in the material, as for example, the difference between the materials at location 6 at depths 1.2 and 1.8 m.

Table 1. In-situ thermal properties enroute of the high tension cable.

Location	Depth, m	k/k_1 x10^3	α/α_1 x10^3	$\rho c/\rho_1 c_1$
1	3.60	2.46	2.55	0.965
	4.20	4.09	2.40	0.971
	4.80	4.11	2.38	0.966
2	4.20	2.46	2.55	0.965
	3.00	1.98	2.05	0.966
	3.60	1.51	1.55	0.974
3	1.20	2.09	2.15	0.970
	1.80	2.19	2.30	0.950
	2.40	2.84	2.95	0.963
4	1.20	4.02	4.20	0.956
	1.80	3.77	4.00	0.942
	2.40	3.63	3.80	0.954
5	1.20	3.71	3.90	0.950
	1.80	3.76	3.90	0.963
	2.40	2.25	2.30	0.977
6	0.60	3.26	3.35	0.979
	1.20	3.08	3.20	0.963
	1.80	2.38	2.50	0.953

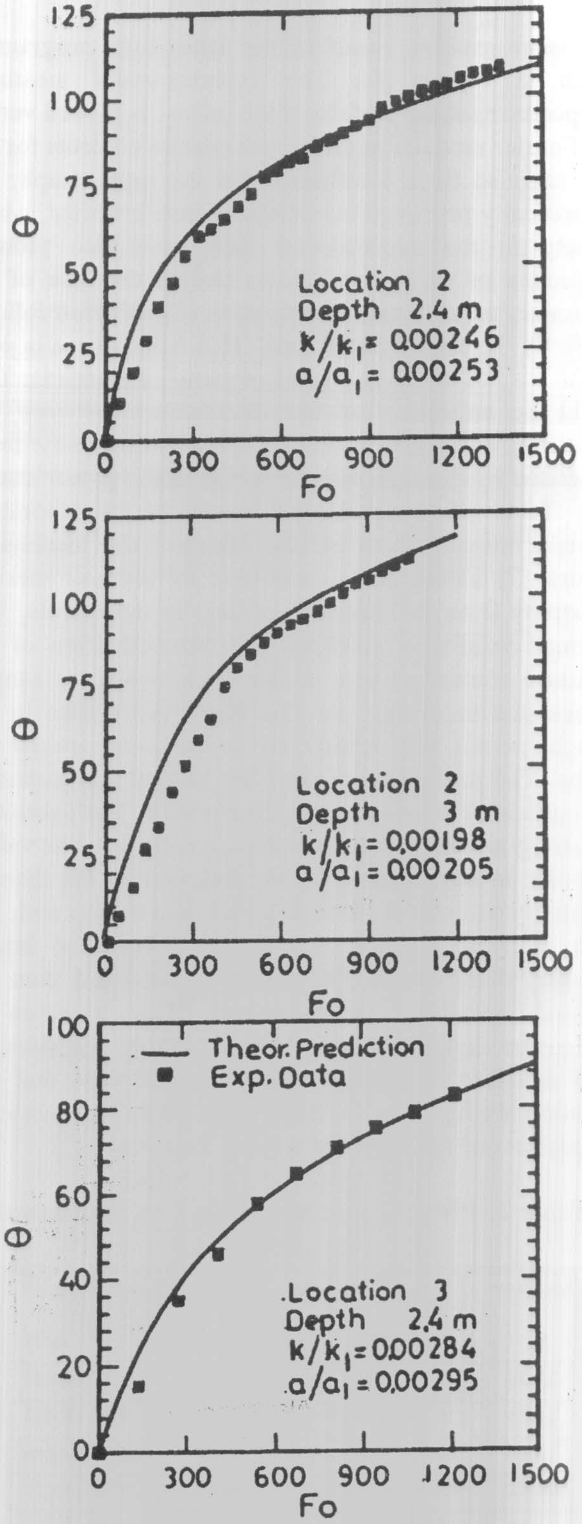

Figure 5. Laboratory experiments.

Laboratory Measurements

A representative result of the laboratory program is given in Figure (6). The experimentally measured temperature, at the surface of the probe, is plotted versus the Fourier modules at different moisture contents for the soil sampled form location 5. On the same graph, the theoretically predicted temperature- time histories, which closely fit the experimental data were also plotted. Inspection of the figure, shows that in the case of the laboratory experiments, the theoretical prediction follows perfectly the experimental data. This coincidence is poor in the case of the in-situ measurements, Figure (5). This could be attributed to the interference between the measuring equipment and the electric and magnetic fields generated by the high tension cables in the power station site. Table (2) compiles the results of the laboratory thermal measurements for the sample from location 5. Figure (7) shows the dependence of the soil thermal resistivity Ω on the moisture content in the sample. The thermal resistivity increases with the reduction of the moisture content up to a certain limit, when the sample cracks due to drying. At this limit, the cracks in the sample permit the outside air to circulate around the probe. The probe is cooled by the natural convection of the outside air moving through the cracks. The calculated resistivity assumes therefore an apparently constant value. In order to have a clue of the variation of the thermal resistivity between the cracking limit and the dry end, i.e. zero moisture content, the sample is crushed into a powder, dried at 110 °C for 24 hours, and then the thermal resistivity is measured. This situation is designated in Table (2) and Figure (7) by the situation zero moisture content. It is important to note that this situation is a hypothetical case, since the soil porous solid properties are different from those in nature.

Table 2. Effect of moisture content on the thermal properties.

Moisture	T_o	k/k_1	k	Ω	α/α_1	$\alpha x10^7$
%	°C	$x10^3$	W/mK	cm K/W		m^2/S
25.24	18.60	4.585	1.7698	56.50	0.0080	8.987
21.33	16.15	4.299	1.6594	60.26	0.0075	8.425
15.44	16.95	4.872	1.8810	53.17	0.0085	9.548
14.41	17.55	4.229	1.6594	60.26	0.0075	8.425
11.15	18.55	4.270	1.6480	60.67	0.0074	8.369
7.26	18.55	4.299	1.6594	60.26	0.0075	8.425
zero	25.30	0.993	0.3832	260.95	0.0017	1.966

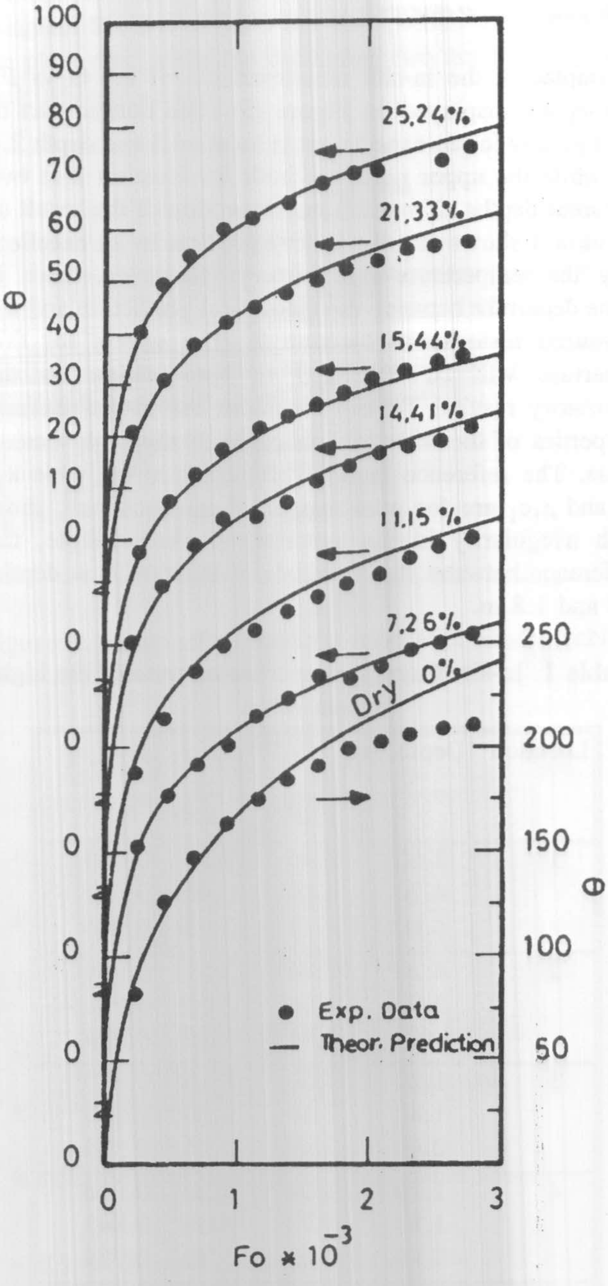

Figure 6. Characteristic of the temperature-time histor with moisture content of the soil at location (5 laboratory experiments.

It was observed that if the heating power is increase the experimental data fall below the theoretic prediction, Figure (8). The reason is that natur convection plays a role beside the conduction in th transfer of heat from the probe to the specimen. Thi phenomena was also evident when the heating power is kept constant and the moisture content in the soil is reduced below the saturation limit. At this state, the pores of the material are partially filled with water. The resistance to the motion of water vapour, water, and air is greatly facilitated, and natural convection can easily be established. The effect of the natural convection can be damped by reducing the power input to the probe. The reduction of the heating power is accompanied by a reduction in the rate of temperature rise as well as the magnitude of the temperatures encountered in the experiment. The smallness of the measured variables has an adverse effect on the measuring accuracy. The outlet from this difficulty is the reformulation of the mathematical model taking into consideration the mode of natural convection.

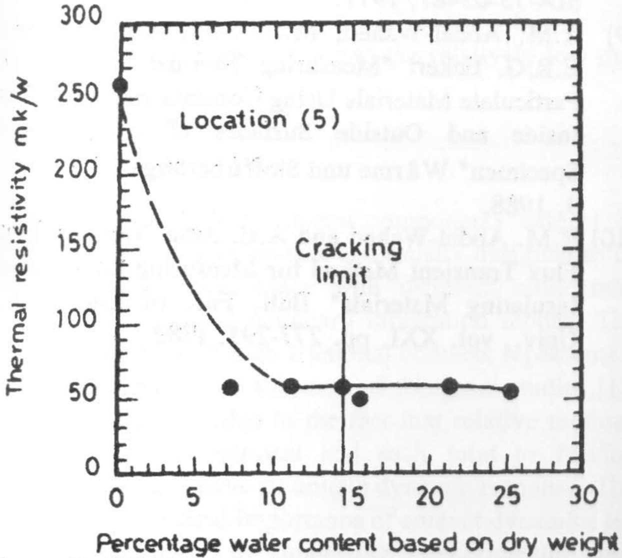

Figure 7. Effect of moisture content on the thermal resistivity.

In order to refrain the user of the present method from resolving the mathematical model each time he wants to measure properties, Figure (9) and (10) were prepared. The results in the figure cover a wide range of the diffusivity ratio α/α_1. The designer has to manufacture the probe according to the geometric proportions proposed in the experimental part of this work. Consulting the theoretical information in Figure (9), he can compute the thermal diffusivity, which when entered in Figure (10), the thermal conductivity can be obtained. It is worth to note that, for $\alpha/\alpha_1 > 20$, the value of the conductivity ratio depends only on the diffusivity ratio. The relation between the conductivity and diffusivity ratios is linear and is represented by the straight line:

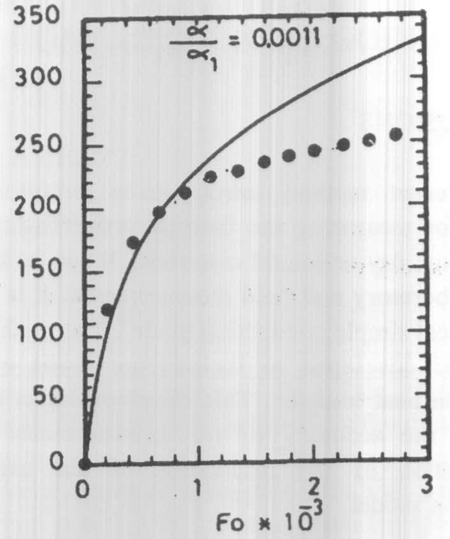

Figure 8. Deviation from pure conduction model due to the natural convection mode of heat transfer.

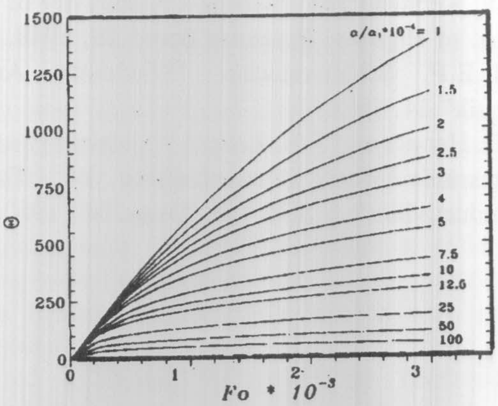

Figure 9. System characteristic design curve.

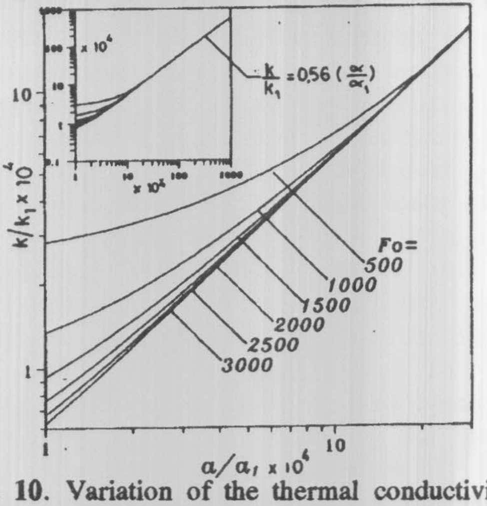

Figure 10. Variation of the thermal conductivity ratio with diffusivity ratio at different Fourier modules.

$$\frac{k}{k_1} = 0.56\left(\frac{\alpha}{\alpha_1}\right) \tag{8}$$

CONCLUSIONS

The present method introduces a simple transient method for measuring the thermal properties of porous and fibrous, dry or humid materials. It can be used both for the laboratory and field measurements. It is based on a theoretical simple conduction model. The method has to be further developed to include the mode of natural convection heat transfer. This development is necessary for both the accuracy of the measurements and the determination of the limitations of the use of the conduction model.

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