# MODIFIED FIGURE OF MERIT TO DESCRIBE THE WAVE PROPAGATION IN WAVE GUIDES

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## ABSTRACT

A newly introduced term "Figure of Merit" is proposed cast and employed to investigate both qualitatively and quantitatively the propagation problem in nonlinear optical waveguides made of either $GeO_2$ (x)-$SiO_2$ (1-x)or Al(x)Ga(1-x)As. Three basic parameters are handled in such design, namely, the optical confinement factor, the transmitted bit rate, and the power-pulse width square product. In this paper the dimensionless analysis is applied and the problem is studied under wide ranges of variation of the affecting parameters as sources parameters, detectors parameters and waveguides parameters. A comparison of the two guide is made under the same set of the affecting parameters.

## 1-INTRODUCTION

The propagation of nonlinear optical pulses in dispersive media in the presence of inhomogeneities had attracted the attention during the last two decades [1-4 for examples]. These pulse- type stationary solutions of nonlinear dispersive wave equations are handled mathematically, physically and experimentally [5,6]. Solitons in optical communications are now considered to have a potential application to high-bit-rate transmission systems. A critical limitation in realizing the full-bandwidth capability of these systems is pulse distortion due to dispersion which is overcome via nonlinear dependence of refractive index on pulse intensity. Here, the dispersion effects result in the broadening of the pulse, while nonlinearity tends to shapen it. It is the appropriate competition of these opposite effects which may lead to a stable solitary solutions of the optical pulse [2].,

In the study of such phenomenon either in planar optical waveguides [7-8] or in circular optical fibers, three major relevant topics of interest are in focus, namely i)the confinement factor [9], ii)the transmitted bit-rate , and iii)the power-pulse width square product. Each topic is studied separately. To the best of our Knowledge, no study has considered the integration of the three topics.

Thus, in the present paper, a newly introduced term "Figure of Merit" is proposed, cast and employed to investigate both quantitatively and qualitatively the propagation problem, with the integration of the three topics, in nonlinear optical waveguides made of either $GeO_2$ -$SiO_2$ or AlGaAs.

## II.BASIC MODEL AND ANALYSIS

### *II.1 On The Confinement Factor*

The confinement factor, $\Gamma$, of optical pulses in nonlinear slab waveguide is cast as [9]

$$\Gamma = \frac{P_f}{P_f + P_s + P_c} \qquad (1)$$

where $P_f$, $P_s$, and $P_c$ are the power flow per unit length in the film, the substrate, and the cover respectively. $\Gamma$ has been correlated and the affecting sets of parameters under the form:

$$\Gamma = \sum_{i=0}^{7} A_i (d/\lambda)^i \qquad (2)$$

where 2d and $\lambda$ are the guide thickness and the optical wavelength respectively. $A_{i,s}$ are functions of the chemical structure of the guide as will as the launched optical pulses and the operating temperature.

The injected optical power density $P_i$ is given as

$$P_i = 2d(P_f + P_s\, P_c\,) \qquad (3)$$

In fact both $P_f$ and $P_s$ and $P_c$ are computed by solving the nonlinear differential equations of the electric fields in the guide, substrate and cover layers with suitable coupling at the interfaces.

The dispersion characteristics of the guide are evaluated based on Sellmeier equations where we have [10]

$$n^2 = 1 + \sum_{i=1}^{3} \frac{A_i \lambda^2}{\lambda^2 - B_i^2} \qquad (4)$$

for $GeO_2$ -$SiO_2$ guide layer, where both $A_{i.s}$ and $B_{i.s}$ are functions of the chemical structure and guide temperature while we have [11]

$$n^2 = C + \frac{D}{\lambda^2 - E} - F\lambda^2 \qquad (5)$$

For (AlGaAs) guide layer, where C,D,E and F are functions of the chemical structure and guide temperature.

### *II.2. On The Bit-Rate*

Based on the model of CSELT [12] and El-Halafawy et.al. [13] a formula for the system bit rate, B , is derived under the form:

$$B_r^2 = 8F_t^2 \log (B_{max}/B_r) \qquad (6)$$

with

$$B_{max} = B_u \exp[-(\sigma L + \sigma_m)] \qquad (7)$$

where $B_u$ is the ultimate value of the bit rate of the system (it depends on the source-detector combination), $\sigma_m$ is the marginal loss is the total guide loss and L is the guide length.

The guide bandwidth due to dispersion cutoff caused by different mechanisms of dispersion in single mode guide is given by $F_t$ [12],

$$F_t = 0.44/(\tau L) \qquad (8)$$

where $\tau$ is the dispersion per unit length of the guide. It is found that both $\tau$ and $\sigma$ are temperature-dependent variables and consequently $B_r$, [13]. The ratio of $B_r$ and $F_t$ is of major interest and is cast as:

$$(B_r/F_t)^2 = 8 \log \frac{(B_{max}/F_t)}{(B_r/F_t)} \qquad (9)$$

The system, in general, operates at the set of parameters at which the above ratio possesses its maximum value. As $F_t$ increases $B_r$ increases also; but as $F_t$ reaches its maximum value (lower orders dispersion terms vanishes), $B_r$ asymptotically reaches its ultimate value $B_{max}$.

### *II.3 On Power-Pulse-Width Square*

The pulse width $\tau_p$ is related with the peak power, $P_o$ of pulse [3] in a manner

$$P\tau_p^2 = S f(\lambda) \qquad (10)$$

where S is the cross sectional area of the fiber and f(λ) is essentially proportional to the group dispersion.

In Ref. [14], the product $P\tau_p^2$ is analyzed under wide range of the affecting parameters in order to minimize that product. It has been derived under the form:

$$P\tau_p^2 = 0.5\epsilon_o v_g n^2 S\psi \qquad (11)$$

where $\epsilon_o = 8.854\times10^{-12}$ F/m, $v_g = c/n$, where c is the velocity of light in vacuum, and $\psi$ is expressed in Ref. [2] as a function of waveguide thickness, laser wavelength, and linear and nonlinear refractive indices.

As Planck's constant is given as

$$h = 6.62517\times10^{-34}\ \text{Watt.sec}^2. \qquad (12)$$

the product has, the dimensions of Planck's constant h, therefore, the product is expressed in terms of dimensionless quantity, $\Gamma_p$. where

$$\Gamma_p = P\tau_p^2/h \qquad (13)$$

in units of Planck's, constant

### *II.4 On The Figure of Merit (FM),*

FM is proposed and cast in a manner to quantitatively describe the propagation process of digital optical signal in nonlinear dispersive medium (waveguide) in an integral way. It couples the merit of relevant topics namely i) the transmitted bit-rate, ii) the confinement factor and iii) the power-pulse-width square. FM is given as

$$FM = (B_r/F_t).\ \Gamma/(P\tau_p^2/h)$$

Based on this criterion, a better propagation process acquires FM of higher value and vise-versa.

## III. RESULTS AND DISCUSSION

Two optical waveguides are taken into consideration when dealing with the variations of FM against the variations of different sets of the affecting parameters namely Germania-doped fibers ($GeO_2$ + $SiO_2$ ) and Al- doped GaAs.

The software that designed to handle the model, employing the set of parameters, compute the confinement factor, Eq. (1), then the transmitted bit rate, Eq. (9), and finally the power-pulse-width square, Eq. (12). Then the computed items are cast in Eq. (13) to calculate FM. Two additional criteria are introduced to study the quality of the propagation process namely four factor formula-figure of merit $FM_4$ and modified figure of Merit $FM_m$ where

$$FM_4 = \left(\frac{B_r}{F_t}\right).\left(\frac{B_r}{B_m}\right).\Gamma.\left(\frac{h}{P\tau_p^2}\right)$$

and
$$F M_m = \left(\frac{B_r}{B_m}\right) . \Gamma . \left(\frac{h}{P}\tau_p^2\right) \qquad (14)$$

The model is processed through the following set of data $0 \leq x \geq 0.4$, $0.5 \leq d, \mu m \leq 1.5$, $2 \leq \tau_o$, P.sec $\leq 1.0 \leq \lambda$, $\mu m \leq 1.6$, $200 \leq T$, °K $\leq 400$, $5 \leq \Delta\lambda$, nm $\leq 50$, $P_i = 2.5 \times 10^4$ Watt/m$^2$. $n_2 = 10^{-9}$ m$^2$/V$^2$, and $\sigma_m = 4$ dB.

Two combinations of special interest of optical source and detector are processed namely:

i. combination made of LED with ($\Delta\lambda = 50$ nm) and APD where $B_u = 1.58 \times 10^{11}$ bit/sec., and

ii combination made of LD($\Delta\lambda = 5$ nm) and APD where $B_u = 6.31 \times 10^{11}$ bit/sec.

Special emphasis is given to two windows of the optical frequencies namely 1.33 $\mu$m and 1.55 $\mu$m. The first window is at dispersion-free zone; while the second window is at minimum spectral loss domain.

The variations of $FM_4$ against the variations of one or more of the affecting parameters are displayed in Figures (1-6). Based on these figures the following general remarks are made without loss of generality:

i- $FM_4$ and d are negatively correlated

ii- As T increases $FM_4$ increases

iii- $FM_4$ of GaAs guide is of higher values compared to $GeO_2$ -doped silica guides.

iv- As x increases $FM_4$ increases.

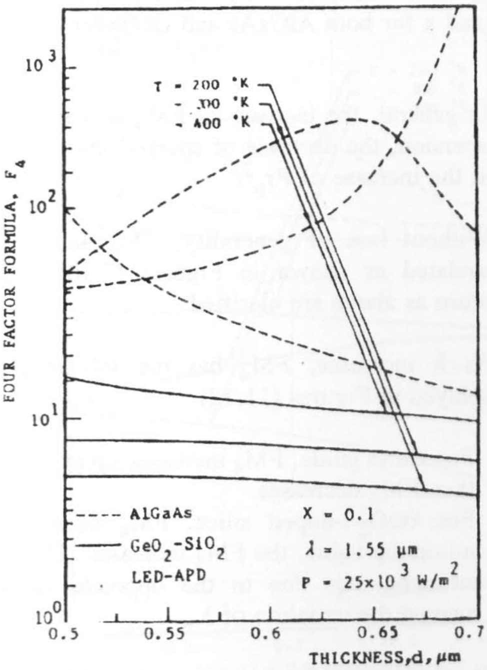

Figure 1. Variation of $F_4$ with d for $X=0.1$, $\lambda=1.55$ $\mu$m and $P=25.10^3$ W/m$^2$ and different values of T for both AlGaAs and $GeO_2$-$SiO_2$ (LED-APD).

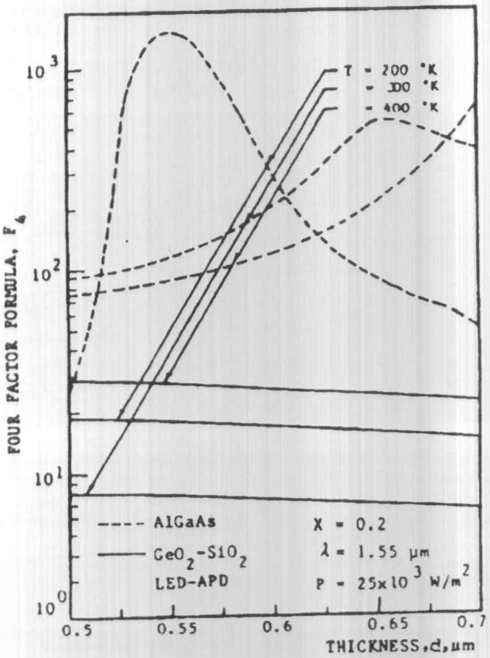

Figure 2. Variation of $F_4$ with d for $X=0.2$, $\lambda=1.55$ $\mu$m and $P=25.10^3$ W/m$^2$ and different values of T for both AlGaAs and $GeO_2$-$SiO_2$ (LED-APD).

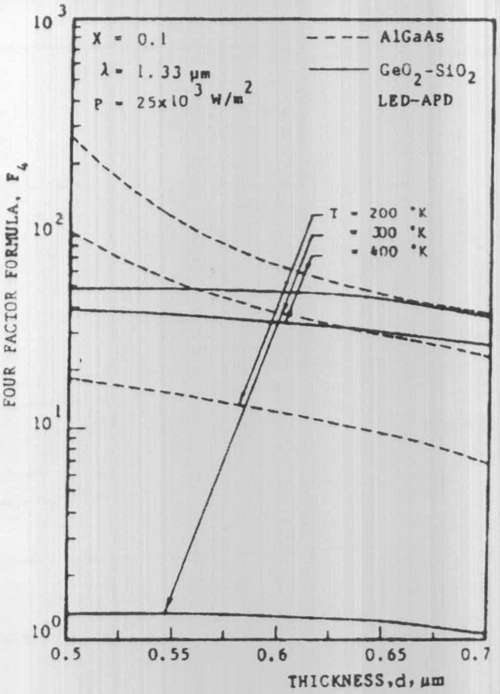

Figure 3. Variation of $F_4$ with d for $X=0.1$, $\lambda=1.33$ $\mu$m and $P=25.10^3$ W/m$^2$ and different values of T for both AlGaAs and $GeO_2$-$SiO_2$ (LED-APD).

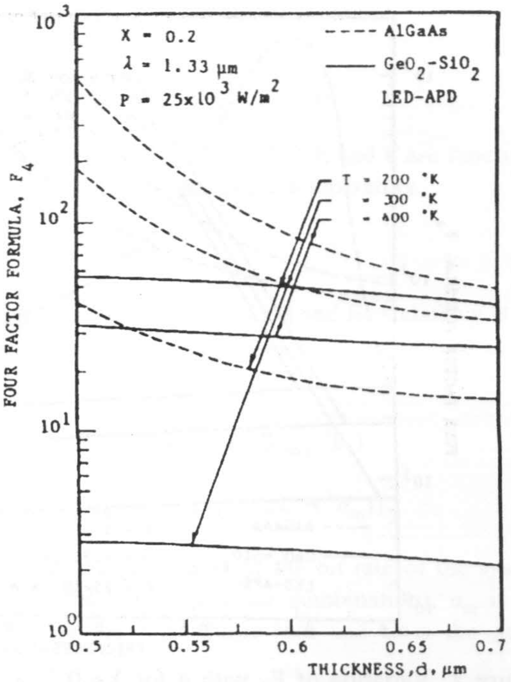

**Figure 4**. Variation of $F_4$ with d for $X=0.2$, $\lambda=1.33\ \mu m$ and $P=25.10^3\ W/m^2$ and different values of T for both AlGaAs and $GeO_2$-$SiO_2$ (LED-APD).

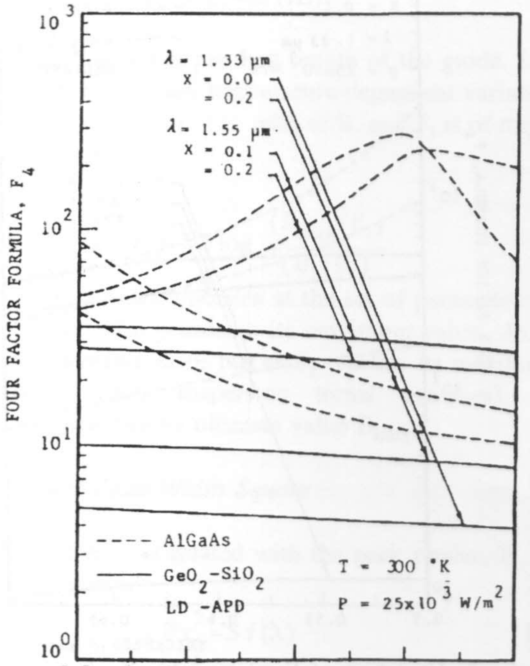

**Figure 5**. Variation of $F_4$ with d for different values of both λ and x for both AlGaAs and $GeO_2$-$SiO_2$ (LD-APD).

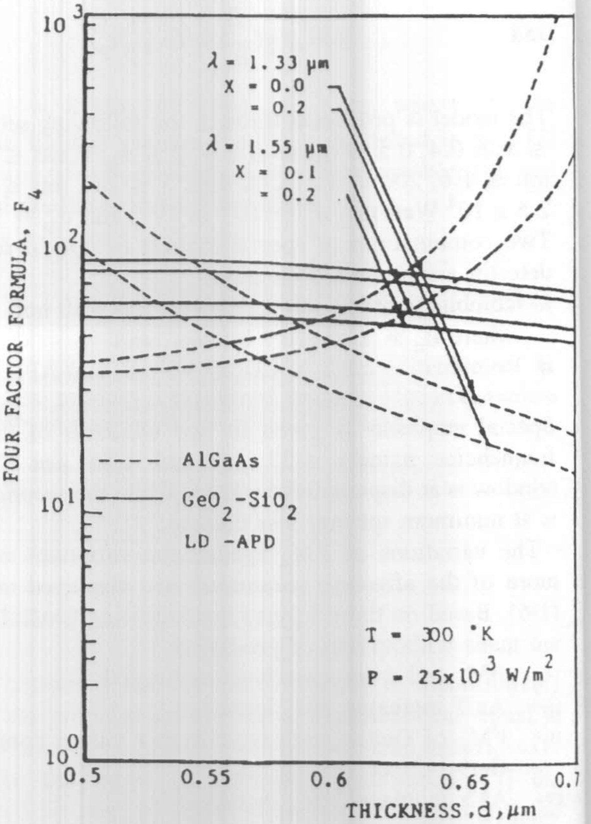

**Figure 6**. Variation of $F_4$ with d for different values of both λ and x for both AlGaAs and $GeO_2$-$SiO_2$ (LD-APD).

In general, the increase of $FM_4$ is due to the decrease of dispersion, the decrease of spectral loss, the increase of Γ, and the increase of $P\tau_p^2$.

Without loss of generality, $FM_4$ and x are positively correlated as shown in Figures (7-10) where the same feature as above are clarified.

As λ increases, $FM_4$ has the following variations as displayed in Figures (11,12).

- i- For GaAs guide, $FM_4$ increases up to a maximum value, then $FM_4$ decreases.
- ii- For $GeO_2$ -doped silica, $FM_4$ decreases down to a minimum value, the $FM_4$ increases. These two opposite situations are due to the opposite variation of $P\tau_p^2$ against the variation of λ.

The same sort of variations are found for the variations of $FM_4$ against the variations of T as portrayed in Figures (13-16), for the same reason as above.

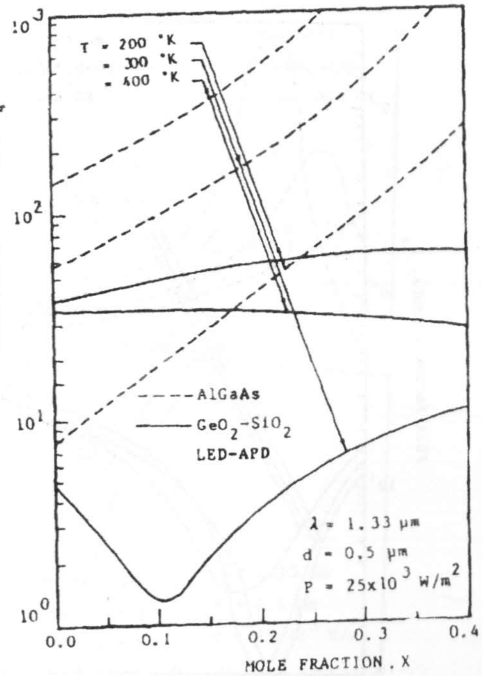

Figure 7. Variation of $F_4$ with x for $\lambda = 1.33 \mu m$, $d = 0.5 \mu u$ and $P = 25.10^3$ W/m$^2$ and different values of T for both AlGaAs and $GeO_2$-$SiO_2$ (LED-APD).

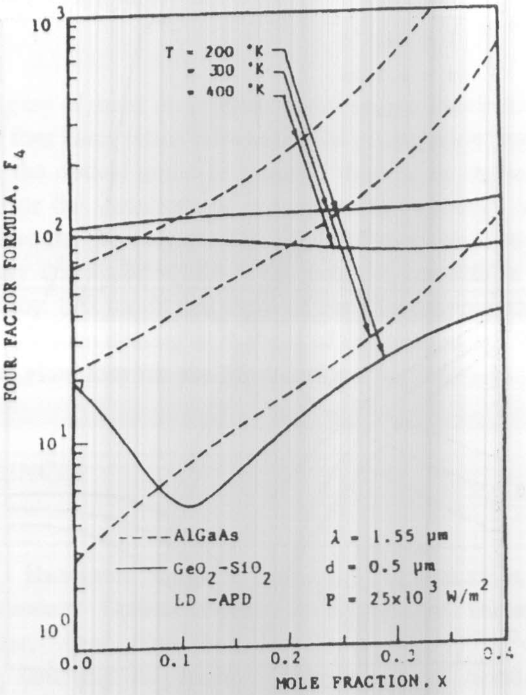

Figure 9. Variation of $F_4$ with x for $\lambda = 1.33$ $\mu m$, $d = 0.5 \mu m$ and $P = 25.10^3$ W/m$^2$ and different values of T for both AlGaAs and $GeO_2$-$SiO_2$ (LD-APD).

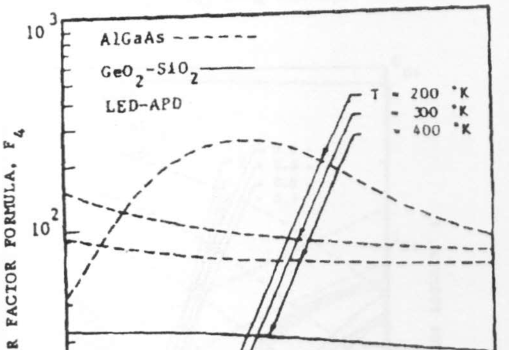

Figure 8. Variation of $F_4$ with x for $\lambda = 1.55$ $\mu m$, $d = 0.5$ $\mu m$ and $P = 25.10^3$ W/m$^2$ and different values of T for both AlGaAs and $GeO_2$-$SiO_2$ (LED-APD).

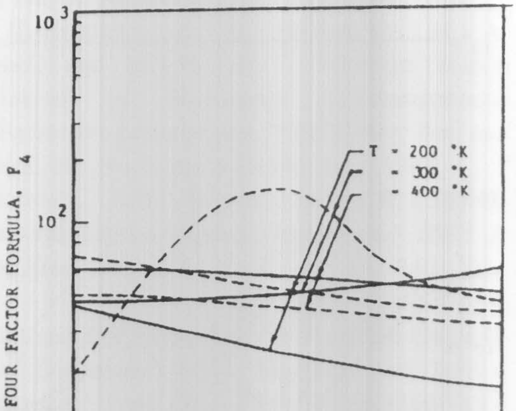

Figure 10. Variation of $F_4$ with x for $\lambda = 1.55 \mu m$, $d = 0.5 \mu m$ and $P = 25.10^3$ W/m$^2$ and different values of T for both AlGaAs and $GeO_2$-$SiO_2$ (LD-APD).

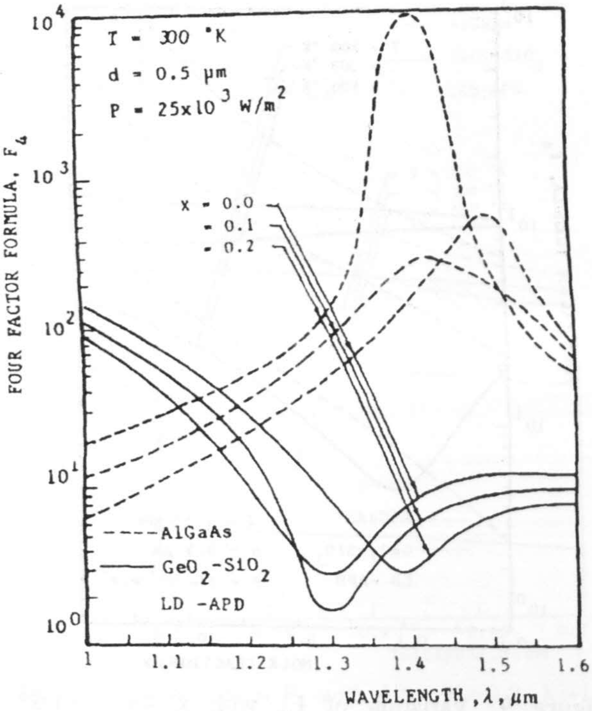

**Figure 11.** Variation of $F_4$ with λ for $T=300°K$, $d=0.5$ $\mu m$ and $P=25.10^3$ $W/m^2$ and different values of T for both AlGaAs and $GeO_2$-$SiO_2$ (LD-APD).

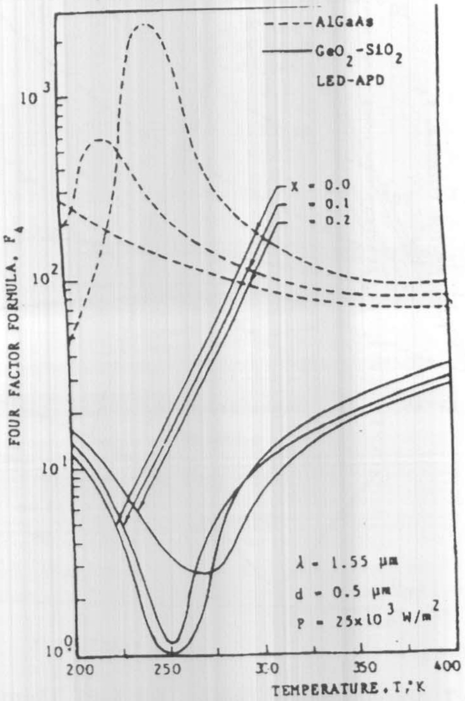

**Figure 13.** Variation of $F_4$ with T for $\lambda=1.55\mu m$, $d=o.5\mu m$ and $P=25.10^3$ $W/m^2$ and different values of T for both AlGaAs and $GeO_2$-$SiO_2$ (LED-APD).

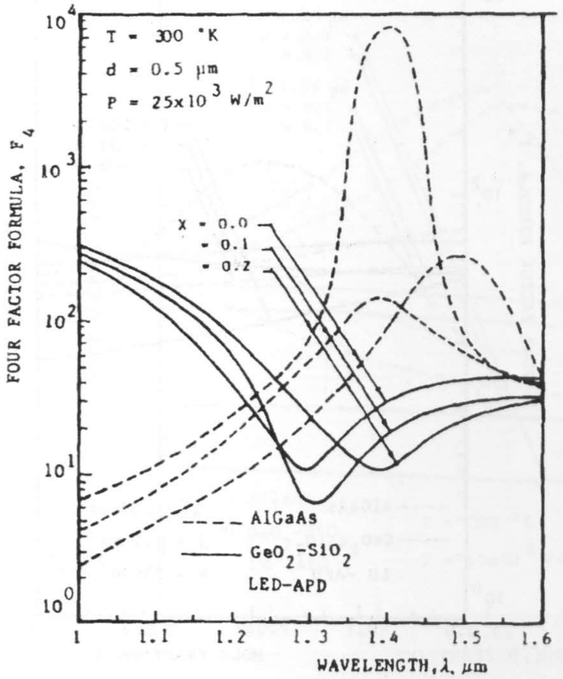

**Figure 12.** Variation of $F_4$ with λ for $T=300°K$, $d=0.5$ $\mu m$ and $P=25.10^3$ $W/m^2$ and different values of T for both AlGaAs and $GeO_2$-$SiO_2$ (LED-APD).

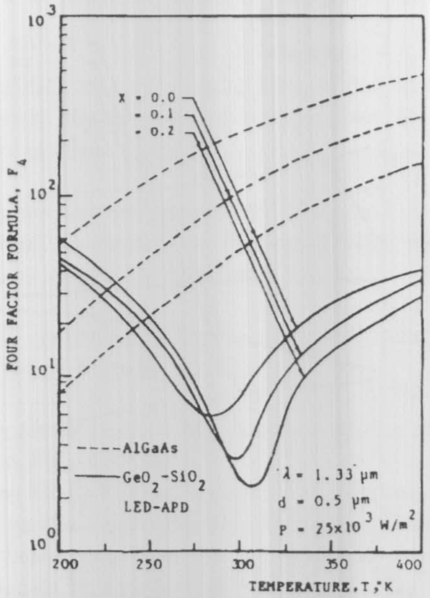

**Figure 14.** Variation of $F_4$ with T for $\lambda=1.33$ $\mu m$, $d=0.5\mu m$ and $P=25.10^3$ $W/m^2$ and different values of x for both AlGaAs and $GeO_2$-$SiO_2$ (LED-APD).

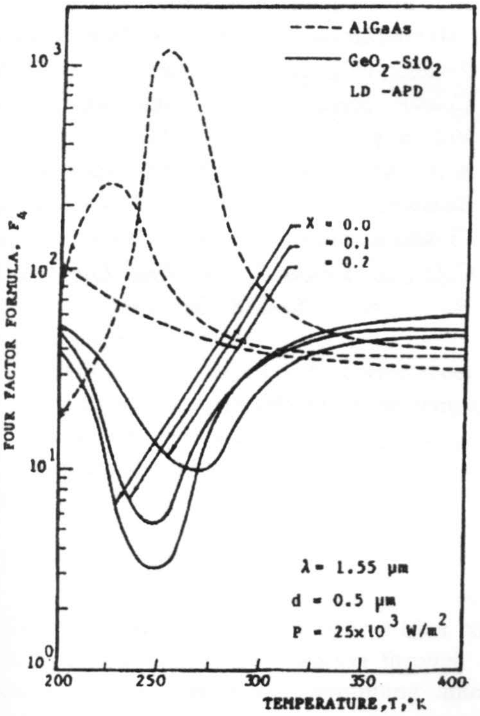

Figure 15. Variation of $F_4$ with T for $\lambda = 1.55\mu m$, $d = 0.5\mu m$ and $P = 25.10^3$ W/m² and different values of T for both AlGaAs and $GeO_2$-$SiO_2$ (LD-APD).

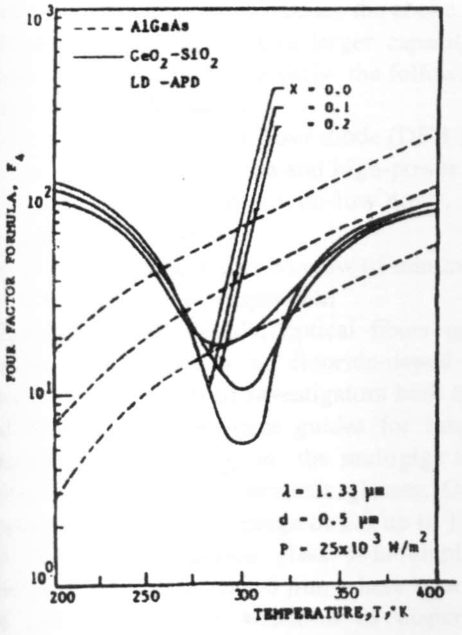

Figure 16. Variation of $F_4$ with T for $\lambda = 1.33\mu m$, $d = 0.5\mu m$ and $P = 25.10^3$ W/m² and different values of T for both AlGaAs and $GeO_2$-$SiO_2$ (LD-APD).

## IV. CONCLUSIONS

The figure of merit is an integrated criterion which couples three of four basis ratios relevant to the propagation problem through the optical guide. In general, there is no monotonic portrait for the variations of it against the variations of the set of affecting parameters. The figure of merit and ratio are negatively correlated while it has positive correlation with other ratios. It is found that some of the affecting parameters yield gains when dealing with some of the basic ratios; while the other yield loss for the other ratios.

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