CROSS BRAING AS AN INTEGRATED SYSTEM

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ABSTRACT

Design of cross bracing members is a common practice for most of the steel structures. Many designers depend on the tension member to resist the full acting load and neglect the existence of the compression member. This procedure changes the indeterminate problem of the bracing cell into a determinate one in which the effect of the areas of members on the force path is not a governing criterion and the lateral stiffness provided by the tension member to the mid-point of the compression one is not considered or existed. In the present study the effect of all member areas is accounted for, the stiffness offered by the tension member at the mid-point of the compression one is considered, and the bracing cell is analized as it exists in the real system.

INTRODUCTION

The bracing cell, shown in Figure (1), is widely used in steel structures to resist laterally acting forces. A common practice used is to neglect the existence of the compression member, this changes the system into a statically determinate one. The tension member is designed to resist the full effect of the acting lateral force and the compression member is scaled as the designed tension one.

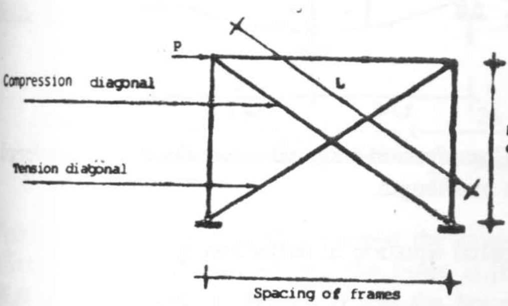

Figure 1. A bracing cell subjected to a lateral force. (P).

In the original system, both the tension and the compression diagonals are acting together along with the other three elements in resisting the lateral force. In the bracing system, the slenderness ratio of the compression member, and also of the tension one, is generally high. Therefore, it is possible that the compression member may practice a case of elastic buckling due to the existing comperssive stresses. Although the system is designed neglecting the compression member, the appearance of the structure will look like it has a serious problems if it buckles and the tension member will suffer a case of an unaccounted for bending or torsional moments based on the mode of buckling of the compression element. This additional moments may easily cause the tension member to practice unsafe stresses.

The analysis and design of the real structural system will lead to a safe, economical, serviceable, and a stable looking bracing cell.

TENSION DIAGONAL

Consider a tension diagonal, as shown in figure (2), subjected to a tensile force , T, and to a lateral load, Q. The member is assumed prismatic with length, L, and having a moment of inertia, I, and a modulus of elasticity, E.

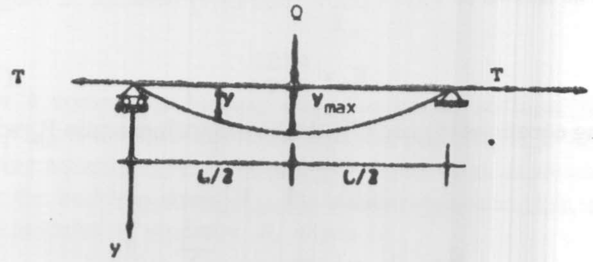

Figure 2. A tension diagonal subjected to a transversal load (Q).

The differential equation of deflection is:

$$\frac{d^2y}{dx^2} - \frac{Ty}{EI} = -\frac{Qx}{2EI} \tag{1}$$

Let $\alpha = (T/EI)^{1/2}$, then equation (1) may be expressed as:

$$\frac{d^2y}{dx^2} - \alpha^2 y = -\frac{Q\alpha^2}{2T}x \tag{2}$$

The general solution for equation (2) is:

$$y = A\ \sinh(\alpha x) + B\cosh(\alpha x) + \frac{Q}{2T}x \quad (3)$$

The boundary conditions to get the constants A and B are:

$$y_{(0)} = 0 \text{ and } y'_{(L/2)} = 0$$

These boundary conditions lead to the solution of the equation which is:

$$y = \frac{Qx}{2T} - \frac{Q\sinh(\alpha x)}{2\alpha T\cosh(\frac{\alpha L}{2})} \quad (4)$$

At mid-length the maximum deflection will be:

$$y_{max} = \frac{QL}{4T}\left[1 - \frac{\tanh\frac{(\alpha L)}{2}}{\frac{\alpha L}{2}}\right] \quad (5)$$

Using this maximum deflection, the lateral stiffness (K_T) of the tensile member is driven as:

$$K_T = \frac{Q}{y_{max}} = \frac{4T}{L}\left[\frac{\frac{\alpha L}{2}}{\frac{\alpha L}{2} - \tanh(\frac{\alpha L}{2})}\right] \quad (6)$$

The stiffness of a member subjected to a mid transversal load as shown in figure (3) is determined as:

$$K_o = \frac{48EI}{L^3} \quad (7)$$

Using equations (6) and (7), the lateral stiffness ratio R_s will be:

$$R_s = \frac{K_T}{K_o} = \frac{1}{3}(\frac{\alpha L}{2})^2\left[\frac{\frac{\alpha L}{2}}{\frac{\alpha l}{2} - \tanh(\frac{\alpha L}{2})}\right] \quad (8)$$

Equation (8) may be expressed using the tensile stress ,f_t, the modulus of elasticity ,E, and the slenderness ratio ,L/r, as:

$$R_s = \frac{1}{12}(\frac{L}{r})^2\frac{f_t}{E}\left[\frac{\frac{1}{2}\frac{L}{r}\sqrt{\frac{f_t}{E}}}{\frac{1}{2}\frac{L}{r}\sqrt{\frac{f_t}{E}} - \tanh(\frac{1}{2}\frac{L}{r}\sqrt{\frac{f_t}{E}})}\right] \quad (9)$$

Equation (9) proves the logical behaviour of increasing th lateral stiffness ratio, R_s, with the increase of the slenderne ratio, L/r, and the tensile stress, f_t.

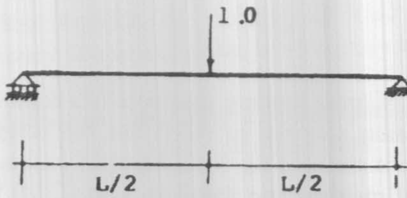

Figure 3. A member subjected to a unit transversal load a mid-length.

COMPRESSION DIAGONAL

Consider a compression diagonal, as shown in figure (4 subjected to a compressive force ,P, and constrained at mi length by a spring with a stiffness ,K_s. The member i prismatic with length ,L, moment of inertia ,I, and modulus of elasticity ,E.

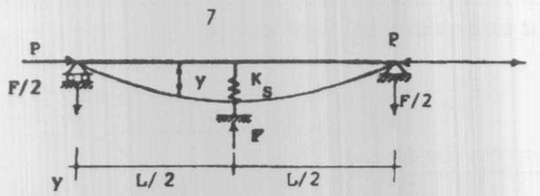

Figure 4. Compression diagonal constrained by a transversa spinning at mid-length.

The differential equation of deflection is:

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{F}{2EI}x \quad (10$$

Let $\beta = (P/EI)^{1/2}$, then equation (10) may be expressed as

$$\frac{d^2y}{dx^2} + \beta^2 y = \frac{F\beta^2}{2P}x \quad (11$$

The general solution of equation (11) is:

$$y = A\sin(\beta x) + B\cos(\beta x) + \frac{F}{2P}x \quad (12$$

The boundary conditions to get A and B are:

$$y_{(0)} = 0 \text{ and } y'_{(L/2)} = 0.$$

These boundary conditions lead to the solution of the equation which is:

$$y = \frac{F x}{2P} - \frac{F \sin(\beta x)}{2\beta P \cos(\frac{\beta L}{2})} \tag{13}$$

At mid length the maximum deflection will be:

$$y_{max} = \frac{F L}{4P}\left[1 - \frac{\tan(\frac{\beta L}{2})}{\frac{\beta L}{2}}\right] \tag{14}$$

Using this maximum deflection, the lateral stiffness (K_s) of the compression member may be driven as:

$$K_s = \frac{4P}{L}\left[\frac{\frac{\beta L}{2}}{\frac{\beta L}{2} - \tan(\frac{\beta L}{2})}\right] = \frac{4\pi^2 EI}{k^2 L^3}\left[\frac{\frac{\pi}{2k}}{\frac{\pi}{2k} - \tan(\frac{\pi}{2k})}\right] \tag{15}$$

Where k is the effective length factor of the member.
Equation (15) coincides with the results driven by Mutton and Trahair [1].
Using equation (14) to get the brace stiffness ratio (K_s/K_o) leads to:

$$\frac{K_s}{K_o} = \frac{1}{3}\left(\frac{\beta L}{2}\right)^2\left[\frac{\frac{\beta L}{2}}{\frac{\beta L}{2} - \tan(\frac{\beta L}{2})}\right] = \frac{\pi^2}{12k^2}\left(\frac{\frac{\pi}{2k}}{\frac{\pi}{2k} - \tan(\frac{\pi}{2k})}\right) \tag{16}$$

Figure (5) shows the relation between the inverse of the effective length factor (1/K) and the brace stiffness ratio (K_s/K_0). It is shown in figure (5) that the brace stiffness factor needed for a second mode buckling is $K_s/K_o = \pi^2/3$ and that the effective length factor (k) may be approximated as:

$$k = \frac{1}{1 + \frac{3K_s}{\pi^2 K_o}} \tag{17}$$

Using the expression of the effective length factor given by equation (17) and Euler buckling stress [2] which is:

$$f_{cr} = \frac{\pi^2 E}{k^2 (\frac{L}{r})^2} \tag{18}$$

leads to the following relation between the buckling stress, f_{cr}, and the brace stiffness ratio (K_s/K_o):

$$f_{cr} = \frac{\pi^2 E}{(\frac{L}{r})^2}\left(1 + \frac{3}{\pi^2}\frac{K_s}{K_o}\right)^2 \tag{19}$$

1.0	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50	2.0
0.0	0.104	0.214	0.329	0.450	0.576	0.709	0.847	0.991	1.142	1.299	$\pi^2/3$

1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95
1.468	1.633	1.811	1.996	2.189	2.390	2.600	2.819	3.049

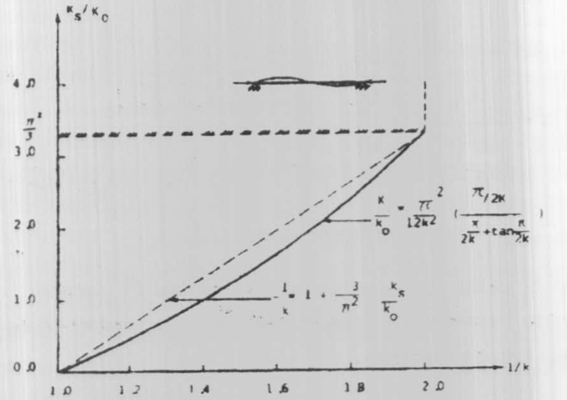

Figure 5. Relation (1/k) and $(\frac{k_s}{k_o})$

For a compatible bracing cell, the lateral stiffness ratio (K_T/K_o) must equal the brace stiffness ratio (K_s/K_o).
Using equations (9), (16), and (19) leads to an expression, for the buckling stress, f_{cr}, the slenderness ratio, L/r, and the modulus of elasticity, E, which is:

$$f_{cr} = \frac{\pi^2 E}{(\frac{L}{r})^2}\left[1 + \frac{f_t(\frac{L}{r})^2}{4\pi^2 E}\frac{\frac{1}{2}\frac{L}{r}\sqrt{\frac{f_t}{E}}}{\frac{1}{2}\frac{L}{r}\sqrt{\frac{f_t}{E}} - \tanh\frac{1}{2}\frac{L}{r}\sqrt{\frac{f_t}{E}}}\right]^2 \tag{20}$$

Equation (16) gives as shown in figure (5) a critical value of K_s/K_o which is $\pi^2/3$ at which the compression diagonal will buckle at the second mode of buckling. This value of K_s/K_o when used with equation (20) leads to a constant ratio between f_{cr} and f_t to cause a second mode of buckling which is:

$$f_t \geq 0.5935\ f_{cr} \qquad (21)$$

This expression is different than that driven by Wang and Borsi [3] which is $f_t = 0.6388\ f_{cr}$ and is close to that introduced by Sritawat and Finch [4] which is $f_t = 0.6\ f_{cr}$

CELL BEHAVIOUR

Consider a bracing cell as shown in figure (6), the force distribution may be driven using the basic structural analysis theories. The member forces can be computed and α, may be given as:

$$\alpha = \frac{\frac{\sin^2\theta}{A_2} + \frac{\cos^2\theta}{A_3} + \frac{1}{A_4} + \frac{1}{A_5}}{\frac{\sin^2\theta}{A_1} + \frac{\sin^2\theta}{A_2} + \frac{\cos^2\theta}{A_3} + \frac{1}{A_4} + \frac{1}{A_5} + \frac{1}{A_6} + \frac{1}{A_7}} \qquad (22)$$

where A_i is the cross sectional area of the i^{th} member. The ratio between the tensile and the compressive forces of the two cross diagonals is:

$$\frac{T}{C} = \frac{1-\alpha}{\alpha} \qquad (23)$$

Equation (23) when used with equation (21) gives a condition relating the geometrical properties, the distribution of areas and the controlling mode of buckling for the compression diagonal. This condition is that the second mode of buckling is governed under the following condition:

$$\alpha \leq 0.6275 \qquad (24)$$

When equation (24) is satisfied the elastic buckling load is increased by a considerable amount which will increase the overall capacity of the bracing cell and the compression diagonal will sustain a major part of the acting force.

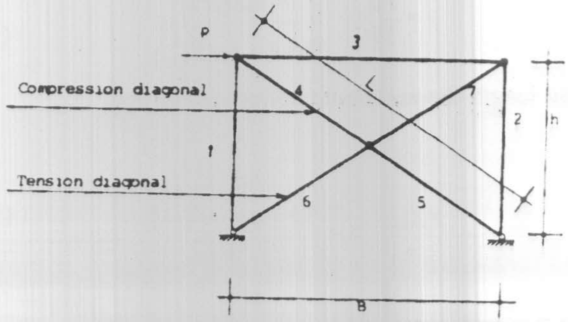

Figure 6 A typical bracing cell.

CONCLUSION

The use of X- bracing is of great importance in the design of steel structures. Criteria are formulated for the general case where the tension and compression braces may have different areas. Results of this study are compared with other solutions. The overall capacity of the bracing cell may be increased considerably by increasing the load carrying capacity of the compression diagonal and the bending rigidity of the tension diagonal.

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