CONTROLLER'S DESIGN FOR SHIP STEERING BY STATE FEEDBACK AND POLE ASSIGNMENT

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ABSTRACT

Based on the state variable feedback and pole assignment concepts, an algorithmic procedure for controllers design with prespecified imposed constraints is presented. The procedure is applied to the steering control of ships overriding the problem of avoiding the state estimator for non-measurable states. Besides, a numerical illustration has been executed by the MATLAB to compute the transient responses of both the open and feedback control loops.

INTRODUCTION

The conventional method for improving system performance by compensating techniques is replaced in modern control theories by the state variable feedback concept. Compensated system requires feedback of only one variable, the output. Of course, in systems compensated through inner feedback loop, more than one dynamic variables are feedback[1].

State variable feedback explores a technique that uses information about all the system's state variables to modify either the control signal or the actuating signal. The former is a version of internal feedback compensation relative to the outer-control loop.

Both forms require all the state variables to be measurable or at least derivable from other information by state reconstructors.

State variable feedback technique is advantageous from the point of view that instrumentation costs may be less expensive than the cost of implementing a complex control algorithm or adding extra actuators.

If so, it would be more economical to measure all the state variables and to use this information to reduce the controller's complexity. In such a case, P-control can be substituted for PD-control if the state variable corresponding to rate is measured.

In addition to cost consideration, the state variable feedback offers significant improvement in performance because the state variables contain a complete description of the plant's dynamics. Moreover, with state variable feedback a better chance of placing the characteristic roots of the closed-loop system in locations that will give desirable performance [2].

The pole assignment problem under constraints was studied by some researchers. A reduced order system with the set of unassigned poles satisfying the constrained conditions may be obtained by removing the characteristic polynomial with the set of assigned poles. This method simplifies the design procedures, whereas the root locus method may be adopted for the proper fixation of the feedback gain[3, 4].

In what concerns the problem of ship dynamics and manoeuvreing, significant analysis, mathematical modelling and numerical estimation of the forces, moments and coefficients are presented in [5-8].

The objectives of this paper is to develop a design procedure of the controller's gain and feedback vector by state feedback and pole assignment applied to the steering control problem. Prespecified constraints are intentionally imposed on the system to exclude building of state estimators.

NOMENCLATURE

a	Translation acceleration of C.G. of ship perpendicular to velocity vector.
a_i	Coefficients of the closed loop characteristic polynomial, i = 1,2...,6
$\underline{A}$	nxn system matrix
$\underline{b}$	control vector
B	Ship's breadth
$\underline{C}^T$	Output vector
d	Longitudinal distance between C.G. of ship and the center of pressure; positive if forward of C.G.
D	Ship's draft
e	Voltage signal
F	Propeller thrust
G(S)	Forward path transfer function
$H_{eq}(S)$	Equivalent feedback transfer function
$\underline{I}$	Unit matrix

J	Polar mass moment of inertia of ship about a vertical axis through C.G. including added moment of inertia due to yaw
K	Controller's gain
K_1, K_2	Parameters of the transfer function of the hydraulic valve
K_3'	Transfer function of the variable stroke pump.
K_3''	Parameter of the rudder cylinder
K_3	$K_3' \cdot K_3''$
K_{CL}	Hydrodynamic lateral force on rudder per unit radian of rudder deflection.
K_D	Total drag force on ship acting in the center of pressure (C.P.)
K_f	Hydrodynamic damping torque coefficient on ship for yawing
K_L	Hydraulic lateral force on ship per unit radian of drift angle; acting in the center of pressure
l	Longitudinal distance between C.G. of ship and point of action of hydrodynamic force on rudder
L	Ship's length
m	Mass of ship including added mass in sway
n	The order of control system
$\underline{Q}$	Controllability matrix
$\underline{r}^T$	State feedback vector
S	Laplace operator
T_1, T_2, T_3, T_a	Time constants of the ship dynamics
u	Input variable
V	Ship's speed
W	Input reference
$\underline{X}(t)$	State variable vector
y	Output variable
α	drift angle
α_1, α_2	Poles of the ship's transfer function $(\Theta_m(s)/\delta_r(s))$
δ_r	Rudder angle
Θ_m	Yaw (heading) angle
τ	Time constant of the hydraulic valve

MATHEMATICAL MODELLING

Consider a ship travelling at a constant speed V with the rudder set to an angle δ_r, the force and moments acting on the ship together with the angles describing its orientation in an inertial coordinate system are shown in Figure (1). The ship executes in this case both translational and rotational motion. Combined sway and yaw motions are considered, since for linearized model the remaining motions, namely surge, pitch, heave and roll are uncoupled from yaw and sway motions.

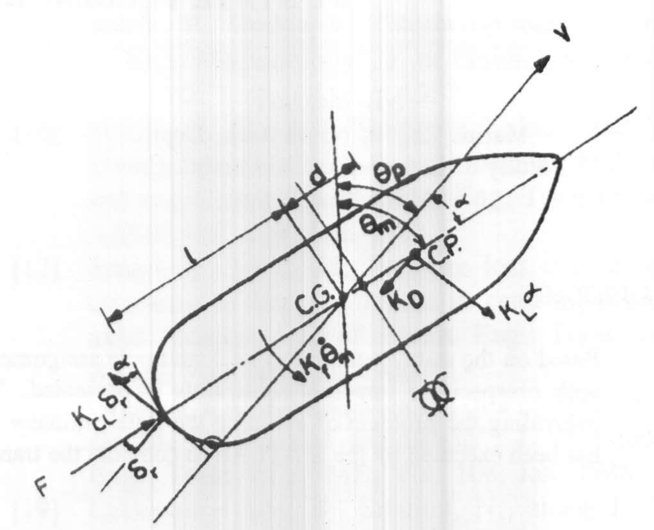

Figure 1. Coordinates, hydrodynamic forces and torques acting on the ship.

Applying d'Alembert's principle in the direction perpendicular to the velocity vector[9], the differential equation for sway is obtained as

$$m a - K_L \alpha - F\cos(90-\alpha) + K_{CL}\delta_r = 0 \qquad (1)$$

Similarly, the yaw equation of motion is obtained by applying d'Alembert's principle for equilibrium of moments about C.G. as

$$J\frac{d^2\theta_m}{dt^2} = (K_L + K_D)d\alpha + K_{CL} l \delta_r - K_f\frac{d\theta_m}{dt} \qquad (2)$$

Linearizing for small values of δ_r and α and transforming into Laplace domain the transfer function of the yaw angle becomes

$$\frac{\theta_m(S)}{\delta_r(S)} = \frac{K_{CL}[m l V S + (K_L + K_D)d + (K_L + F) l]}{S\{J m V s^2 + [J(K_L + F) + K_f m V]S + K_f(K_L + F) - mV(K_L + K_D)d\}} \qquad (3)$$

According to [10,11], the following time constants were defined as

$$T_1 = \frac{m l V}{h} \quad (s)$$

$$T_2 = \sqrt{\frac{J(K_L + F) + K_f\, m\, V}{K_{CL} \cdot h}} \quad (s)$$

$$T_3 = \sqrt[3]{\frac{J\, m\, V}{K_{CL} \cdot h}} \quad (s)$$

$$T_a = \frac{[K_f(K_L + F) - mV(K_L + K_D)d]}{K_{CL} \cdot h} \quad (s)$$

where

$$h = [(K_L + K_D)d + (K_L + F)\, l]$$

and the transfer function of the yaw angle Θ_m becomes

$$\frac{\theta_m(S)}{\delta_r(S)} = \frac{T_1 S + 1}{T_3^3 S(S+\alpha_1)(S+\alpha_2)} \qquad (4)$$

where

$$\alpha_{1,2} = \frac{-T_2^2 \pm \sqrt{T_2^4 - 4T_a\, T_3^3}}{2T_3^3} \quad (s^{-1})$$

and

$$T_2^4 > 4\, T_a\, T_3^3$$

Figure (2) represents the decomposed block diagram of equation (4).

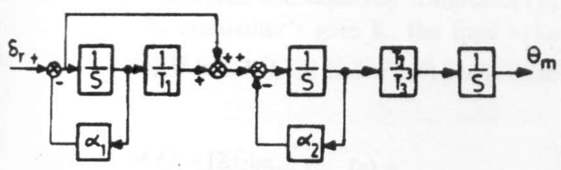

Figure 2. Decomposed block diagram of equation (4).

A suggested schematic electrohydraulic controller for ship steering is indicated in Figure (3). The helm position is sensed by a linear variable differential transformer (LVDT), which supplies an electrical signal to the input of the transformer. The amplifier output drives the electrohydraulic servo valve, which by using a cylinder, controls the stroke of the variable displacement pump. The pump permits both positive and negative stroke positions meaning that the output flow can be reversed in direction. Fluid from the pump enters one or the other of the two rudder cylinders, which acting on the rudder arm, controls the rudder position. A second LVDT senses the rudder position and provides the primary feedback of the system. An additional LVDT supplies pump stroke information to the amplifier, this feedback is necessary to achieve absolute stability.

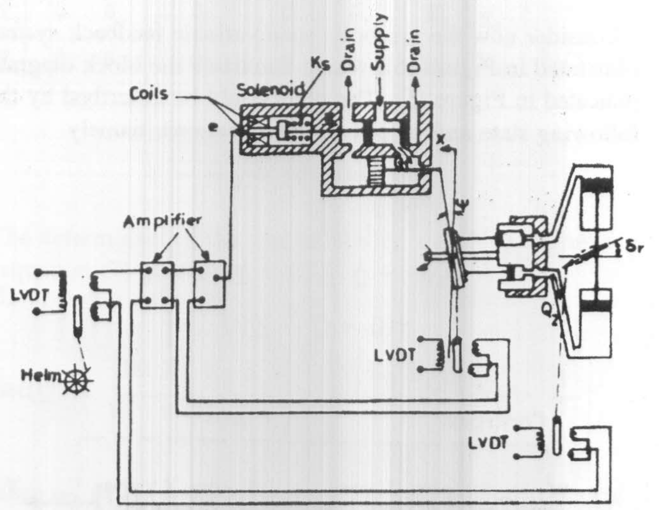

Figure 3. Closed-loop electrohydraulic controller for ship steering.

The dynamic analysis of the electrohydraulic controller was carried out and the final transfer functions are represented in the block diagram shown in Figure (4) [9].

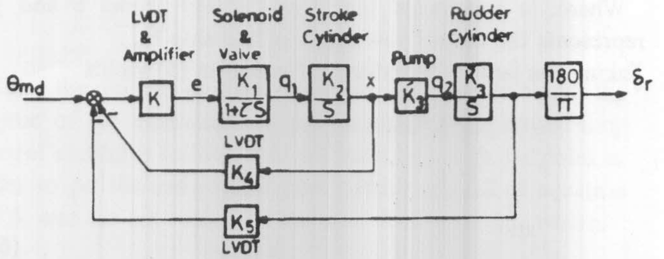

Figure 4. Block diagram of the electrohydraulic controller.

In order to control the direction of the ship during steering, a sensing gyro with gain K_6 is to be fed in the major feedback of the automatic closed-loop of directional control.

The overall block diagram of the ship dynamics with controller is indicated in Figure (5).

It is required to evaluate the controller's gain K, the two minor feedbacks of the LVDT'S (K_4 and K_5) and the gyroscopic gain K_6.

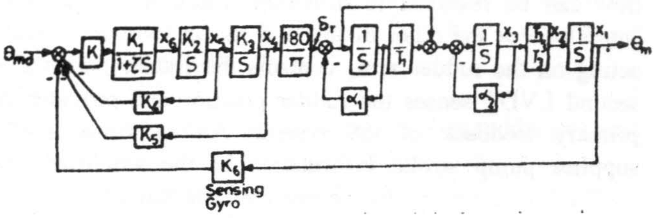

Figure 5. Overall block diagram of the ship dynamics and controller.

Consider now the symbolic state variable feedback system illustrated in Figure (6), which simulates the block diagram indicated in Figure (5). The plant could be described by the following state and output matrix equations, namely

$$\left.\begin{aligned} \dot{\underline{X}}(t) &= \underline{A}.\underline{X}(t) + \underline{b}.u(t) \\ y(t) &= \underline{C}^{T}.\underline{X}(t) \end{aligned}\right\} \quad (5)$$

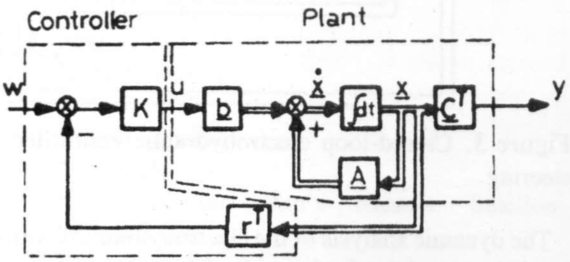

Figure 6. Symbolic state variable feedback of the block diagram in Figure (5).

Where, u represents the input voltage signal e and y represents the output yaw angle of the ship Θ_m.

Taking the laplace transform of equation (5) yields

$$\left.\begin{aligned} &\underline{X}(S) = (S\underline{I} - \underline{A})^{-1}.\underline{b}.u(S) \\ &\text{and} \\ &\frac{Y(S)}{U(S)} = \frac{\underline{c}^{T}.\text{adj}(S\underline{I} - \underline{A}).\underline{b}}{\det(S\underline{I} - \underline{A})} = \frac{N_G(S)}{D_G(S)} \end{aligned}\right\} \quad (6)$$

Where $N_G(S)$ and $D_G(S)$ represent the numerator and the denominator of the transfer function G(S) of the plant stated in equation (6) respectively.

In order to design the controller's gain K and the state feedback vector r^T, the controllability of the plant should be first examined through the controllability matrix $\underline{Q}$, where,

$$\underline{Q} = (\underline{b}\ \underline{A}.\underline{b}\ \underline{A}^{2}.\underline{b} \ldots\ldots \underline{A}^{n-1}.\underline{b}) \quad (7)$$

The controllability matrix $\underline{Q}$ should be non-singular, or

$$\det(\underline{Q}) \neq 0$$

From Figure (6), it could be written that

$$u = K(w - \underline{r}^{T}.\underline{X}) \quad (8)$$

Where w represents the reference signal.

Substituting equation (8) into equation (5) and transforming into Laplace domain, we get

$$\underline{X}(S) = K\left[S\underline{I} - (\underline{A} - K.\underline{b}.\underline{r}^{T})\right]^{-1}.\underline{b}.W(S) \quad (9)$$

$$\frac{Y(S)}{W(S)} = \frac{K.\underline{C}^{T}.\text{adj}\left[S\underline{I} - (\underline{A} - K.\underline{b}.\underline{r}^{T})\right].\underline{b}}{\det\left[S\underline{I} - (\underline{A} - K.\underline{b}.\underline{r}^{T})\right]} = \frac{N_c(S)}{D_c(S)} \quad (10)$$

Where, $N_c(s)$ and $D_c(S)$ are the numerator and denominator polynomials of the closed-loop respectively.

Similarly, the control system can be represented in an alternative way as shown in Figure (7).

From Figure (6). the feedback signal $\sigma(S)$ could be expressed as

$$\sigma(S) = \underline{r}^{T}.(S\underline{I} - \underline{A})^{-1}\ \underline{b}.u(S) \quad (11)$$

From Figure (7), it can be deduced that

$$H_{eq}(S) = \frac{\sigma(s)}{Y(s)} = \frac{\underline{r}^{T}.\text{adj}(S\underline{I} - \underline{A}).\underline{b}}{\underline{C}^{T}.\text{adj}(S\underline{I} - \underline{A}).\underline{b}} = \frac{N_H(s)}{D_H(s)} \quad (12)$$

and similarly;

$$\frac{Y(S)}{W(S)} = \frac{K.G(S)}{1 + K.G(S).H_{eq}(s)} \tag{13}$$

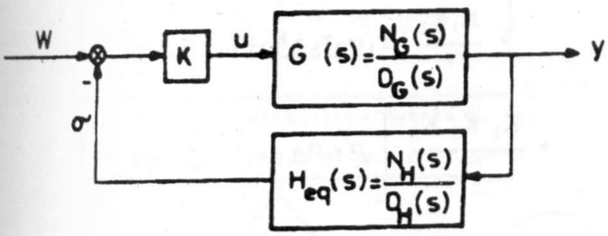

Figure 7. Alternative block diagram of the control system indicated in Figure (6).

It should be noted that, the numerator of equation (6) equals the denominator of equation (12), i.e. $N_G(S) = D_H(S)$. Hence, equation (13) is transformed into the form

$$\frac{Y(s)}{W(S)} = \frac{K.N_G(S)}{D_G(S) + K.N_H(S)} = \frac{K.N_G(S)}{D_c(S)} \tag{14}$$

where, $D_c(S)$ represents the characteristic polynomial of the closed-loop system, i.e.

$$D_c(S) = D_G(S) + K.N_H(S)$$

$$= \det\left[S\underline{I} - (\underline{A} - K.\underline{b}.\underline{r}^T)\right]$$

$$= \det(S\underline{I} - \underline{A}) + K.\underline{r}^T.\mathrm{adj}(S\underline{I} - \underline{A}).\underline{b} \tag{15}$$

or,

$$N_H(S) = \frac{1}{K}\left[D_c(s) - D_G(s)\right] = \underline{r}^T.\mathrm{adj}(S\underline{I} - \underline{A}).\underline{b} \tag{16}$$

By pole assignment of the closed-loop control system $D_c(S)$ could be determined. If the gain K could be evaluated, the state feedback vector r^T will be consequently computed[12].

In order to obtain the controller's gain K, the final value theorem with zero steady state error is applied to equation (14), i.e.

$$\lim_{S \to 0} \frac{K.N_G(S)}{D_C(S)} = 1$$

or,

$$K = \lim_{S \to 0} \frac{D_c(s)}{N_G(S)} \tag{17}$$

Referring to control system under discussion which is indicated in Figure (5) and applying equation (5) to this system yields.

$$\begin{bmatrix} \dot X_1 \\ \dot X_2 \\ \dot X_3 \\ \dot X_4 \\ \dot X_5 \\ \dot X_6 \end{bmatrix} = \begin{bmatrix} 0 & T_1/T_3^3 & 0 & 0 & 0 & 0 \\ 0 & -\alpha_2 & 1/T_1 - \alpha_1 & 180/\pi & 0 & 0 \\ 0 & 0 & -\alpha_1 & 180/\pi & 0 & 0 \\ 0 & 0 & 0 & 0 & K_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_2 \\ 0 & 0 & 0 & 0 & 0 & -1/\tau \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ K_1/\tau \end{bmatrix} e$$

$$Y = \Theta_m = (1\ 0\ 0\ 0\ 0\ 0)\ \underline{X}$$

The determinant of the controllability matrix $\underline{Q}$ expressed in equation (7) gives,

$$\det(\underline{Q}) = \left[\frac{180 K_1\ K_2\ K_3}{\pi\ \tau T_3}\right]^3 \left[\frac{K_1^3 K_2^2 K_3}{\tau^3}\right] \left[\alpha_2 - \frac{1}{T_1}\right]$$

Since $\alpha_2 \neq 1/T_1$, this means that the controllability matrix is non - singular, i.e. the system is already controllable.

The open-loop transfer function can be determined from equation (6) as

$$\frac{Y(S)}{U(S)} = \frac{\theta_m(s)}{E(S)} = \frac{N_G(S)}{D_G(S)} = \frac{\frac{180 K_1 K_2 K_3}{\pi T_3^3 \tau} T_1 (S + \frac{1}{T_1})}{S^3 (S + \alpha_1)(S + \alpha_2)(S + 1/\tau)} \tag{18}$$

Since the number of state variables is six, which equals the degree of the characteristic polynomial of the closed-loop control system it follows that we have to assign six poles in order to get the controllers gain K(with the aid of equation (17)), and the six values of the state feedbacks $\underline{r}^T$, where,

$$\underline{r}^T = (r_1\ r_2\ r_3\ r_4\ r_5\ r_6)$$

In other words, the degree of freedom of assumptions here is six. It is aimed to investigate the behaviour of the automatic control system with intentionally prespecified constraints namely omitting arbitrarily some elements of the state feedback vector r^T.

The state X_3 is non-measurable and it is intended to avoid the state estimator, while instead of feeding back the state, X_2 which is proportional to the rotational velocity of yaw, it may be preferable to feedback the state X_1 by adopting a sensing gyro for the yaw angle. In addition, let us ignore also feeding back of the state X_6. To summarize, instead of feeding back six states, three state variables when feedback may be adequate and may meet the needs of time domain specifications. It follows that three of the state feedbacks namely r_2, r_3 and r_6 should be nullified. The state feedback vector consequently is reduced to

$$\underline{r}^T = (r_1 \quad 0 \quad 0 \quad r_4 \quad r_5 \quad 0) \tag{19}$$

The absence of three of the state feedbacks r_2, r_3 and r_6 necessitates that assigning the six poles of the closed-loop characteristic polynomial fails in this case, because it will yield six feedbacks.

In order to override this difficulty, only three poles of the characteristic equation have to be assigned together with the three nullified state feedbacks $(r_2=r_3=r_6=0)$

Let the characteristic polynomial be

$$D_C(s) = S^6 + a_1 S^5 + a_2 S^4 + a_3 S^3 + a_4 S^2 + a_5 S + a_6$$

According to equation (17), the controller's gain K is given by

$$K = \frac{\pi T_3^3 \tau}{180 K_1 K_2 K_3} a_6 \tag{20}$$

Applying equation (16) gives

$$N_H(S) = \frac{180 K_1 K_2 K_3}{\pi T_3^3 \tau a_6} [(a_1 - \alpha_1 - \alpha_2 - \frac{1}{\tau}) S^5 +$$

$$+(a_2 - \alpha_1\alpha_2 - \frac{\alpha_1+\alpha_2}{\tau}) S^4 + (a_3 - \frac{\alpha_1\alpha_2}{\tau}) S^3 + a_4 S^2 + a_5 S + a_6]$$

$$= \frac{K_1 K_2}{\tau} r_5 S^4 + \frac{K_1 K_2}{\tau} [(\alpha_1 + \alpha_2) r_5 + K_3 r_4] S^3$$

$$+ \frac{K_1 K_2}{\tau} [\alpha_1 \alpha_2 r_5 + K_3(\alpha_1 + \alpha_2) r_4] S^2$$

$$+ \frac{K_1 K_2 K_3}{\tau} \left[\alpha_1 \alpha_2 r_4 + \frac{180}{\pi} \frac{T_1}{T_3^3} r_1 \right] S$$

$$+ \frac{180 K_1 K_2 K_3}{\pi T_3^3 \tau} r_1 \tag{21}$$

Equating the coefficients of S^i, $i = 0,1,2,\ldots,5$ in both sides of equation (21), the following relationships are deduced

$$r_1 = 1 \tag{22}$$

$$r_4 = \frac{1}{\alpha_1 \alpha_2} \frac{180}{\pi T_3^3} \left[\frac{a_5}{a_6} - T_1 \right] \tag{23}$$

$$K_3(\alpha_1 + \alpha_2) r_4 + \alpha_1 \alpha_2 r_5 = \frac{180 K_3}{\pi T_3^3} \frac{a_4}{a_6} \tag{24}$$

$$K_3 r_4 + (\alpha_1 + \alpha_2) r_5 = \frac{180 K_3}{\pi T_3^3 a_6} \left(a_3 - \frac{\alpha_1 \alpha_2}{\tau} \right) \tag{25}$$

$$r_5 = \frac{180 K_3}{\pi T_3^3 a_6} \left[a_2 - \alpha_1 \alpha_2 - \frac{\alpha_1 + \alpha_2}{\pi} \right] \tag{26}$$

$$a_1 = \alpha + \alpha_2 + \frac{1}{\tau} \tag{27}$$

It is obvious that the value of the state feedback r_1 (the gyroscopic gain K_6) is always unity irrespective of the system parameters, i.e.

$$r_1 = K_6 = 1$$

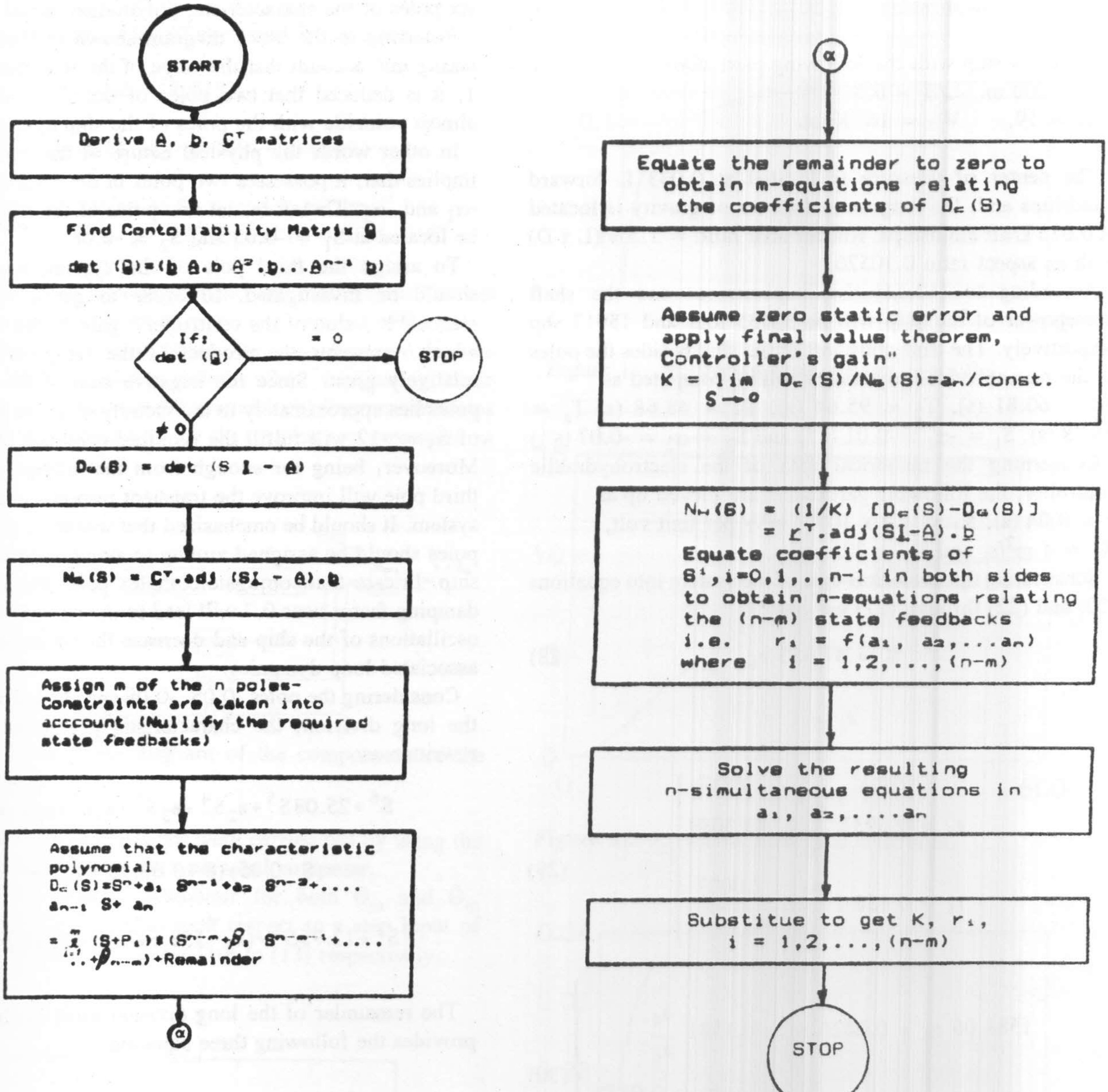

Figure 8. Logical procedure for controller's design with constraints.

Besides, it is worth mentioning that the coefficient a_1 represents the negative sum of the poles of the open-loop transfer function and equals the negative sum of the poles of the ship dynamics plus the reciprocal of the time constant of the hydraulic valve.

Eliminating both r_4 and r_5 in equations (23) to (26), only two equations in terms of the coefficients a_j, $j=2,3,\ldots,6$ are obtained.

The resulting two equations involve five unknowns-the coefficients a_j, $j=2,3,\ldots,6$-necessitate to solve driving another three equations. These equations can be derived by substituting the proposed three poles into the characteristic polynomial of the closed-loop control system.

Solving the above mentioned five simultaneous equations, the unknowns a_2 through a_6 could be determined. Hence, substituting these values of the coefficients in equations (20), (23) and (26), the controller's gain K and the state feedbacks r_4 and r_5 could be calculated as well.

The algorithmic procedure of this approach is pictorially visualized indicated in Figure (8).

Numerical Illustration:

Consider a ship with the following particulars:
L = 200 m, L/B = 6.5
L/D = 19, V = 16 Knots

The center of pressure is located at 0.033 L forward amidships and, the longitudinal center of gravity is located at 0.015 L aft amidships. Rudder area ratio = 1.5% (L x D) with an aspect ratio 0.10526.

According to [10,11] the displacement and the shaft horsepower of the ship will be 50135 tons and 15917 shp respectively. The time constants of the ship besides the poles of the marine vehicle dynamics will be computed as T_1 = 60.81 (s), T_2 = 95.64 (s), T_3 = 48.68 (s), T_a = 75.78 (s), $S_1 = -\alpha_1 = -0.01$ (s^{-1}) and $S_2 = -\alpha_2 = -0.07$ (s^{-1})

Concerning the numerical data of the electrohydraulic controller, the following particulars are picked up as τ = 0.04 (s), $K_1 = 1.3 \times 10^{-3}$ m^3/s per unit volt, K_2 = 4 m^2/s, and K_3 = 325 rad/s.

Substituting the numerical data of the system into equations (20) and (22) through (27) we get:

$$K = 47.655\, a_6 \tag{28}$$

$$r_1 = 1$$

$$\left.\begin{aligned} r_4 &= 0.6932\,\frac{a_5}{a_6} - 43.3089 \\ r_5 &= 0.158\left(\frac{a_2 - 2.0007}{a_6}\right) \end{aligned}\right\} \tag{29}$$

$$\left.\begin{aligned} 1581.06\, r_4 + 0.0426\, r_5 &= 0.1581\,\frac{a_4}{a_6} \\ 19763.25\, r_4 + 4.8648\, r_5 &= 0.1581\left(\frac{a_3 - 0.0175}{a_6}\right) \end{aligned}\right\} \tag{30}$$

$$a_1 = 25.08 \tag{31}$$

To eliminate r_4 and r_5 from equations (30), equations (29) are substituted into equations (30), it results:

$$7\times10^{-4} a_2 - a_4 + 113.9451 a_5 - 7121.5714 a_6 = 1.4\times10^{-3} \tag{32}$$

$$0.08 a_2 - a_3 + 1424.3143 a_5 - 89019.6429 a_6 = 0.1426 \tag{33}$$

In accordance with equation (31) the negative sum of the six poles of the characteristic polynomial should be 25.08.

Referring to the block diagram shown in Figure (5) and taking into account that the value of the state feedback r_1 = 1, it is deduced that two poles of the closed-loop system almost coincide with the poles of the ship dynamics.

In other words the physical nature of the control system implies that, it possesses two poles in the neighbourhood of $-\alpha_1$ and $-\alpha_2$. Therefore, let two poles of the control system be located at S_1 = -0.05 and S_2 = -0.08.

To assign the third pole of the system, equation (28) should be investigated. In order to get a considerably reasonable value of the controller's gain K, the value of a_6 which represents the product of the six poles should be relatively great. Since the negative sum of the remaining poles lies approximately in the vicinity of 25, an assumption of S_3 = -12 will fulfill the required condition of the gain. Moreover, being for enough from the imaginary axis, the third pole will improve the transient response of the control system. It should be emphasized that whenever possible real poles should be assigned aiming to avoid oscillations of the ship. In case that conjugate complex poles are inevitable, a damping factor near 0.7 will lead to the minimization of the oscillations of the ship and decrease the settling time in the associated loop dynamics.

Considering the poles -0.05, -0.08 and -12 and performing the long division, the characteristic polynomial could be rewritten as

$$S^6 + 25.08 S^5 + a_2 S^4 + a_3 S^3 + a_4 S^2 + a_5 S + a_6$$

$$= (S + 0.05)(S + 0.08)(S + 12) *$$

$$\left[S^3 + 12.95 S^2 + (a_2 - 158.6475) S + 20.8333/a_6\right]$$

The remainder of the long division when equated to zero provides the following three equations

$$12.13\, a_2 - a_3 + 20.8333 a_6 = 1904.0924 \tag{34}$$

$$1.564\, a_2 - a_4 + 252.7083\, a_6 = 247.5031 \tag{35}$$

$$0.048\, a_2 - a_5 + 32.5833 a_6 = 7.6151 \tag{36}$$

Solving equations (32) to (35) simultaneously yields the values of the coefficients a_2 through a_6. The obtained values of the coefficients are as follows:-

a_2 = 158.8128 , a_3 = 22.308

a_4 = 0.8894 , a_5 = 0.0091

and $a_6 = 3.639 \times 10^{-5}$

Hence, the poles of the characteristic polynomial are

$-0.05, -0.08, -12.0, -12.9372,$

$-0.0064 + j0.0042$ and $-0.0064 - j0.0042$

The numerical values of K, r_4 and r_5 become,

$K = 17.34 \times 10^{-4}$, $r_4 = 130.05$ and $r_5 = 68.6 \times 10^4$

Evidently, these results express only mathematical values which are excessively far from the performance of the actual components. This is attributed to the absence of some LVDT's and amplifiers, which when readjusting these numerical values in an equivalent block diagram simulates the actual physical nature of the system's components as shown in Figure (9).

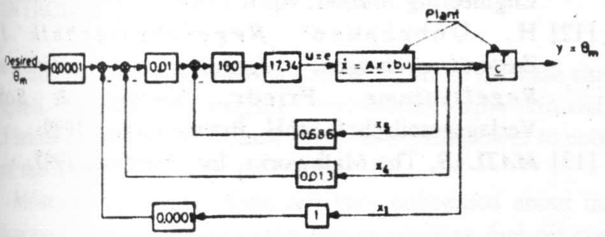

Figure 9. Final block diagram of the components of the control system.

The preceding computations were carried out by using the package MATLAB [13] on a personal computer.

Finally, the dynamic behaviour for both Θ_m and $\dot{\Theta}_m$ without and with controller with respect to a step input of (10°) are displayed in Figures (10) to (13) respectively.

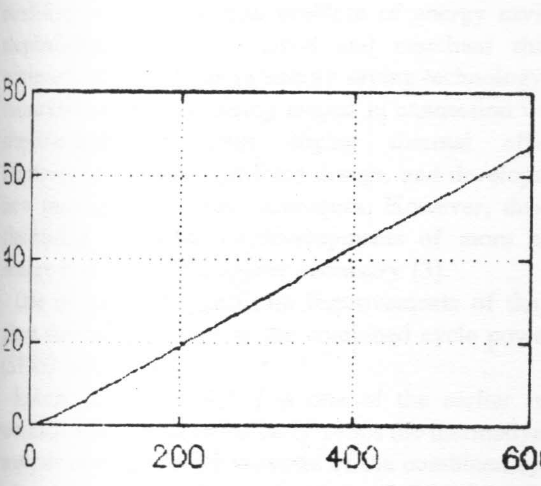

Figure 10. θ_m versus time without controller.

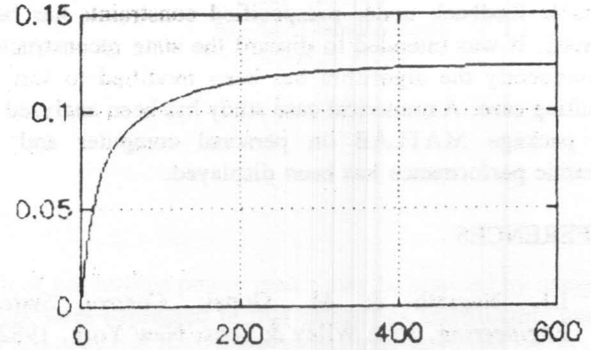

Figure 11.. θ_m versus time without controller.

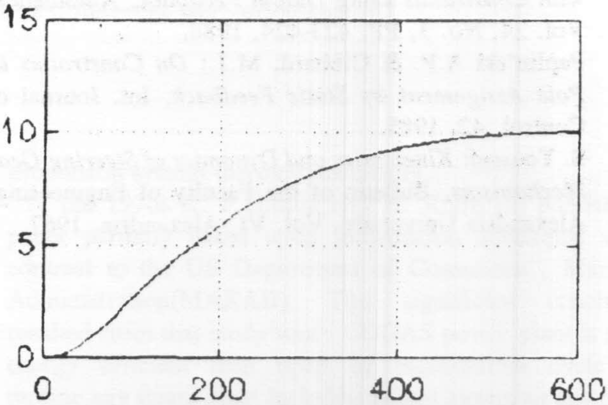

Figure 12. θ_m versus time with controller.

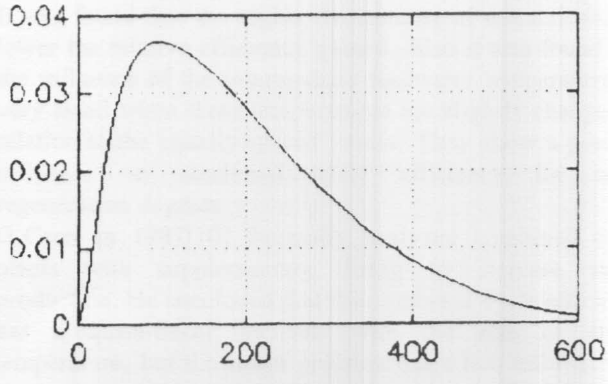

Figure 13. $\dot{\theta}_m$ versus time with controller.

CONCLUSION

The modern approach of state variable feedback and pole placement has been applied to the problem of directional control of ships.

A design procedure of the controller's gain and the state variable feedback under prespecified constraints has been derived. It was intended to discard the state reconstructor, consequently the algorithm has been modified to suit the resulting case. A numerical case study has been analyzed by the package MATLAB on personal computer and the dynamic performance has been displayed.

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