AN EXPERIMENTAL STUDY OF SEEPAGE TOWARDS AN ARTESIAN WELL USING A HORIZONTAL HELE-SHAW MODEL

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ABSTRACT

A horizontal Hele-Shaw model is developed to solve problem of steady radial flow to an artesian well in order to verify its efficiency to handle horizontal flow problems. The groundwater flow through a confined aquifer is imitated by a laminar flow of a viscous oil through a narrow interspace between two parallel horizontal plates. Experimental results agree very well with the Dupuit exact equation. The Hele-Shaw model proved to be a versatile tool for horizontal steady flow investigations.

NOTATION

a	half spacing between the two horizontal prespex plates of the experimental model
g	gravity acceleration
h	piezometric head at a radial distance "r" from a well
h_w	water depth in well
H_o	height of the original piezometric surface
k	permeability coefficient
Q	well discharge
r	radial distance from a well
r_w	well radius
R_o	well radius of influence
Y	thickness of confined aquifer
υ	kinematic viscosity

INTRODUCTION

The use of the Hele-Shaw model is based on the fact that a two-dimensional laminar flow of groundwater through a porous soil can be expressed by the same differential equation as the laminar flow of a viscous fluid through a narrow interspace between two parallel plates.

Hele-Shaw apparatus was mainly used in vertical position as a groundwater flow model to study hydrologic balance (Dietz [2], [3], Todd [9] and Marino [6]). It has been modified to make it possible to simulate precipitation, irrigation and evaporation effects on the groundwater table (Krul and Liefrinck [5]). Santing [10] discussed its adaptability to study of sea-water intrusion problems in coastal aquifers. The model was also employed to facilitate subdrainage studies (Hathoot et al [4] and Sobeih [8]). The Hele-Show model proved to be a versatile tool for groundwater investigations. Its analogy to flow in porous media made it superior to a sand model because of the ease of reading and recording data and the avoidance of a variable capillary zone.

Horizontal Hele-Shaw model can represent a certain areal extent of an aquifer. The interspace between the plates imitates the aquifer, the plate spacing is related to the value of the permeability of the aquifer, and the viscous liquid in the interspace represents the groundwater. According to the Authors knowledge, models of this type have been used little and very few researches have been founded in this field (Santing [7] and Columbus [1]). Seepage towards an artesian well is a well known horizontal flow problem (Figure (1)). Dupuit [10] developed an exact empirical formula to determine the well discharge, Q, from an artesian well in the steady state condition:

$$Q = 2 \pi k Y \frac{H_o - h_w}{\ln(R_o / r_w)} \qquad (1)$$

where

k	=	the permeability coefficient
Y	=	thickness of the confined aquifer
H_o	=	height of the original piezometric surface above the impervious stratum
h_w	=	water depth in the well
R_o	=	well radius of influence
r_w	=	well radius

Using piezometer reading around the well, Eq. (1) appears in the form:

$$Q = 2 \pi k Y \frac{H_o - h}{\ln(R_o / r)} \qquad (2)$$

where h = piezometric head at a radial distance r from the well.

The aquifer chosen for the study is assumed to be homogeneous and isotropic and the flow towards the well is radial everywhere.

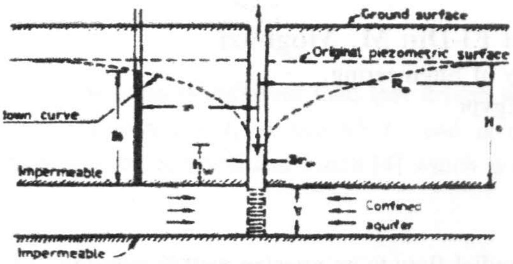
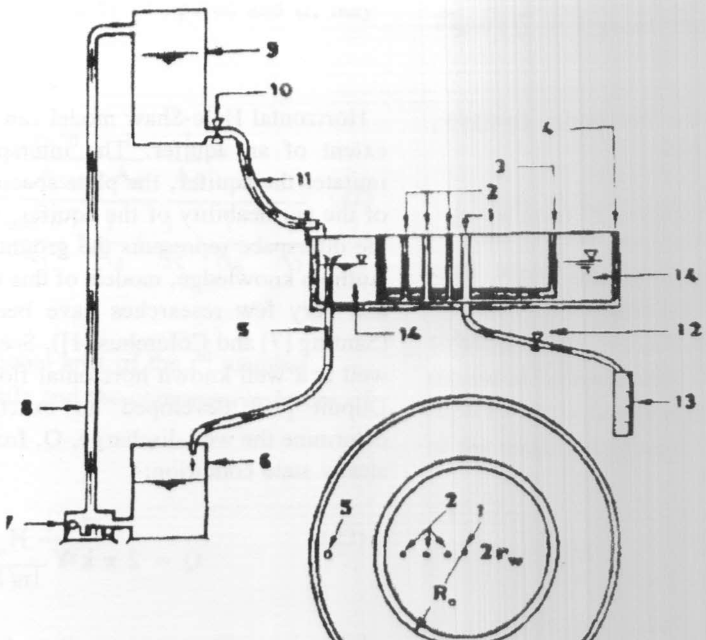


Figure 1. Steady radial flow towards a well penetrating a confined aquifer.

The main objective of this research is to develop a proper horizontal Hele-Shaw model in order to verify its efficiency and accuracy, compared to the Dupuit exact equation, to handle the problem of horizontal groundwater flow towards an artesian well



1_Well

2_Piezometers

3_Internal cylinder

4_Outer cylinder

5_Over flow tube

6_Collected tank

7_Centrifugal pump

8_Feeder tube

9_Supply tank

10_Control valve

11_Feeder tube

12_Control discharge valve

13_Graduated tube

14_Viscous oil

Figure 2. A schematic diagram of the horizontal Hele-Shaw model.

DESCRIPTION OF THE MODEL

Figure (2) illustrates a schematic diagram of the horizontal Hele-Shaw model used to simulate the problem. It consists of two prespex cylinders (10 mm thick.) to represent radial flow towards the well. They are mounted on 15 mm thick. steel base. The outer radius of the inner cylinder represents the well radius of influence, R_o, while the outer cylinder represents a constant head reservoir to represent the original piezometric head of the aquifer H_o. The narrow spacing between the bottom of the two cylinders is kept constant, to represent the confined aquifer thickness Y, with the aid of individually prepared 1.5 mm thick. washers made out of fiber (Klingarite type). The two bottoms of cylinders joined with the washers by brass bolts. A glass tube 10 mm diameter is fitted to the centre of the inner cylinder to represent the artesian well diameter $2r_w$. A viscous oil (supper 7500-20 w /50) is used to represent groundwater. The discharge from the well is withdrawn by a discharge tube connected to the bottom of the outer cylinder and opposite to the well. The well discharge is regulated by a valve and collected into a graduated tube. An overflow tube is used to make it possible to keep constant head in the outer cylinder. To facilitate the reading of data vertical strips of millimeter papers are placed on the outer cylinder. In order to observe any change in the piezometric head during experiments, a group of small diameter piezometers is used as shown in figure (2). A small centrifugal pump is used to lift the viscous oil from a collected tank to an elevated tank in order to feed the constant head reservoir.

MODEL DIMENSIONS

Four inner cylinder have different radii (15, 20, 25 and 30 cms) have been chosen to represent different values of R_o. The well radius r_w is kept constant throughout all experiments and equal to 5.0 mm. Four values of the original piezometric head H_o (4.85, 7.35, 9.85 and 12.35 cms) have been chosen.

DESCRIPTION OF THE EXPERIMENTS WORK

The oil level in the constant head reservoir should be kept constant during any experiment to be ensure that the steady state condition is reached. This is obtained when the inlet flow to the reservoir is equal to the well discharge by using the control discharge valve. Variations in well discharge Q are obtained, for each value of R_o and H_o, by using the control valve. Then, the corresponding reading for both the water depth in the well h_w and the piezometers h are recorded. It should be noted that surrounding temperature should be recorded during each experiment in order to determine the corresponding kinematic viscosity of the oil υ. Although a uniform temperature was achieved in most of the experimental runs, at times the initial temperature varied from the final temperature by from one to several tenths of degree. An average between the initial and final temperatures was therefore considered as the representative temperature. The permeability coefficient of the aquifer k is then obtained from the formula (Todd [10]):

$$k = \frac{a^2 g}{3 \nu} \tag{3}$$

where a = half spacing between the two horizontal prespex plates (spacing between bottoms of the two cylinders)

g = the gravity acceleration

A summary of the experimental records is listed in Table (I) for different values of R_o.

ANALYSIS OF RESULTS

Figures (3) to (6) show the relationship between the measured well discharge Q and drawdown in the well H_o-h_w for radius of influence R_o equal to 15, 20, 25 and 30 cms respectively. A comparison between the experimental and the theoretical results, calculated by the Dupuit equation (Eq. (1)), is also shown in figures (3) to (6). It can be seen that an excellent agreement is obtained between both results. However, there are very small deviations are noticed at some positions. This may be due to probable observational errors occur in keeping a uniform temperature throughout experiment as explained earlier. Therefore, a careful control of temperature is essential.

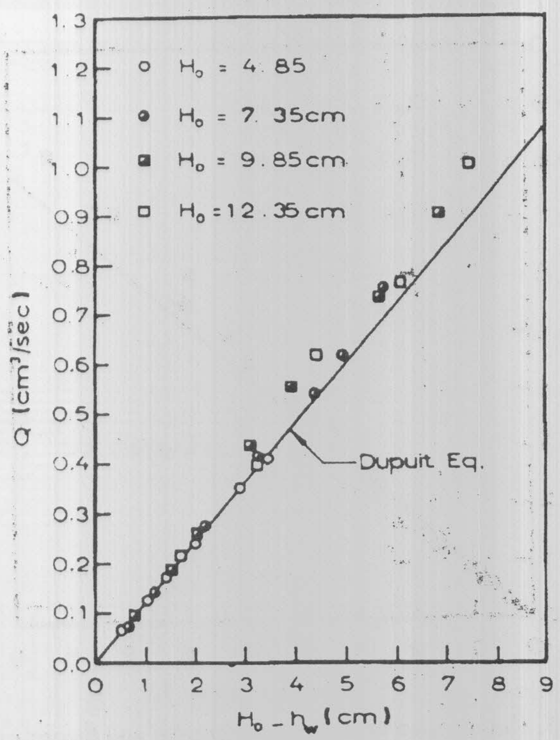


Figure 3. Comparison between experimental and theoretical results for R_o = 15 cm.

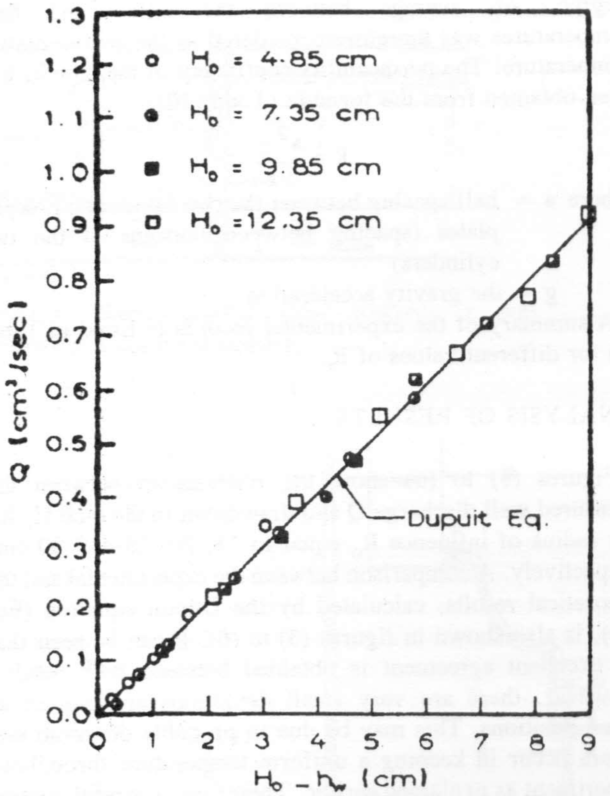


Figure 4. Comparison between experimental and theoretical results for R_o = 20 cm.

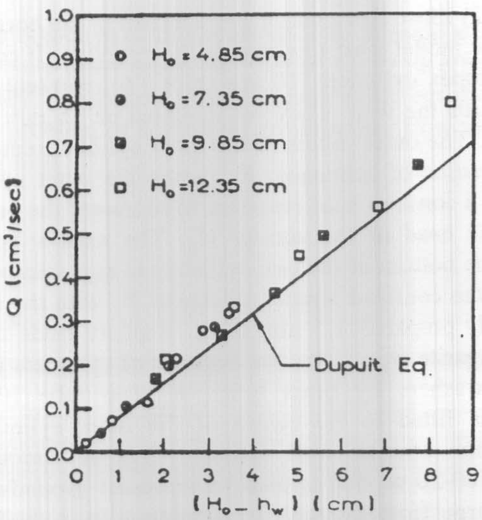


Figure 6. Comparison between experimental theoretical results for R_o = 30 cm.

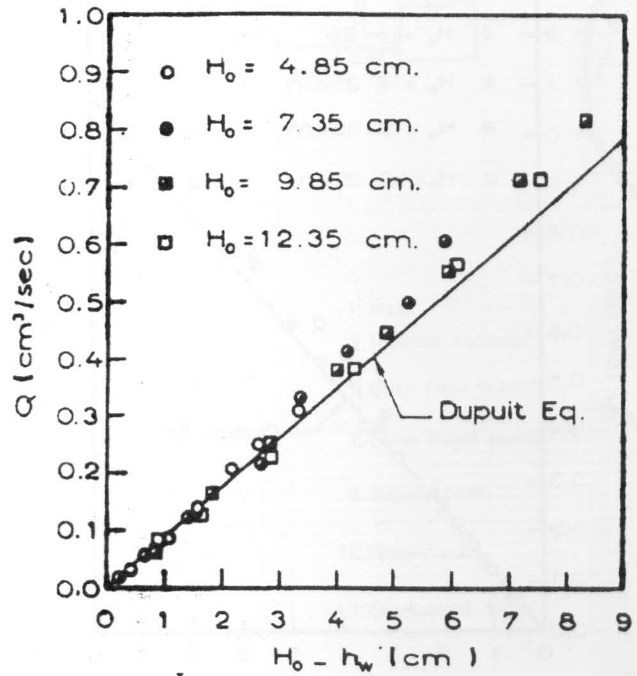


Figure 5. Comparison between experimental and theoretical results for R_o = 25 cm.

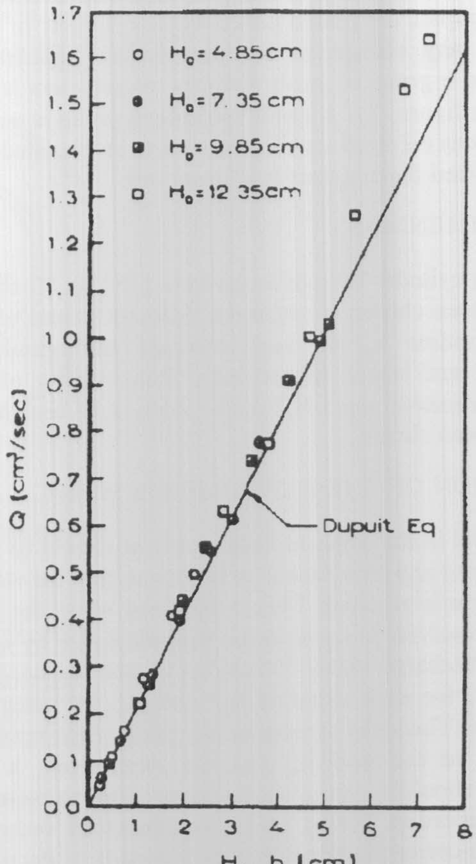


Figure 7. Comparison between experimental and theoretical results for the drawdown in the piezometer (R_o=15cm, r=2cm).

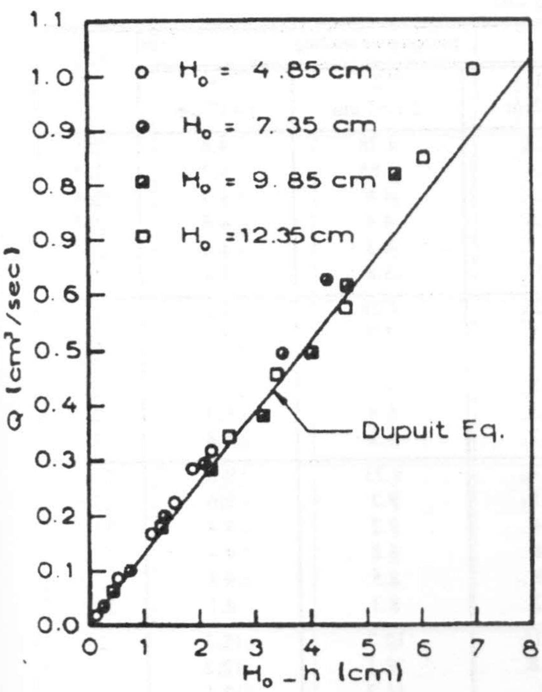


Figure 8. Comparison between experimental and theoretical results for the drawdown in the piezometer (R_0 = 30 cm, r = 2 cm).

Table (I) lists reading of piezometers which are fitted at different radial distances r from the well. Some of these records and the corresponding well discharge, for a piezometer at r=2 cm, are plotted in Figures (7) and (8) for R_o equal to 15 and 30 cms respectively. These figures show also a comparison between the experimental and the theoretical results using equation (2). It can be noticed that a very good agreement is obtained at different values of H_o and R_o.

CONCLUSIONS AND RECOMMENDATIONS

1. Experimental results agree very well with those obtained by the Dupuit equation for a well fully penetrating a horizontal confined aquifer.
2. The horizontal Hele-Shaw model proved to be a versatile tool for simulating purely horizontal flow problems.
3. A careful control of the surrounding temperature is essential during the experiments.
4. The horizontal Hele-Shaw model can be easily modified to study the effect of the replenishment of the groundwater by rainfall or the loss by evapo-transpiration on the flow towards an artesian well.

Table (I-a). R_o = 15 cm

H_o cm	h_w cm	Volume cm^3	time min	time sec	piezometer teading cm 1 r=2cm	2 r=7 cm	3 r=12 cm	C°
	4.3	10	3	11	4.5	4.6	4.7	27°
	3.8	10	1	29	4.3	4.5	4.6	27°
	3.4	10	-	58	4	4.4	4.5	27°
4.85	2.8	20	1	11	3.7	4.3	4.5	27°
	1.9	20	-	48	3.1	4	4.5	27°
	1.4	10	-	20	2.7	4.4	4.4	27°
	6.8	10	2	15	7	7.1	7.2	25.5
	6.2	10	1	13	6.7	7	7.2	25.5
7.35	5.1	10	-	34	6.0	6.8	7.1	25.5
	4.0	10	-	24	5.4	6.6	7.0	25.5
	3.0	10	-	18	4.7	6.3	6.8	25.5
	2.4	10	-	16	4.3	6.0	6.8	25.5
	9.1	10	1	41	9.4	9.6	9.7	27
	8.3	10	-	52	8.9	9.4	9.6	27
9.85	7.8	10	-	37	8.6	9.3	9.6	27
	6.7	20	-	45	7.9	9.0	9.5	27
	5.9	20	-	36	7.5	8.8	9.4	27
	4.2	20	-	27	6.4	8.4	9.3	27
	11.6	10	1	50	11.8	12	12.1	26.5
	10.6	10	-	45	11.2	11.8	12	26.5
	9.1	10	-	24	10.4	11.4	11.8	26.5
12.35	7.9	10	-	16	9.6	11.1	11.7	26.5
	6.3	10	-	13	8.6	10.8	11.6	26.5
	4.9	10	-	10	7.6	10.4	11.5	26.5

Table (I-b). $R_o = 20$ cm

H_o cm	h_w cm	$Volume_w$ cm^3	time min	time sec	piezometer teading cm 1 r=2cm	2 r=7 cm	3 r=12 cm	C°
4.85	4.6	5	3	40	4.7	4.78	4.8	26°
	4.2	5	1	4	4.4	4.65	4.7	26°
	3.8	5	-	38	4.3	4.6	4.7	26°
	3.2	5	-	26	3.8	4.4	4.6	26°
	2.8	5	-	23	3.7	4.3	4.6	26°
	1.8	10	-	28	3	3.8	4.4	26°
7.35	7	5	5	32	7.25	7.28	7.3	25
	6.6	5	1	10	6.8	7.2	7.3	25
	6.0	5	-	37	6.6	7.0	7.2	25
	4.8	10	-	40	5.8	6.7	7.1	25
	4.0	10	-	30	5.3	6.4	6.8	25
	3.2	10	-	25	4.7	6.2	6.8	25
9.85	9.5	5	2	47	9.7	9.75	9.8	25.5
	8.7	10	1	28	9.1	9.3	9.6	25.5
	7.6	10	-	44	8.4	9.2	9.4	25.5
	5.6	10	-	31	7.8	8.8	9.4	25.5
	5.2	10	-	21	6.8	8.5	9.3	25.5
	4.1	10	-	16	6.2	8.3	9.1	25.5
12.35	12.1	5	4	30	12.15	12.2	12.3	26
	11.4	5	1	4	11.8	12.1	12.2	26
	10.2	5	-	22	11	11.7	12.1	26
	8.7	10	-	26	10	11.3	11.9	26
	7.2	10	-	18	9.2	10.9	11.7	26
	5.8	10	-	15	8.3	10.6	11.5	26

Table (I-c). $R_o = 25$ cm

H_o cm	h_w cm	$Volume_w$ cm^3	time min	time sec	piezometer teading cm 1 r=2cm	2 r=7 cm	3 r=12 cm	C°
4.85	4.4	5	4	13	4.6	4.7	4.8	25
	3.7	5	1	1	4.1	4.4	4.6	25
	3.2	5	-	34	3.7	4.3	4.6	25
	2.7	5	-	23	3.4	4.1	4.5	25
	2.2	5	-	20	3.0	3.9	4.4	25
	1.5	5	-	16	2.6	3.8	4.3	25
7.35	7.1	5	6	47	7.15	7.2	7.3	25
	6.7	5	1	26	6.8	7.1	7.2	25
	5.9	5	-	39	6.4	6.8	7.0	25
	4.8	5	-	24	5.8	6.5	6.9	25
	4.0	5	-	15	5.2	6.3	6.7	25
	3.2	5	-	15	4.5	6.0	6.5	25
9.85	9.5	5	4	55	9.65	9.7	9.8	24.5°
	9.0	5	1	34	9.6	9.7	9.8	24.5°
	8.0	5	-	33	8.5	9.2	9.4	24.5°
	7.0	5	-	20	8.1	8.9	9.4	24.5°
	5.8	5	-	13	7.2	8.6	9.2	24.5°
	5.0	5	-	11	6.5	8.3	9.0	24.5°
12.35	12	5	5	57	12.15	12.2	12.3	24.5°
	11.4	5	1	14	11.7	12.0	12.2	24.5°
	10.7	5	-	43	11.2	11.8	12.0	24.5°
	9.5	5	-	23	10.4	11.4	11.8	24.5°
	8.0	5	-	13	9.3	11.0	11.6	24.5°
	6.3	5	-	9	8.4	10.5	11.3	24.5°

Table (I-d). R_o = 30 cm

H_o cm	h_w cm	$Volume_w$ cm^3	time min	time sec	piezometer teading cm 1 r=2cm	2 r=7 cm	3 r=12 cm	C°
4.85	4.6	5	4	27	4.7	4.75	4.8	24.5
	4.0	5	-	58	4.3	4.55	4.6	24.5
	3.2	5	-	31	3.7	4.3	4.4	24.5
	2.6	5	-	22	3.3	4.0	4.3	24.5
	2.0	5	-	17	2.9	3.8	4.2	24.5
	1.5	5	-	15	2.7	3.7	4.1	24.5
7.35	6.8	5	1	53	7.1	7.25	7.3	25
	6.2	5	-	46	6.6	7.0	7.2	25
	5.3	5	-	25	6.0	6.7	6.9	25
	4.2	5	-	17	5.3	6.3	6.7	25
	3.3	5	-	11	4.5	5.9	6.4	25
	2.4	5	-	10	3.8	5.6	6.3	25
9.85	9.6	5	4	15	9.6	9.65	9.7	24.5°
	9.0	5	1	7	9.3	9.5	9.6	24.5°
	7.9	5	-	26	8.5	9.3	9.4	24.5°
	6.5	5	-	18	7.6	8.7	9.0	24.5°
	5.4	5	-	14	6.7	8.3	8.9	24.5°
	4.2	5	-	10	5.8	7.8	8.6	24.5°
12.35	12.0	5	2	50	12.2	12.25	12.3	24.5°
	11.3	5	-	52	11.6	11.8	12	24.5°
	10.3	5	-	25	11	11.6	11.8	24.5°
	8.7	5	-	15	9.8	11.0	11.5	24.5°
	7.2	5	-	11	8.8	10.6	11.2	24.5°
	5.5	5	-	9	7.5	10	10.9	24.5°

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