

# SWITCHED RELUCTANCE MOTOR CHARACTERISTICS BASED ON A NON-LINEAR MODEL

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## ABSTRACT

This paper presents a detailed computer study of Switched Reluctance (SR) motors based on a non-linear model, which proved to be very accurate in predicting the performance of a wide range of SR drives. The effect of design parameters on the steady state characteristics of a doubly salient machine has been investigated. The paper concludes with recommendations for optimum choice of the different parameters according to the type of application.

## INTRODUCTION

Over the last three decades or so, there has been a great interest in developing new types of electrical machines fed from solid state power converters in order to obtain a better performance over wide ranges of torque and speed. More recently Switched Reluctance (SR) drive systems have emerged as an attractive and competitive solution for variable speed applications/1/.

Basically, an SR system consists of a simple reluctance machine (doubly/1/or/singly/2/salient), a power electronic switching circuit/3/ (bifilar, non-bifilar, H-circuit, ...) and a position sensor (or sensorless /4/).

The torque is developed by the tendency for the magnetic circuit to adopt a configuration of maximum inductance. The current is commutated to the appropriate phase winding according to the position of the rotor with respect to the stator in order to optimise its performance. The torque produced is independent of the direction of current flow.

The well documented design methods available for conventional machines are not readily applicable to the design of SR systems. This is because the magnetic circuit is heavily saturated under normal operation, and because there are many dependent design parameters to choose from, as well as the interactive effects between the machine and the feeding electronic switching circuit. The task becomes even more difficult if the optimum design is aimed for.

It must be noted that SR systems are significantly different from variable reluctance stepping motors. Thus where as stepping motors are current fed, low-power positioning devices which operate under open loop control, SR Systems are voltage fed, high-power variable speed drives which operate under closed loop control /5/. These differences suggest that any attempt to use the design methods readily available for stepping motors to design SR systems will not give satisfactory results.

The object of this paper is to review the SR design methods available to date and concentrate on an accurate computer simulation program developed for doubly salient SR drive systems which takes into account the saturation of iron and the form of the switching circuit used. Using this program an extensive study was made on the effect of many design parameters on the output torque capability of the machine.

A graphical optimisation routine /6/ was used to search for the optimum parameters keeping the rms current density, peak flux density and peak flux current under control throughout the optimisation procedure.

The machine design parameters which were studied in this paper are:

Air-gap length, split ratio, different pole combinations, number of phases, ranges of stator and rotor pole arcs, number of turns per phase, and switching angles.

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The paper concludes with recommendations for optimum choice of the different parameters according to the load characteristics.

## DESIGN METHODS

There is no single design method for all types of electrical machines. The design methods of conventional machines have been developed over many years and have now become a set of well documented design procedures. These methods are not applicable to SR Systems as the principle of operation is totally different, the flux is pulsating not rotating or stationary, they are excited from a power electronic circuit, they operate with wide range of speeds and outputs, and the flux density is relatively high under normal operation.

For SR systems there are many design methods available. Each method has its merits, but there is only one distinct method which has been proved reliable /9,11/. The following is a brief review of the design methods available to date.

### 1. scaling

In this method the parameters of a new machine are scaled from a well designed original machine. It is fast and easy but it requires an original machine of the same structure. The scaling equations must be derived for each new configuration and an optimum design cannot be guaranteed /10/.

### 2. Linear methods

The machine is represented by a linear model in which the iron throughout is assumed to be infinitely permeable so that the inductance profile is linear. The calculation of the average torque is based upon the change of system co-energy when the rotor moves a complete rotor pole pitch. Linear analysis is used for better understanding of the relative effect of the machine parameters on its performance, but fails badly for realistic design procedure /6/.

### 3. Finite element (FE) techniques

Finite Element methods have been applied to calculate the torque output from SR machines with different structures /7,8/, but to the best knowledge of

the authors, general design methods based on FE have not been yet developed. Also the cost FE package is still high.

## 4- Non-Linear methods

Saturation in Sr machines is complex and affects performance of the system to great extent. It cannot be treated using a saturation factor as applied in conventional machine /5/. On the other hand a non-linear model which takes into account the combined effect of the main magnetic circuit saturation and local saturation has to be developed.

There are many non-linear models already developed for SR systems. Each one uses different design equations. The accuracy of the model depends upon the assumptions made in deriving these equations. So the degree of the model accuracy may be sacrificed to reduce the computing time.

## SIMULATION TECHNIQUES

A set of computer programs is written to solve the mathematical equations which represent both the machine and the switching circuit.

In deriving the mathematical model the following assumptions are made:

- 1- Iron losses are not taken into account.
- 2- Mutual coupling between phases is neglected.
- 3- Steady state operation is assumed.
- 4- No skin effect in the windings.
- 5- A non-bifilar switching circuit of the types shown in Figure (1) is used.

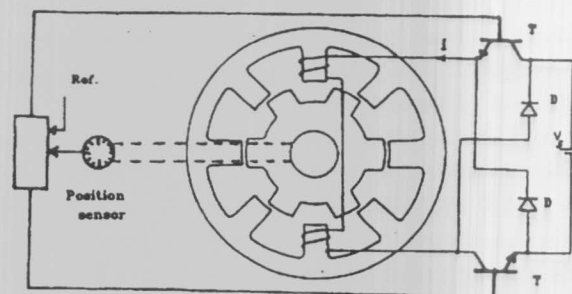
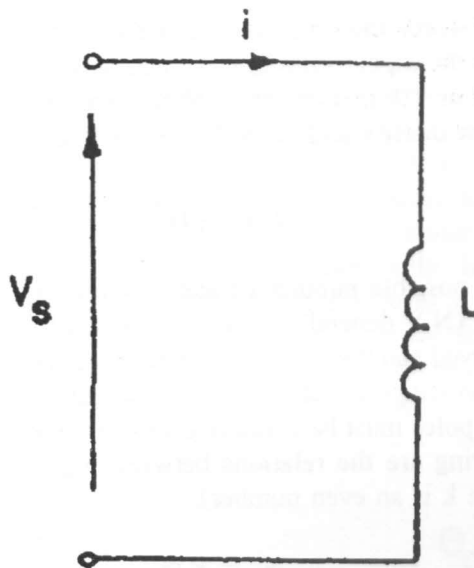


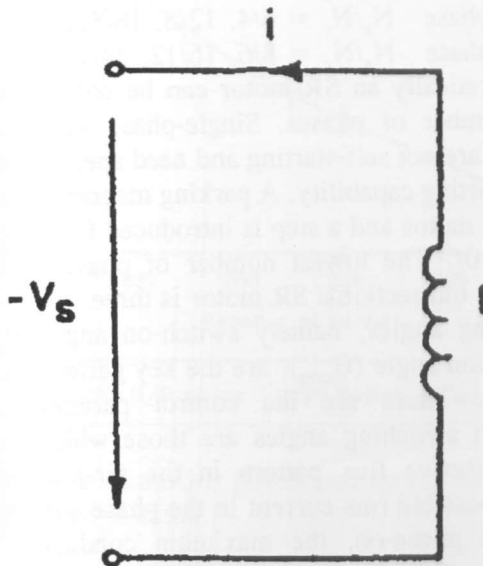
Figure 1. Basic circuit of 1 4-phase SR system.

The equivalent circuits of one phase are shown in Figure (2-a) during conduction period and (b) during switch-off period.

The voltage equations for (a) and (b) are given respectively by:



(a)



(b)

Figure 2. Equivalent circuit of one phase of an SR motor  
 a- during conduction period  
 b- during switch-off period.

$$V_s - 2 V_T = iR + N \frac{d \phi (\theta, i)}{dt} \quad (1)$$

$$-V_s - 2 V_D = iR + N \frac{d \phi (\theta, i)}{dt} \quad (2)$$

Under steady state conditions the model equations can be written as follows:

$$N \frac{d \phi (\theta, i)}{d\theta} = 1 / \Omega (V_s - 2 V_T - iR) \quad (3)$$

$$N \frac{d \phi (\theta, i)}{d\theta} = 1 / \Omega (-V_s - 2 V_D - iR) \quad (4)$$

Where

- $V_s$  is the dc supply voltage
- $V_T$  is transistor saturation voltage
- $V_D$  is diode forward voltage drop
- $R$  is the phase winding resistance
- $N$  is the number of turns per phase
- $\phi$  is the flux per pole
- $\theta$  is the rotor angle
- $\Omega$  is the angular velocity

The model requires flux-linkage or inductance to be known at all values of both  $i$  and  $\theta$ .

The inductance variation table is synthesised from details of the machine geometry. The minimum inductance value was analytically estimated in terms of the geometric proportions of the machine by constructing an approximate field pattern consisting of line segments and circular arcs. The maximum inductance was calculated using the B-H data of the iron used. Intermediate inductance values at different rotor angles were obtained using a piece-wise approximation /9/. This method was modified /10/ by linear considering the saturation of the minimum inductance and adding sinusoidal components to the values of the linear variations of the inductance for better predictions of the intermediate inductance values. The solution of the previous equations requires a table of current values as functions of both rotor angle  $\theta$  and flux  $\Phi$ ,  $i (\theta, \Phi)$ . This table with equally spaced angles and flux is obtained by inverting the input table  $\Phi (\theta, i)$ . The model equations are then numerically solved using the Runge-Kutta fourth order integration method. Intermediate values of  $i (\theta, \Phi)$  required through the course of numerical integration are obtained using quadratic interpolation for  $\Phi$  and linear interpolation for  $\theta$  / 11/ .

An optimisation method was used/6/, in which the

output from a given machine configuration at a given speed is maximised subject to certain constraints. These are the peak flux density (B), the rms current density (J) and the peak current ( $I_m$ ). This method will give a perfect match between the magnetic and electric characteristics of the machine and considers the ratings of the switching devices.

Figure (3) shows a simplified flowchart of the computer algorithm used for the study presented in the following section.

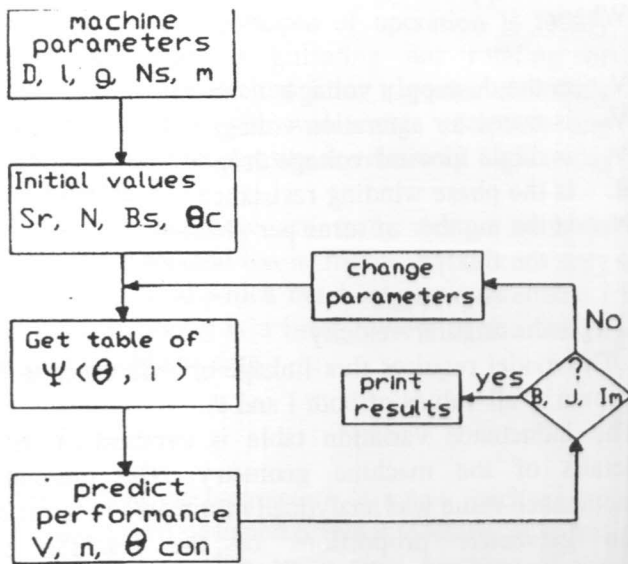


Figure 3. Simplified flowchart of the computer algorithm.

SIMULATION RESULTS

All the results are obtained for a doubly salient SR motor. D90L frame, with outer stator diameter  $D = 0.140$  m, and a core length of  $0.114$  m. The mode of operation is single pulse, voltage-fed from a non-bifilar, two transistors per phase,  $285$  V dc supply and a speed of  $1000$  rpm. The imposed constraints are:  $B = 2$  T,  $J = 5$  A/mm<sup>2</sup> and  $I_m < 3$  I.

It is clear that the output can be increased by increasing the working current density (if the method of cooling permits) and by allowing the machine to work with a higher peak flux density (if the saturation content can be accommodated). To deal with other frame sizes, both B and J must be readjusted. As the machine gets bigger, its efficiency increases, or in

other words the small machines are electrically limited while the bigger ones are magnetically limited. It is found in [10] that an approximate relation between current density and outer diameters is given by:

$$J \propto 1/\sqrt{D}$$

The possible number of stator poles ( $N_s$ ) and rotor poles ( $N_r$ ) depend on the number of phases employed and the number of stator poles per phase. The windings on at least two diametrically opposite stator poles must be connected to form one phase. The following are the relations between  $N_s$ ,  $N_r$ ,  $m$  (where k is an even number):

$$N_s = k m$$

$$N_r = N_s - k$$

The possible combinations of stator and rotor poles for 2, 3 and 4 phases are:

For 2-phase  $N_s/N_r = 4/2, 8/4, 12/6, \dots$

For 3-phase  $N_s/N_r = 6/4, 12/8, 18/12, \dots$

For 4-phase  $N_s/N_r = 8/6, 16/12, 24/18, \dots$

Theoretically an SR motor can be constructed with any number of phases. Single-phase and two-phase motors are not self-starting and need special means for their starting capability. A parking magnet is used for a 1-phase motor and a step is introduced for the 2-phase motor [10]. The lowest number of phases for a self-starting, bidirectional SR motor is three.

Switching angles, namely switch-on angle ( $\theta_o$ ), switch-off angle ( $\theta_{co}$ ), and conduction angle ( $\theta_{con}$ ), are the key parameters in SR motor systems. These are the control parameters. The optimum switching angles are those which give the most effective flux pattern in the air-gap with the lowest possible rms current in the phase winding. For one phase-on, the maximum conduction angle ( $\theta_{con}$ ) (fully open) is given by:

$$\theta_{con} = 2 \pi / (mN_r)$$

According to the above equation, the conduction angles for the 2, 3 and 4 phase motors are  $90^\circ$ ,  $45^\circ$ , and  $15^\circ$  respectively.

For a given magnetic structure there will be an optimum number of turns which maximise the output under the previous chosen constraints. The effective

changing the number of turns on the output of a 4-phase motor with 15° conduction angle is shown in Figure (4). Also switch-on angle curve is shown.

Figure (5) shows the relation between maximum output power and the split ratio for 2,3, and 4 phase SR motors. It can be noticed that the 2-phase motor has the lowest output while the 4-phase motor has the highest. The split ratio increases with increasing number of phase. Also the number of turns at optimum split ratio of the 4-phase motor is approximately 25% less than that required by the 3-phase motor at its optimum split ratio.

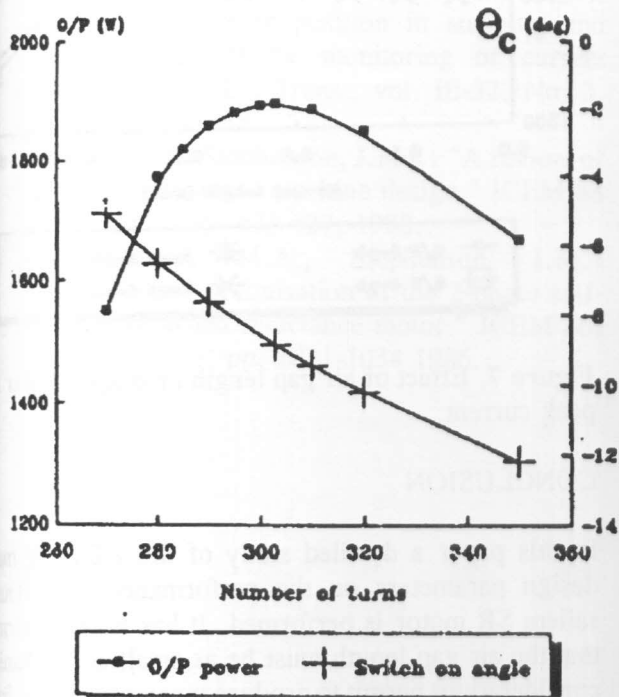


Figure 4. 4- Phase O/P power and switch on angle for different no. of turns.

Note that iron losses were neglected in the model and increasing the number of phases will increase the supply switching frequency and hence increase the iron losses. It is therefore not reliable to study higher number of phases without considering the iron losses.

Pole arcs are very important parameters in the design of SR motors. Optimum values for better starting performance are not the optimum for better running performance. Using the linearised inductance profile it

can be shown that pole arcs must satisfy the following conditions for self starting from any position and reversible rotation [1/]:

$$\tau_r / m \leq \beta_s \leq \tau_s - \tau_r / m \quad (9)$$

$$\beta_r = \beta_s + \sigma \quad (10)$$

where

$\tau_r, \tau_s$  are the rotor and stator pole pitches ( $\tau_r = 2\pi/N_r, \tau_s = 2\pi/N_s$ )

$\beta_s$  and  $\beta_r$  are stator and rotor pole arcs respectively.  
 $\sigma$  is the dead zone defined as the region of zero inductance variations.

To satisfy the condition of self starting and reversible rotation the range of possible pole arcs are:

For 3- phase  $30^\circ \leq \beta_s \leq 30^\circ$

For 4- phase  $15^\circ \leq \beta_s \leq 30^\circ$

The 3-phase motor is inflexible in its design due to the starting requirements. For practical design the non-linear variation of inductance will allow the lower limit of stator pole arc to be smaller than that obtained from the previous inequalities. Also, the presence of a dead zone is not desirable [10/]. The lower limit of stator pole arc will increase the available space copper, but on the other hand saturation problems arise.

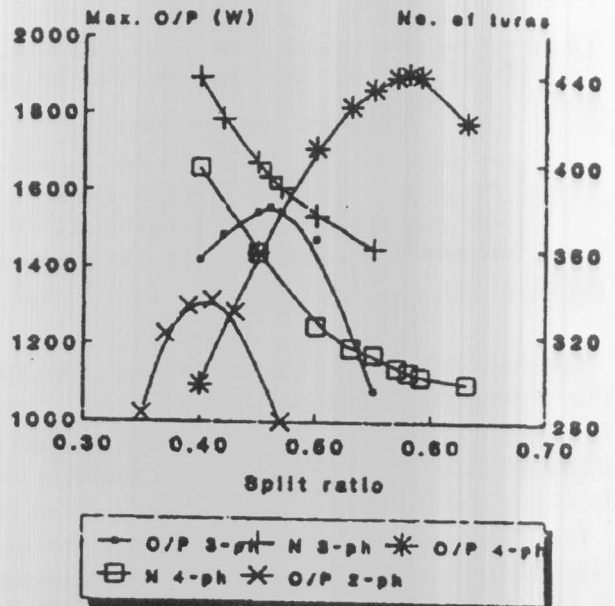


Figure 5. Maximum O/P and number of turns as a function of split ratio for 2,3 and 4 phase SR motor.

Referring to Figure (5) and according to the above analysis the low output of both 2 and 3-phase motors compared to that obtained from a 4-phase motor can be attributed to the fact that the optimum stator pole arc from the point of view of better starting performance is not the optimum one for better running performance.

The effect of stator pole arcs on the running performance is shown in Figure (6). The optimum stator pole arc for the 3-phase is 20° while its optimum value for the 4-phase is 15°. The ratio of  $\beta_s/\tau_s$  is 0.33 for both machines. This is not the case for stepping motors/5/where this optimum ratio is in the range of 0.4-0.45.

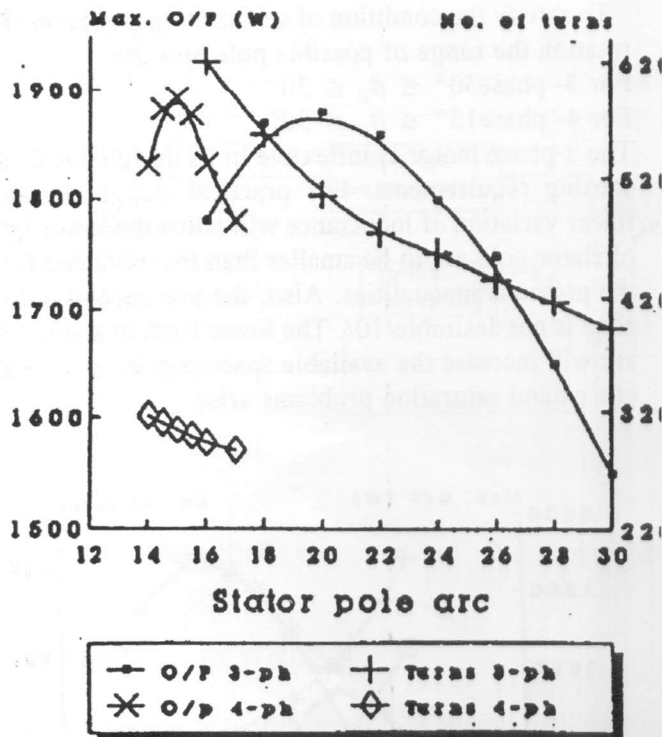


Figure 6. Effect of stator pole arc.

The maximum output of the 3-phase motor is close to that of the 4-phase motor when the starting constraint is relaxed. Practically, the fringing flux, which was neglected in the linear theory will give a starting torque from any position even with a stator pole arc of 20°

The effect of the air-gap length on the output obtained from 3-phase and 4-phase SR motors is shown in Figure (7). This figure confirms the common belief that the superior performance of SR systems is attributed to its small air-gap length. SO, it is desirable

to use as small an air-gap as mechanical considerations permit. Also the variation of peak current with air-gap length is shown in the same figure.

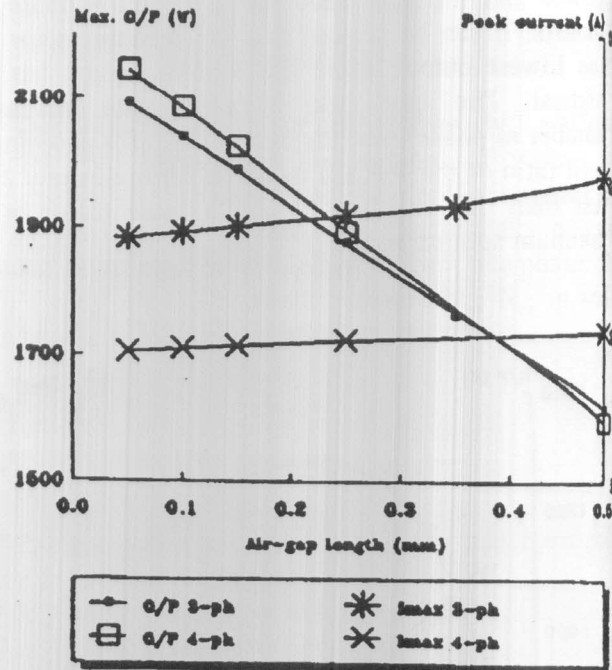


Figure 7. Effect of air gap length on output power and peak current.

CONCLUSION

In this paper a detailed study of the effect of many design parameters on the performance of a double salient SR motor is performed. It has been confirmed that the air gap length must be as small as mechanical considerations permit to produce maximum output from a given frame size. Also, it has been shown that a 4-phase design is superior to the 3-phase one. The speed ratio is an important parameter and it is a function of the number of phases, the speed ranges and the type of application. The optimum ratio of stator pole arc to stator pole pitch is less than that obtained using stepping motors design methods.

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