PREDESIGN OF A BUILT-UP FLEXURAL ELEMENT "MINIMUM COST CONCEPT"

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ABSTRACT

An optimum predesign concept for a built up flexural element is presented. This concept is based on minimizing the total cost of an element taking into account the material cost and the expensive cost of the modern technology of surface treatments (coating, painting). Actually, these surface treatments are commonly used for either fire protection and/or corrosion prevention.

NOTATIONS

- Z_x elastic modulus of a section with respect to major axis x.
- I_x moment of Inertia with respect to major axis x.
- A_w Area of web.
- A_f Area of flange.
- A Total area of section.
- t_w web thickness.
- t_f flange thickness.
- h height of section
- h_w height of web.
- S element surface area per unit length.

INTRODUCTION

The aim of structural design is to fulfil some conditions to produce a safe and economical structure. The use of a built up section for some structure elements is a usual solution for satisfying these conditions, and also, for aesthetic requirements. The materials used for the protection against fire and the prevention of corrosion have an important influence on the economy of the design. Thus, the total cost of a structure could be represented by the cost of the two following parameters:

1. Cost of structural material per unit weight (including fabrication and erection).
2. Cost of material used for element surface protection per unit area (including application).

In order to reduce the cost of painting and/or coating the element surface area must be minimized, but the balance between the reduced costs of surface treatment and the cost of material weight must be achieved.

In fact, the economy of a structure depends on many factors. Not only the initial cost of a structure gives the economy requirements but also, the cost of maintenance plays an important part and constitutes a major share of the project cost. The modern technology offers nowadays materials for surface treatment with initial relative expensive cost, and minimum maintenance requirements.

Therefore, the optimum design of a built up section, based on the concept of minimum weight only [1,2], does not fulfil the economy requirement. In the present work, the two parameters; element weight and element surface area treatment are introduced in a minimum cost predesign concept.

WORK OBJECTIVE

It is proposed here to elaborate a predimensining design formula satisfying both the static and the kinematic requirements of a given flexural element. The predesign problem of a beam element could be represented as follows:

- Knowing the applied bending moment M for a given beam, one can obtain the elastic modulus of section (Z_x) which satisfies the minimum resistance requirement.
- For a given maximum allowable deflection value, according to the current specifications, one can

obtain the required moment of Inertia (I_x) for the beam section which satisfies the deflection condition. It is well known that the optimum design of a beam element is governed either by Z_x or by I_x. In this study, predesign charts expressing the variation of the optimum height of a given beam as function of the required Z_x or I_x are presented. These charts take into account the optimum section of the beam and consider a general cost study based on minimizing a function of both the beam weight and its surface area.

HYPOTHESES

The general hypotheses concerning the resistance of materials in elastic deformation are considered here. All calculations are being oriented to the study of a symmetric I-section, and neglecting the local flange rigidity. Finally the height of the section (h) is taken as: $h = h_w + t_f$ (Figure (1)), in order to simplify the analytical approach [2].

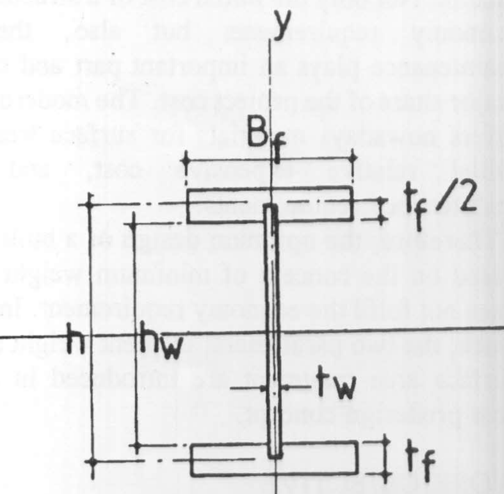


Figure 1. Symetric I-section.

THEORETICAL MODEL

a) Optimum height satisfying resistance condition.

The elastic modulus (Z_x) which satisfies the static requirement is supposed to be known.

From Figure (1), the moment of inertia (I_x) of t section can be written as;

$$I_x = \frac{h^2}{12}(3A - 2A_w) \quad (1)$$

Rearranging relation (1), the total area of the cros section (A) is expressed as follows;

$$A = \frac{4I_x}{h^2} + \frac{2}{3}A_w \quad (2)$$

substituting the value of (I_x) by $\frac{Z_x h}{2}$ in relation (2), the total area becomes:

$$A = \frac{2Z_x}{h} + \frac{2}{3}A_w \quad (3)$$

From the current specification [3], the stabilit requirement of web, is governed by the condition;

$$t_w = \alpha\, h \quad (4)$$

where α is a constant depends on web stiffenin condition.

Then, the total area of relation (3) could be represente in term of the elastic modules Z_x and the height h:

$$A = \frac{2Z_x}{h} + \frac{2\alpha h^2}{3} \quad (5)$$

For the case of unstiffened web Figure (2), the surfac area per unit length of the element is simpl represented by the relation:,

$$S = 2h + 4B_f \quad (6)$$

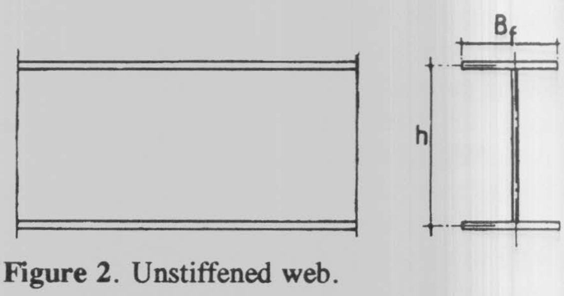


Figure 2. Unstiffened web.

and for the case of a stiffened web as shown in figure (3), the surface area is written as follows:

$$S = 2h + 4B_f + \frac{2B_f}{\gamma}$$

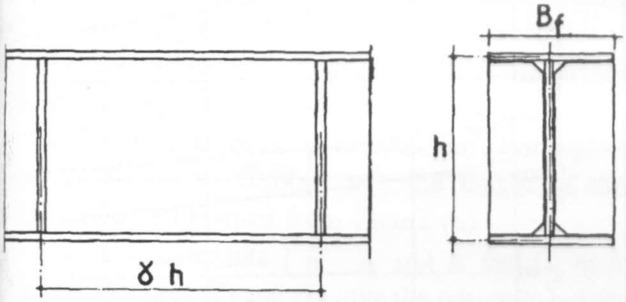


Figure 3. Stiffened web.

where $\gamma > 0$,

To build a general relation for stiffened and unstiffened webs, a scaler η is introduced:

$$S = 2h + 4B_f + \frac{2\eta B_f}{\gamma} \quad (7)$$

where;

$\eta = 0$ for the case of unstiffened web and $\eta = 1.0$ for the case of a stiffened web.
Substituting the value of B_f by β h, then,

$$S = h(2 + 4\beta + \frac{2\eta\beta}{\gamma}) \quad (8)$$

where; β which represents the width-height ratio, satisfies the lateral stability requirements of the section.
From relations (5) and (8), the total cost (C_t) of a unit length of a flexure element is;

$$C_t = C_s.d.A + C_p.S$$

or;

$$C_t = \frac{2C_s.d.Z_x}{h} + \frac{2}{3}C_s \,.\, d \,.\, \alpha \,.\, h^2 + (2 + 3\beta + \frac{2\eta\beta}{\gamma})C_p \,.\, h \quad (9)$$

where;

(C_s) is the cost per unit weight. (representing structural material, fabrication, erection, and transport), (C_p) is the cost per unit area (representing coating and/or painting material for surface treatment and the applications) and (d) is the structural material density.
Expression (9) passes with a unique minimum at $\frac{dC_t}{dh} = 0$, this leads directly to the cubic equation:

$$a\,h^3 + bh^2 + c = 0 \quad (10)$$

where;

$$a = \frac{4dC_s\alpha}{3},$$

$$b = C_p(2 + 4\beta + \frac{2\eta\beta}{\gamma}) \text{ and } C = -2\,d\,C_s\,Z_x. \quad (11)$$

A complete description for the analytical solution of the cubic equation (10) is presented in [4].
In case of a neglected surface treatment cost ($C_p = 0$) i.e. case of ordinary painting, the analytical solution of equation (10) becomes;

$$h = \frac{\sqrt[3]{-c}}{a}$$

or

$$h = \frac{\sqrt[3]{3Z_x}}{2\alpha} \quad (12)$$

b) Optimum height satisfying deflection condition

The moment of inertia (I_x) which fulfils the kinematic requirement is supposed to be known.
The total area (A), given by relation (2), can be rearranged as follows:

$$A = \frac{4I_x}{h^2} + \frac{2\alpha h^2}{3} \quad (13)$$

In this case, the total cost per unit length (C_t) which satisfies the deflection condition is obtained using relations (8) and (13):

$$C_t = \frac{4C_s \cdot d \cdot I_x}{h^2} + \frac{2}{3}C_s \cdot d \cdot \alpha \cdot h^2 + (2 + 3\beta + \frac{2\eta\beta}{\gamma})C_p \cdot h \qquad (14)$$

The optimum height, corresponding to the minimum cost is obtained at $\frac{dC_t}{dh} = 0$, which leads to the quardic equation;

$$a\,h^4 + bh^3 + c = 0\,, \qquad (15)$$

where; a and b are given by (11), and in this case;

$$c = -8\,d.\,C_s\,I_x \qquad (16)$$

The solution of the quardic equation (15) is described analytically in [4].

For the case where the cost of coating is neglected ($C_p = 0$), , the solution of equation (15) becomes:

$$h = \sqrt[4]{\frac{6I_x}{\alpha}} \qquad (17)$$

SYSTEMATIC APPLICATIONS

Equations (10) and (15), which determine the optimum height and satisfy the resistance and deflection conditions respectively, are solved here numerically. For the case of unstiffened web ($\eta = 0$) the height obtained from equations (10) and (15) is compared, in Figure (4), with that of the corresponding standard IPE200 and IPE400 profiles. The comparison is made taking into account the value Z_x, I_x, α, and β of the standard profiles. From Figure (4), it is clear that if the static condition is dominant in the design, the height of the standard profiles is practically optimum for low values of C_p only. On the other hand, if the deflection condition is dominant in the design, the height of the standard profiles does not present the optimum solution even for ordinary painting cost.

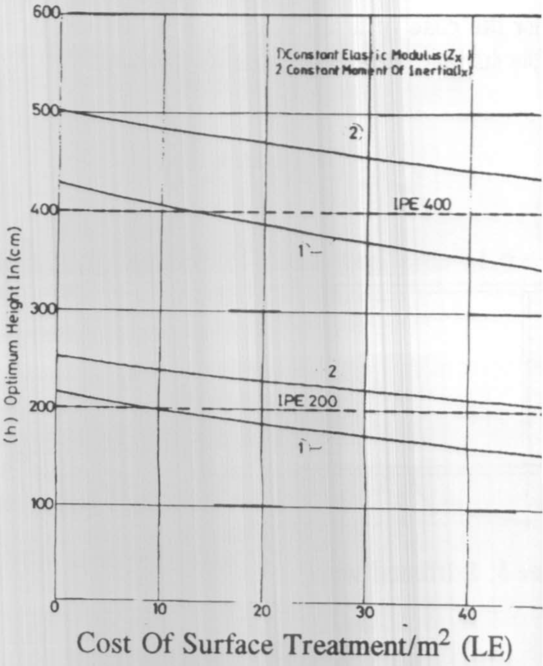


Figure 4. Variation of the optimum height surface treatment cost.

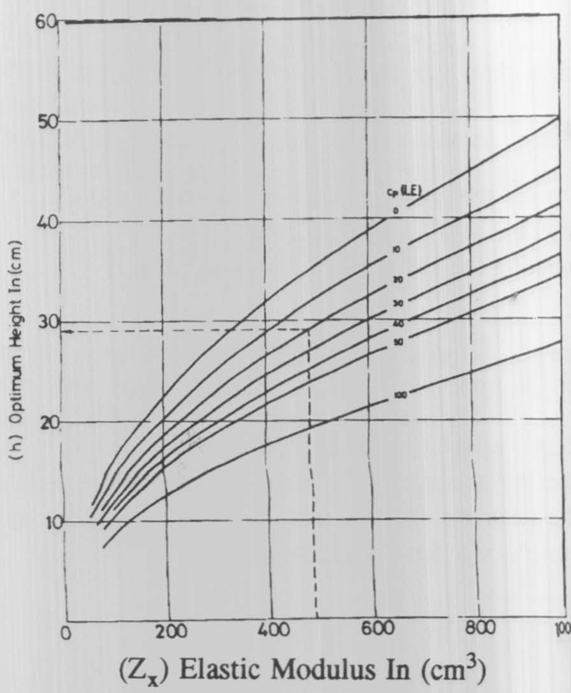
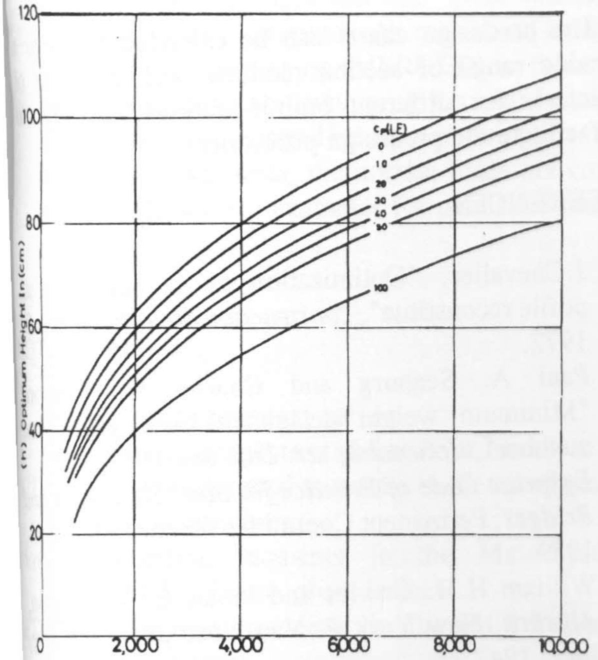


Figure 5. Variation of the optimum height (h) with the elastic modulus (Z_x) (Z_x up to 1000 cm^3).

Another systematic application is effectuated in order to construct predesign charts. This application is made for the case of unstiffened web ($\eta = 0$), for which $\alpha = \frac{1}{85}$ as recommended by the Egyptian code [3] and considering $\beta = 0.25$. The variation of the optimum height as a function of the required elastic modulus is presented in Figure (5), taking the cost of surface treatment per square meter (C_p) as a parameter.

In this figure Z_x is taken up to 1000 cm^3. For higher values of Z_x up to 10,000 cm^3, the height of the section could be obtained from Figure (6).

For the same constants (η , α and β and C_p as a parameter, Figures (7) and (8) give the optimum height of the section as a function of the required moment of inertia I_x.



(Z_x) Elastic Modulus In (cm^3)

igure 6. Variation of the optimum height (h) with the astic modulus (Z_x) (Z_x up to 10,000 cm^3).

n Figure (7), I_x is taken up to 10,000 cm^4 and the her values up to 100,000 cm^4 are presented in ure (8).

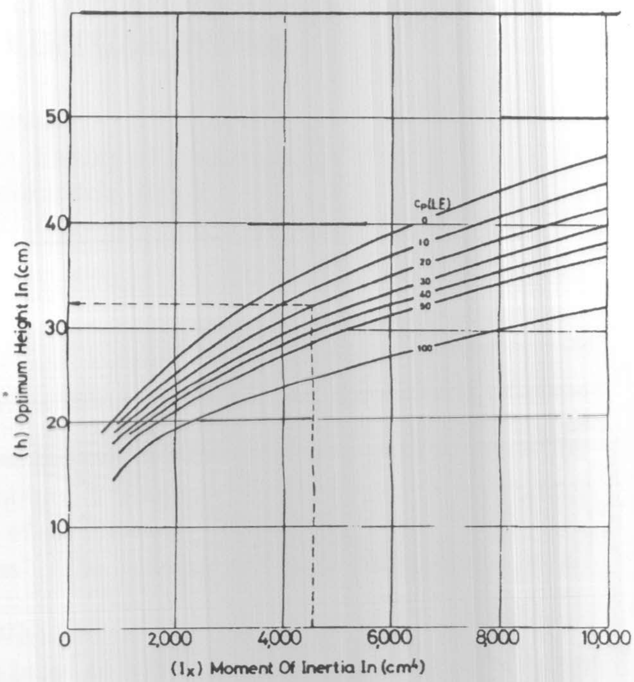


Figure 7. Variation of the optimum height (h) with the moment of Inertia (I_x) (I_x up to 10,000 cm^4).

It is noticed from these figures that the optimum height decreases as the cost of surface treatment (C_p) increases.

SIMPLE PREDESIGN FORMULAS

For the case of ordinary painting, where the cost of surface treatment is practically negligible, the optimum height of a section is estimated by expressions (12) and (17). It is important to note that the optimum height determined by these expressions, which is also obtained in [1], is independent not only on the current price for the structural material, but also on the conditions governing the lateral stability requirement of element section (β).

Two cases are presented here with respect to the stiffening conditions.

* case of unstiffened web, ($\alpha = \frac{1}{85}$), the section height is given by either:

$$h_1 = 5.03 \sqrt[3]{Z_x}$$

or

$$h_2 = 4.75 \sqrt[4]{I_x}$$

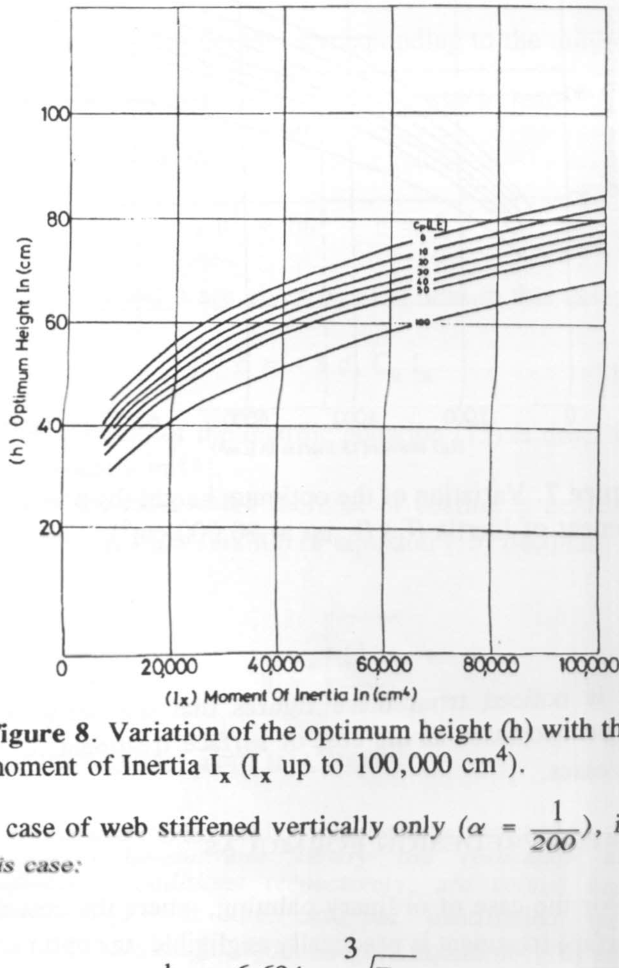


Figure 8. Variation of the optimum height (h) with the moment of Inertia I_x (I_x up to 100,000 cm^4).

* case of web stiffened vertically only ($\alpha = \frac{1}{200}$), *in this case:*

$$h_1 = 6.694 \sqrt[3]{Z_x}$$

or

$$h_2 = 5.886 \sqrt[4]{I_x}$$

The optimum height (h) in each case is taken as the maximum value of h_1 and h_2.

CONCLUSION

An optimum concept is presented for th predimensionning of a built up flexural element. Th concept is based on balancing the cost of structur material weight and the cost of surface treatment fo fire protection and/or corrosion prevention. The majo conclusions of the study are:

1. If the deformation is the dominant in the design (i.e. crane girders, crane bridges, special floor caring machines) the optimum height obtained i always bigger than that of the standard (IPE Profiles.
2. If the resistance is the dominant in the design, and for (C_p) more than 10 LE the optimum height i smaller than that of the standard, (IPE). profiles.
3. From the predesign charts, the optimum heigh *decreases with the increase of the surface* treatmen cost.
4. The predesign charts can be extended to cover a wide range of section modules and moments of interia for different built I-sections in order to facilitate the predesign procedure.

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