# EFFECT OF TROUGH MOTIONS ON PERFORMANCE OF OSCILLATING CONVEYORS

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## **ABSTRACT**

The conveying advance depends on performance of the coveyor. This work presents an analysis of the performance of oscillating conveyors. The conveyor trough mechanism is driven by either a follower-point or a coupler-point of a four-bar planar linkages. Also, different proposed motions to the trough are studied. These types of motions are selected to be simple harmonic, cycloidal, parabolic, 3-4-5-polynomial or eight-polynomial. The presented formulations can be used to synthesize other planar mechanisms. The results showed that, in order to improve the conveying performance, it is better for such conveyors to be driven by a coupler-point on a drag-link driving mechanism. Also, it is preferable that the trough motion should be parabolic.

## NOMENCLATURES:

Coefficients

1 & C

Aac	Coefficients
g	Gravitational acceleration
L	Stroke length of the trough
r	Link length
R	Distance between the crank-pivot and
	coupler-point p or follower-point f
t	Time
μ	Coefficient of friction
,	Position components of a point on through and stationary coordinates respectively
φ	Total crank angle which corresponds to an inflection point
β	Total angle of stroke
θ	Angular displacement of a link or crank angle
ω	Angular velocity of a link
σ&λ	Sign factors
α	Angle of the coupler or follower link

# Subscripts:

- ()c Conveying stroke of the trough
- ()e End of the complete cycle to the trough
- ()i Coupler-point p, follower-point f or link number

()cr Crank

- ()L Load
- ()o Initial value
- ()r Relative value
- ()ri Initial relative value
- ()s Start of the operating cycle of the trough
- ()t Trough or return stroke of the trough

## Abbreviations

C-R Crank-rocker, D-L: Drag-link and conv.: Conveyor

## 1. INTRODUCTION

Conveying machines are used widely in industry. Several studies on these machines can be found in [1-14]. In [1,2], the dynamic analysis of nine-link and twelve-link adjustable mechanism has been presented. General characteristics of one group of the conveying machines (conveyors) are given in [3,4]. The conveyed load rides on the trough of the belt, chain and screw conveyors are given in [5-7]. While the relative motion of the conveyed load with the trough of shaking conveyors or oscillating conveyors are given in [8-14]. The performance of the driving mechanism of an oscillating conveyors in which its trough is suspended

by levers has been carried out by computer aided design in [11]. Performance of such conveyors has been given in [12]. This performance is measured by the conveying advance. Useful design nomograms for such conveyors can be found in [13]. The purpose of this work is to be point out the effect of the trough motions on the performance of two types of oscillating conveyors. The first (type I) is conveyor with variable pressure on the trough and the second (type II) is conveyor with constant pressure on the trough.

## 2. MODEL FORMULATION

#### 2.1. Relative motion

The acceleration components of the relative motion of the trough with the conveyed load are given by:

$$\ddot{x}_r = \ddot{x}_L - \ddot{x}_t$$

$$\ddot{y}_r = \ddot{y}_L - \ddot{y}_t$$
(1)

Where:  $\ddot{x}_t$  and  $\ddot{y}_t$  are given by:

$$\ddot{\mathbf{x}}_{t} = \ddot{\mathbf{X}}_{t} \cos \theta_{t} + \ddot{\mathbf{Y}}_{t} \sin \theta_{t}$$

$$\ddot{\mathbf{v}}_{t} = \ddot{\mathbf{Y}}_{t} \cos \theta_{t} + \ddot{\mathbf{X}}_{t} \sin \theta_{t}$$
(2)

By similar method, the relative velocity and displacement components  $\dot{x}_r$ ,  $\dot{y}_r$  and  $x_r$ ,  $y_r$  respectively can be achieved.

# 2.2. Conveyor Performance

The performance of the conveyors is measured by the conveying advance q [12] which is the sum of the relative displacement component  $x_r$  at end of complete cycle to the trough and is given by:

$$q = \sum_{\theta_{cr} = \theta_{s}}^{\theta_{e}} x_{r}$$
 (3)

x<sub>r</sub> has been derived as follows:

i) Within pure riding stage

$$x_r = 0$$

ii) Within sliding stage

$$x_{r} = (C/2) t^{2} + \mu k y_{t} - x_{t} + V_{1} t + S_{1}$$
where,
$$C = g (\mu k \cos \theta_{t} - \sin \theta_{t}),$$

$$V_{1} = -C t - \mu k \dot{y}_{t} + \dot{x}_{t} + \dot{x}_{ri},$$

$$S_{1} = -(C/2)t^{2} - \mu k y_{t} + x_{t} - V_{1} t + x_{ri},$$

$$k = -sgn (\dot{x}_{r})$$

# iii) Within flight stage

During this stage, the relative displacement is derive by:

$$x_r = -x_t - (g/2) t^2 \sin \theta_t + V_2 t + S_2$$

and

$$y_r = -y_t - (g/2) t^2 \cos \theta_t + V_3 t + S_3$$
 (6)

Where,

$$V_{2} = \dot{y}_{t} + g t \sin \theta_{t} + \dot{x}_{n},$$

$$V_{3} = \dot{x}_{t} + g t \cos \theta_{t} + \dot{y}_{n},$$

$$S_{2} = x_{t} + (g/2) t^{2} \sin \theta_{t} - V_{2} t + x_{n},$$
and 
$$S_{3} = y_{t} + (g/2) t^{2} \cos \theta_{t} - V_{3} t + y_{n}$$

More details of these equations can be found in [12].

# 2.3. Trough Motions

The trough is driven by either four-bar linkages or by different proposed motions. In case of the trough is driven by a four-bar linkages (crank-rocker of drag-link mechanism), the connection between the trough and the driving mechanism is by either coupler point or follower-point, Figures (1) and (2). The trough motion can be formulated in general as:-

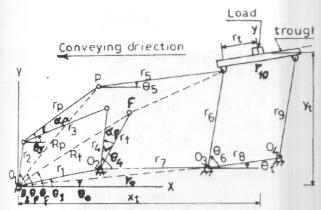


Figure 1. Type I conveyor.

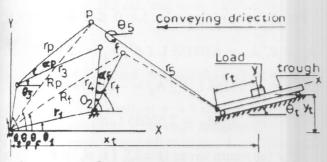


Figure 2. Type II conveyor.

a) For coupler-point and follower-point connections:

$$\mathbf{x_t} = \mathbf{R_i} \, \cos \, \theta_i \, + \, \mathbf{r_k} \, \cos \, \theta_k \, + \, \mathbf{r_t} \, \cos \, \theta_t \qquad (7)$$
 and

$$y_t = R_i \sin \theta_i + r_k \sin \theta_k + r_t \sin \theta_t$$
 (8)

Where :  $R_i$  and  $\theta_i$  are expressed by:

$$R_{i} = (1/\cos \theta_{i}) (r_{n} \cos \theta_{n} + r_{i} \cos \phi_{i}), \qquad (9)$$

$$\theta_{i} = \tan^{-1}\left(\frac{r_{n}\sin\theta_{n} + r_{i}\sin\theta_{i}}{r_{n}\cos\theta_{n} + r_{i}\cos\theta_{i}}\right)$$
 (10)

$$\phi_i = \theta_m + \sigma \alpha_i, k = 5,$$

$$\theta_{k,k+1} = \tan^{-1}\left[\frac{B(C\sin\theta_i - \sin\theta_o) - \sqrt{1 - B^2} \left(C\cos\theta_i - \cos\theta_o\right)}{\sqrt{1 - B^2} \left(C\sin\theta_i - \sin\theta_o\right) + B(C\cos\theta_i - \cos\theta_o)}\right],$$

$$B = (C_{k+1}^{2} - C_{k}^{2} + \lambda A^{2}) / (2 C_{k} A),$$

$$\lambda = \begin{cases} 1 & \text{for } \theta_{k+1} \\ -1 & \text{for } \theta_k \end{cases}$$

$$A = \sqrt{1 + C(C - 2\cos(\theta_i - \theta_o))}$$

$$C = R_i/r_o$$
,  $C_k = r_k/r_o$ ,  $C_{k+1} = r_{k+1}/r_o$ 

and

i=p, m=3, n= 2,  $\sigma$ =1 for coupler-point connection,

 $i=f, m=4, n=1, \sigma=-1$  for follower-point connection.

# b) For other proposed motions:

In this case the trough is driven by either parabolic, simple harmonic, cycloidal, 3-4-5-polynomial or eight-polynomial motion [15]. Equation of each motion is formulated as follows;

$$x_{t} = A_{o} + \sum_{j=1}^{N} A_{j} f_{j}(\theta)$$
 (11)

Where,  $A_0$ ,  $A_j$  and  $f_j$  ( $\theta$ ) are coefficients and functions of the trough motion respectively.

The coefficients and functions are derived by using the equation of motion and boundary values of the trough motion as follows;

#### i) Parabolic motion

Period of the conveying stroke ( $\beta = \beta_c$ ) is divided by two parts as follows;

- For 
$$0 \le \theta \le \phi_c$$
,  $(\phi_c = C_c \beta_c)$ ,  $A_0 = A_1 = 0$  and  $A_2 = C_c L/\phi_c^2$ 

$$-\text{For}\phi_{c} \le \theta \le \beta_{c}, (\beta = \beta_{c}), A_{o} = L(1-1/C_{c}), A_{1} = 2L/(C_{c}\beta),$$

and 
$$A_2 = -L/\beta^2$$

Also period of the return stroke ( $\beta = \beta_t$ ) is divided by two parts as follows;

- For 
$$\beta_c \le \theta \le (\beta_c + \phi_t)$$
, i.e.  $0 \le \theta \le \phi_t(\theta_t = C_t \beta_t)$ ,

$$A_0 = L, A_1 = 0 \text{ and } A_2 = C_t / \phi_t^2$$

-For 
$$\phi_t \le \theta \le \beta(\beta = \beta_t)$$
,  $A_o = L/C_t$ ,  $A_1 = -2L/(C_t\beta) = -2L/\phi_t$ 

and  $A_2 = L/(\phi, \beta)$ 

The functions, for the whole cycle, are given by

$$f_1(\theta) = \theta$$
, and  $f_2(\theta) = \theta^2$ 

# ii) Simple harmonic motion

- During the conveying stroke, i.e.  $0 \le \theta \le \beta(\beta = \beta_c)$ ) the coefficients are given as;

$$A_o = L / 2$$
, and  $A_1 = -A_o$ 

- For the return stroke, i.e.  $\beta_c \le \theta \le 2 \pi$  or  $0 \le \theta \le \beta_t$ , are

$$A_o = L / 2$$
, and  $A_1 = A_o$ 

While for whole cycle, the functions are only one and given by:

$$f_1(\theta) = \cos(\pi\theta/\beta)$$

$$\beta = \frac{\beta_c \text{ for the conveying stroke}}{\beta_t \text{ for the return stroke}}$$

# iii) Cycloidal motion

- For the conveying stroke ( $\beta = \beta_c$ ), i.e.  $0 \le \theta \le \beta_c$ ,

$$A_0 = 0$$
,  $A_1 = L/\beta$ , and  $A_2 = -L/(2\pi)$ 

- For the return stroke  $(\beta = \beta_t)$ , i.e.  $\beta_c \le \theta \le 2 \pi$ ,

$$A_0 = L, A_1 = -L /\beta \text{ and } A_2 = L/(2\pi)$$

While the functions for the whole cycle are:

$$f_1(\theta) = \theta$$
, and  $f_2(\theta) = \sin(2 \pi \theta / \beta)$ 

iv) 3-4-5-Polynomial motion [15]

- For the conveying stroke, i.e.  $0 \le \theta \le \beta$ ,  $(\beta = \beta_c)$ ,

$$A_o = 0$$
,  $A_1 = 10 L/\beta^3$ ,  $A_2 = -15 L/\beta^4$ , and  $A_3 = 6 L/\beta^5$ 

- For the return stroke  $(\beta = \beta_t)$ , i.e.  $\beta_c \le \theta \le 2 \pi \alpha$   $0 \le \theta \le \beta_t$ ,

$$A_{o}=L,\,A_{1}=$$
 - 10 L  $/\beta^{3},\,A_{2}=$  15 L/ $\beta^{4},\,$  and 
$$A_{3}=$$
 - 6 L/  $\beta^{5}$ 

The functions, for the whole cycle are

$$f_1(\theta) = \theta_3, f_2(\theta) = \theta^4 \text{ and } f_3(\theta) = \theta^5$$

- v) Eight- Polynomial motion [15]
- For the conveying stroke, i.e.  $0 \le \theta \le \beta$ ,  $(\beta = \beta_c)$ ,

$$A_o = 0$$
,  $A_1 = 6.09755 L / \beta^3$ ,  $A_2 = -20.7804 L/\beta^5$   
 $A_3 = 26.73155 L/\beta^6$ ,  $A_4 = -13.60965 L/\beta^7$  and

- For the return stroke (\$\beta = \beta\_{t}\$), i.e. \$\beta\_{c} \leq \theta \leq 2 \pi of

 $A_5 = 2.56095 L/\beta^8$ 

$$0 \le \theta \le \beta_t$$
,  $A_o = L$ ,  $A_2 = -2.63415 L/\beta^3$ ,

$$A_3 = 3.1706 \text{ L/}\beta^6$$
,  $A_4 = -6.87795 \text{ L/}\beta^7$  and

$$A_5 = 2.56095 L/\beta^8$$

Whereas the functions for the whole cycle are

$$f_1(\theta) = \theta^3$$
,  $f_2(\theta) = \theta^5$ ,  $f_3(\theta) = \theta^6$ ,  
 $f_4(\theta) = \theta^7$  and  $f_5(\theta) = \theta^8$ 

It should be noted that, for the previous proposed motions the following function is considered

$$\beta_{\rm c} + \beta_{\rm t} = 2 \pi$$

## 3. IMPLEMENTATION

The following dimensions and initial values are taken into account;

- For the crank-rocker driving mechanism

 $r_1 = 100 \text{ cm}, r_2 = 50 \text{ cm}, r_3 = 160 \text{ cm} \text{ and } r_4 = 120 \text{ cm}$ 

-For the drag-link driving mechanism

 $r_1 = 50 \text{cm}, r_2 = 100 \text{ cm}, r_3 = 160 \text{cm} \text{ and } r_4 = 120 \text{ cm}$ 

- For the trough mechanism

$$r_6 = r_9 = 250$$
 cm,  $r_8 = 50$  cm,  $r_t = 0.5$   $r_8$  and  $\theta_t = \theta_1 = 0$ 

- For the coupler-link

$$r_p = 130 \text{ to } 190 \text{ cm}, \text{ and } \alpha_p = -20^{\circ} \text{ to } 20^{\circ}$$

- For the follower-link

$$r_f = 100$$
 to 140 cm, and  $\alpha_f = -20^{\circ}$  to  $20^{\circ}$ 

Moreover;

 $r_5 = 240 \text{cm}, r_7 = 170 \text{ cm}, \theta_0 = 0$ , and the angular velocity

of the crank  $\omega_2 = \text{constant} = 4.5 \text{ r/s (c.c.w)}$ 

- The initial values for each stage are zero.
- For the trough proposed motions, L = 100 cm, factors  $C_c$  and  $C_t$  are selected as;

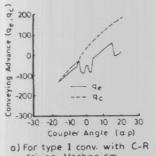
$$0 < C_c < 1$$
 and  $C_t = 1 - C_c$ 

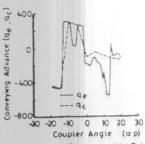
#### 4. RESULTS AND DISCUSSION:

The conveying advance is supposed to increase in the negative direction of x-axis (desired direction), Figures (1) and (2). Results of the presented analysis are discussed as follows:

- 1- If the two investigated conveyors are driven by motions of coupler- or follower-point on either crank-rocker or drag-link mechanism;
- i) For type I conveyor, the conveying advance  $q_c$  and  $q_e$  are not influenced by variation in  $\alpha_f$ . While for type II conveyor,  $q_c$  and  $q_e$  are sensitive due to variation in  $\alpha_f$ , Figures (6) and (7).
- ii) When using the crank-rocker as driving mechanism, Figures (3-a), (4), and (6) show that both  $q_c$  and  $q_e$  decrease with increasing either in  $\alpha_p$ ,  $r_p$  or  $r_f$ .
- iii) Using the drag-link as driving mechanism for type I conveyor, Figures (3-b) and (5-a, b) indicate

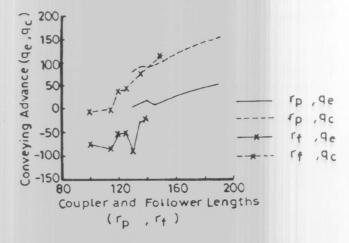
that,  $q_c$  and  $q_e$  are sensitive by different rates due to variation in  $\alpha_p$  and  $r_p$ . Whereas increasing  $r_f$  leads to decreasing in  $q_c$  and  $q_e$  by different rates.



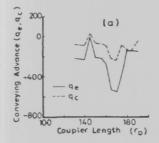


For type I conv. with C-R b) For type I conv. with D-L driving Mechanism.

Figure 3.



**Figure 4**. For type I conv. with C-R driving Mechanism.



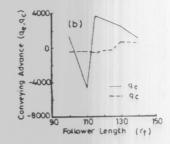


Figure 5. For type I conv. with D-L driving Mechanism.

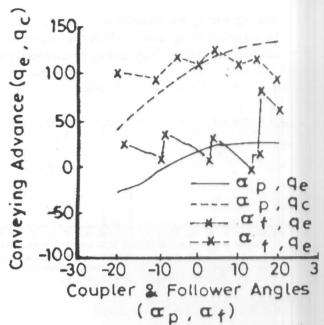


Figure 6. For type II conv. with C-R driving Mechanism.

iv) For type II conveyor which is driven by drag-link mechanism, Figures (7) and (9) show that to increase both  $q_e$  and  $q_e$ ,  $r_p$  and  $r_f$  should be decreased and  $\alpha_p$  should be increased as much as possible.

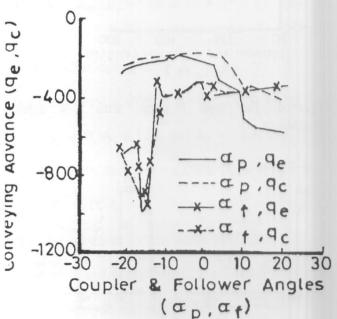


Figure 7. For type II conv. with D-L driving Mechanism.

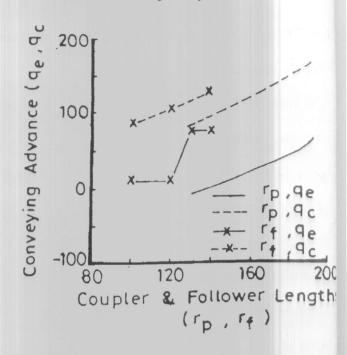


Figure 8. For type II conv. with C-R driving Mechanism.

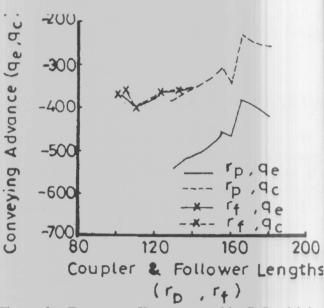
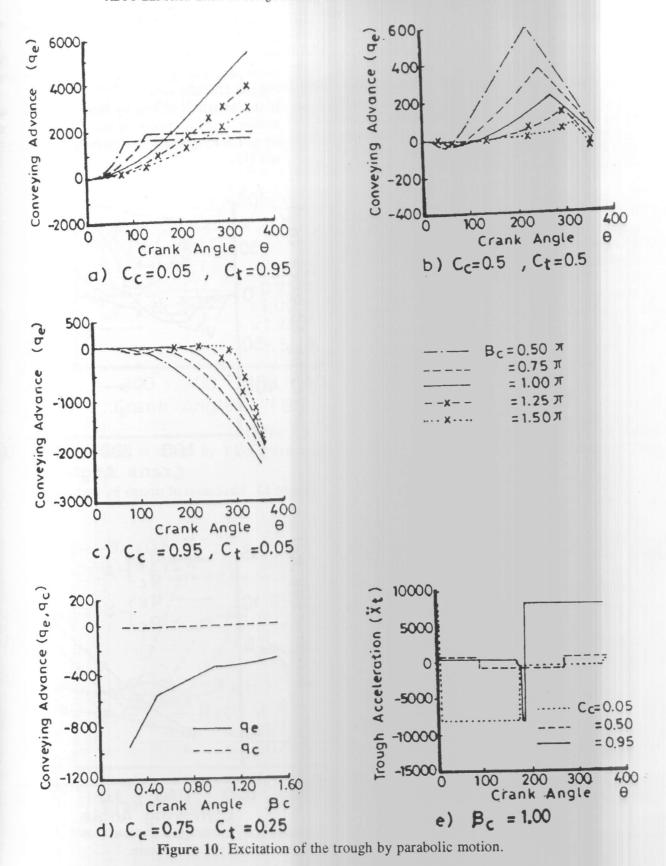


Figure 9. For type II conv. with D-L driving Mechanism.

The values of  $q_c$  and  $q_e$  (obtained from the figures), which are transmitted by using drag-link driving mechanism, are greater that those transmitted by the case of the crank-rocker driving mechanism.



# 2- Various proposed motions

- i) If the trough is driven by the parabolic motion,  $q_c$  and  $q_e$  depend on the position of the inflection point. From Figure (10) one can observe that, with increasing and decreasing the acceleration and deceleration intervals respectively (by changing the factor  $C_c$  with different values of the conveying angle  $\beta_c$ ),  $q_c$  and  $q_c$  occur in desired direction of the conveying load. It was found that, to increase the conveying advance,  $C_c$  should be increased and  $\beta_c$  decreased.
- ii) If the trough is driven by the simple harmonic motion, one can see that, as the conveying angle  $\beta_c$  decrease,  $q_c$  is increased and  $q_e$  is decreased, Figures (11) and (12).

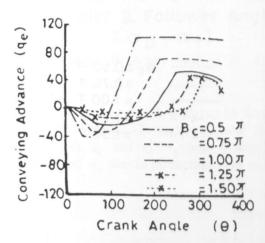


Figure 11. Excitation of trough by simple harmonic motion.

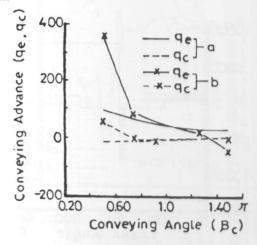


Figure 12 a- for simple harmonic motion. b- For eight-polynomial motion.

- iii) If the trough is driven by the cycloidal motion changing  $\beta_c$  in the range of  $0.5~\pi \le \beta_c \le 1.5~\pi$  leads to  $q_c$  is increased in opposite direction of the conveying and  $q_c$  remains approximately zero, Figures (13) and (14).
- iv) If the trough is driven by the 3-4-5-polynomia motion, as  $\beta_c$  decreases in the range of 0.5  $\pi \le \beta_c \le 1.5 \pi$ ,  $q_c$  and  $q_e$  are increased, Figures (14) and (15).

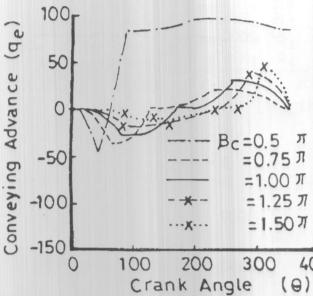


Figure 13. Excitation of trough by cycloidal motion.

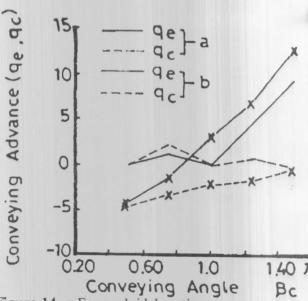


Figure 14. a-For cycloidal motion. b- For 3-4-5 polynomial motion.

v) If the trough is driven by the eight-polynomial motion, changing  $\beta_c$  in the range of  $0.5\pi \le \beta_c \le 1.5\pi$  indicates that increasing  $q_e$ ,  $\beta_c$  should be increased. Also to increase  $q_c$ ,  $\beta_c$  should be decreased until 0.75  $\pi$ , Figures (12) and (16).

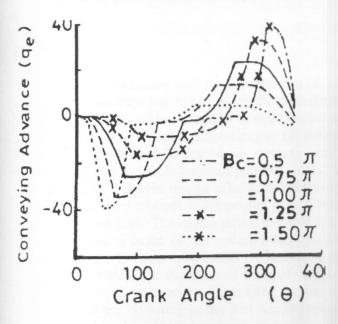


Figure 15. Excitation of trough by 3-4-5 polynomial motion.

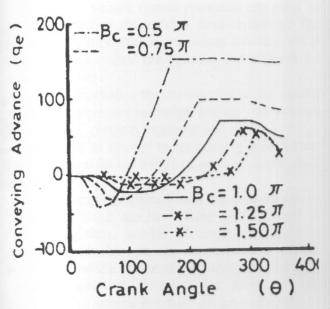


Figure 16. Excitation of trough by eight-polynomial motion.

## 5. CONCLUSION

This work presents a general analysis of the performance of an oscillating conveyor. Various motions are selected to drive the trough of such conveyor. The following conclusion can be drawn;

- 1- For the investigated oscillating conveyors, it was found that, the drag-link driving mechanism is preferable than the crank-rocker driving mechanism. Also, the connection between the driving mechanism and the trough should be at the coupler-point.
- 2- The results of the proposed motions to the trough indicated that, the trough parabolic motion with non-symmetrical acceleration pattern is better than other exciting motions, since it increases in the conveying advance in the desired direction. Hence, the driving mechanism which generates parabolic motion to the trough should be selected.

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