# OMPUTER ANALYSIS OF STRUCTURES LOADED NORMAL TO THEIR PLANES BY THE METHOD OF VIRTUAL WORK

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#### BSTRACT

In a previous paper [1], the anther gave a computer program, in "BASIC" for the analysis of plane frames with, or without arches, under one or several cases of loading acting in their plane using the well known method of virtual work. In this paper the computer analysis of these structures under forces acting normal to their plane is given. The loads may be concentrated at the joints, or distributed in any manner over the member axes. The structure may be composed of several bents. The program given herein may be applied to the solution of "Pannelled Beams Floors". The bending and torsional moments in all members can be readilly obtained. A main program named (VWN) which solves all problems is given in (Appendix 1). Appendix (2), gives the (DATAVWN) program necessary to obtain the (DATA) program for each problem. As mentioned before in the paper of VW1, the necessary (DATA) program for any problem is to be (MERGED) in the main program (VWN), and when (RUN), we get the final result. Notice that the main program is to be saved in the form ["VWN. BAS", A] as mentioned before 1. The examples illustrated here are in two parts. Part (1) shows three examples for loads acting on one and continuous two bents, the normal loads are concentrated or uniformly distributed on the member axes. Part (2) shows two examples for pannelled-beam floors, neglecting torsion, and considering it.

#### **WOTATIONS**

	No of redundant values.	A (I)	Average moment in the additional moment
	No of moments at the ends of members		Mo(I) diagram
112	No of torsional moments in the members = no of members	D (I)	Displacement coefficients from Mo, MTO & M (I) diagrams
lo(I)	Bending moments in the Main-System (M.S.) due to given loads	S (I,J)	Displacement coefficients from the appropriate M (I.J) diagrams.
ITo(I)	Torsional moments in the (M.S.) due to		
	given loads	INTRODUCTION	
$I_1(I,J)$	Bending moments in the (M.S.) due to		
	redundant values $X(I) = 1$	For loads acting normal to the plane of the structures,	
(LD).TI	Torsional moments in the (MS) due to	the internal stress action on any section will consist of	

For loads acting normal to the plane of the structures, the internal stress action on any section will consist of a moment, the vector of which lies in the plane of the structure, together with a shearing force Q acting normal to this plane. If the acting moment vector in the plane of the structure is resolved into a vector along the axis of the member, and one normal to this axis, we get the torsion or twisting moment and the bending moment acting there on.

Elastic torsion length of member (I) = L1(I)

Gives the position of the C.G. of the

redundant values X(I) = 1

Value [EI/JG] for member (I)

Length of member (I)

\* R(I)

R(I)

The bending moment vector, normal to the axis will follow the right-hand-screw rule ((R.H.S.) Also, the direction of sight for the member moment vectors are shown. This enables to know whether the acting moments produce tension on the top side of the plane or not. Bending moments producing tension on the top face of the plane of the structures will be considered negative.

The moment vector, parallel to the axis of the member will give the torsional moment therein. If this vector seems if considered as on axial force to give a tensile force, then this torsional moment will be considered positive.

The shearing force Q at any section will consist of two equal and opposite forces, normal to the plane of structure. When the force acting on left is downwards, and that acting on right is upwards, the shear Q is said to be positive.

# Main System (M.S.)

For the frames subjected to loads normal to their plane, the end supports are supposed to be totally fixed. This is the case in examples 1,2, and 3, part (1). In all cases, however, the movement taking place in the frame will consist of a translation normal to the plane and a rotation vector in the plane. In other wards there will be no translation in the plane itself of the frame. It will be possible then, to obtain a statically determinale main-system (M.S.) by producing a cut section in each bent in the frame, usually at the mid point in any member. This will not be the case generally in part (2) dealing with pannelled beam roofs. Hence, there will be three unknown redundant values acting on each side of the cut section. These values consist of two equal and opposite unit moment vectors parallel and normal to the axis of the member, together with unit equal and opposite shears. For convenience, the unit moment vectors will be along the horizontal axis x-x, and the vertical axis y-y. Hence they will be MX=1 and MY=1.

The Shear vector Q will consist of two equal and opposite forces acting on both sides of the cut section.

# Straining Actions

The straining actions on any member DC Figure (can be obtained from the moment vectors  $(M_x)$  at  $(M_y)$  acting at C by resolving these moment vector along the member axis and normal to it. This gives the torsional moment and the bending moment at section C respectively in terms of the angle of inclination with the horizontal direction.

These moments will be: Figure (b).

$$M^{c} = -M_{x} \cdot \sin \theta - M_{y} \cos \theta$$

$$T^{c} = -M_{x}. \cos \theta + M_{y} \sin \theta$$

For the member shown in Figure (b), T represents a positive twisting moment, and M represents a negative bending moment producing tension on the top face of the frame.

The torsional or twisting moment T in the member will have a constant value for the full length CD. If, however, the shearing force at C equals Q acting downwards, then the bending moment at D will be:

$$M^d = M^c - Q^c. L^{cd}$$

Where  $L^{cd}$  = Length of member CD.

If still there are further loads acting normal to Cd, whether concentrated or distributed in any manner, we have to add the bending moment of these loads about D we their proper sign.

For member CD to the right of the cut-section, the positive values of the moment vectors will be as shown in Figure (c). The resulting bending and torsional moments at C will be:

$$M^{c} = -M_{x}. \sin \theta + M_{y} \cos \theta$$
 (4)

$$T^{c} = -M_{x}. \cos \theta + M_{y} \sin \theta$$
 (5)

Here M will be positive and T will be negative.

If a negative force Q acts upwards at C, the twisting moment will remain constant, where as the bending moment M will increase by Q.L, such that.

$$M^{d} = M^{c} + Q^{c}. L^{cd}$$

To this we still have to add the moment about D

moduced by any loads acting on CD. This will be lustrated by the given examples.

## Steps of Solution

It is assumed that the main programs (VWN), and DATAVWN) are stored in the computer to be used. The first of these programs when saved, shall be saved as: SAVE "VWN. BAS", A

This enables to be used later with the order:

MERGE "VWN" when required.

Now, the steps of solutio will be as follows:

Load the "DATAVWN" program. Enter the necessary data from the corresponding diagrams containing:

N, R, L1 (I), LT (I), Mo (I), A (I),

C (I), M1 (I,J), MTO (I), MTI (I, J)

- 2 Save the program thus obtained as: SAVE "DATAVWNn", where n referse to the number of the example.
- 3- Merge this program with the main program (VWN), from this order:

## Merge "VWN"

Notice that unless the main program (VWN) has been saved in the proper form

"VWN. BAS", A the order (MERGE) will not be fulfilled. In this case call the program, and re-save it properly.

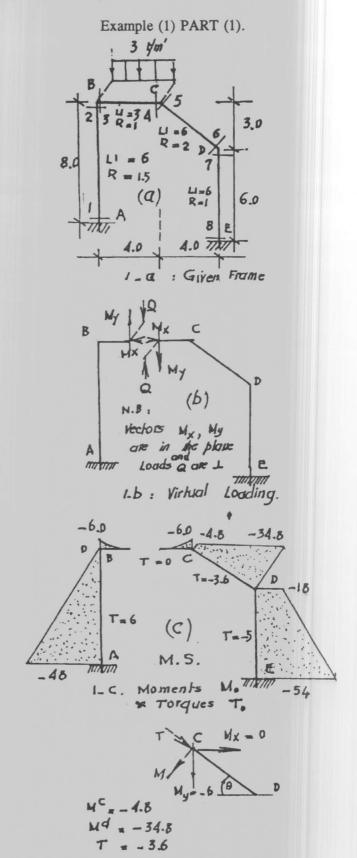
This gives the final program, which when "RUN" shows the bending and twisting moments in the frame the to the given case of loading.

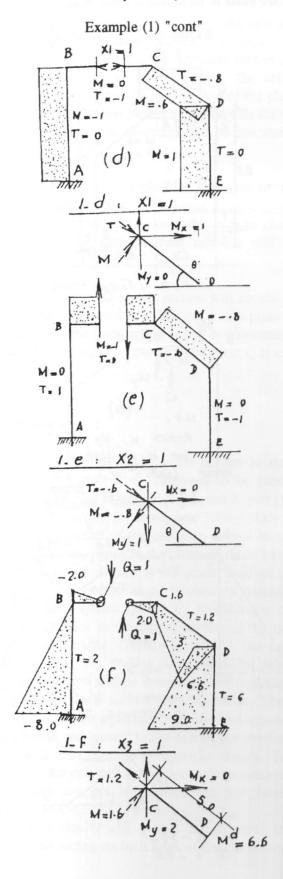
### Examples:

Part (1)

#### Example (1)

This example shows a single-bent frame A B C D E loaded with a uniform load  $\rho=3t/m$  normal to the plane of the frame, and acting on member BC only, the rest of the frame being unloaded.





Since there is only one bent in the frame, one of section is only needed therein, say at the midd section in member BC to get a statically determine main system for the frame. Figure (1-a) shows the given loaded frame, its dimensions, the L1 and r. fit the members and the positions (1) to (8) for the sections at which M will be obtained.

Figure (1-b) shows the unit moment vectors  $M_x = 1$ ,  $M_y = \pm 1$ ,  $Q = \pm 1$  which will be imposed on the frame at the cut-section. Figure (1-c) gives the bending moments  $M_o$ , and torsion To, on the members due the given loading on the Main-System (M.S.). It seen that the moments (Mo) produce tension on the toface of the frame, and hence will be negative. It shown these moments are drawn outside the frame.

In all members except the inclined member (CD),  $\mathfrak{g}$  moments  $(M_x)$  and  $(M_y)$  will given directly the bending moment M and the twisting moment T in the member

In member CD, the bending moment M<sup>c</sup> and the twisting moment T<sup>c</sup> are obtained from equations (and (5) as follows:

$$\cos \theta = 0.8$$
,  $\sin \theta = 0.6$   
 $M^{c} = 0 - 6.0 (8.0) = -4.8 \text{ tm}$   
 $T = 0 - 6.0 (0.6) = -3.6 \text{ tm}$ 

At section D; the shearing force Q at c will be:

$$Q = -6 tons$$

Then, from equation (6) we get:

$$M^d = -4.8 - 6.0 (5) = -34.8 \text{ t.m}$$

As mentioned before, the torsional moment T remain constant in CD.

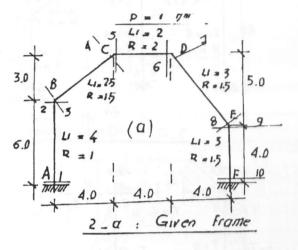
Figures (1-d) to (1-f) give the bending moments and the torsional moments T, for the redundant 100

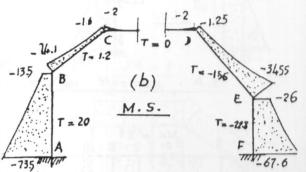
Now, all the required data, for prog (DATAVWNI) can be obtained from these figures thus we obtain the data program.

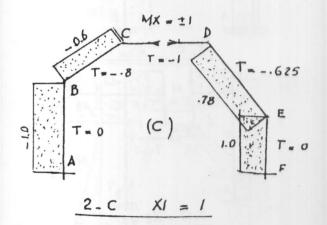
The number (1) referse to the member of example Enter the order (MERGE "VWN") to get the f program. Order (RUN) will give the final results M and T.

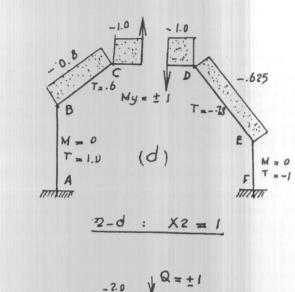
# Example (2)

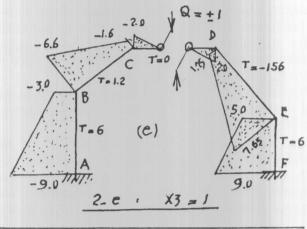
Uniform normal pressure.

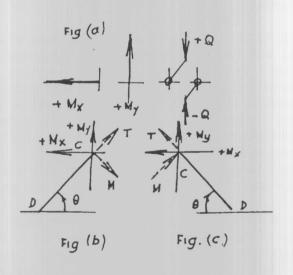












## Example (2)

The frame here is subjected to a uniform pressure acting on all members normal to the plane of the frame, which still consists of one bent. The (M.S.) will then be obtained by introducing one cut section, say at the mid-section of member CD. Figure (2-a) shows the given frame, the values L1 and r for all members, and the positions of the sections at which M is to be calculated.

The bending moments Mo, and twisting moments for all members are shown in Figure (2-b). The values of the shearing force Q will be 2, 7, 2, 8.4 at sections C,B,D, and E respectively.

The uniform load p = 1 t/m, acting on the frame will add a moment  $M = -pL^2/2$  on the far ends of members. This enables to get the values of (M) shown in the figure. This uniform load acting normal to the plane will give a second-degree parabolic bending moment on each member, with a maximum value of (pL2/8), and an average value of (pL2/12)

Figures (2-c) to (2-e) give the bending moments M, and torsional moments T to virtual loadings  $X1 = \pm 1$   $X2 = \pm 1$ ,  $X3 = \pm 1$ .

This enables to get all data required for the "DATAVWN 2" program which when "MERGED" with the original "VWN" program gives the final program from which all bending moments and torsional moments in the frame are obtained, as shown

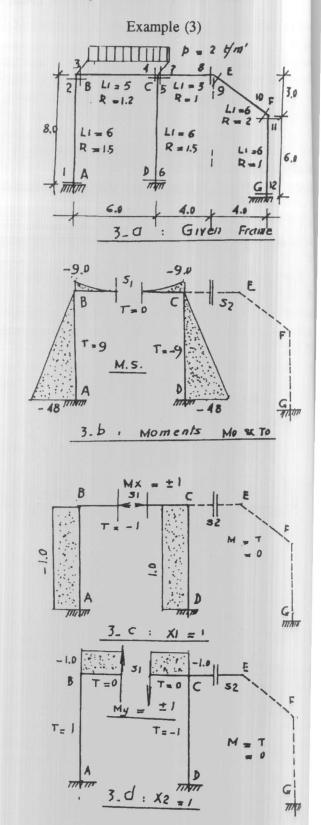
### Example (3)

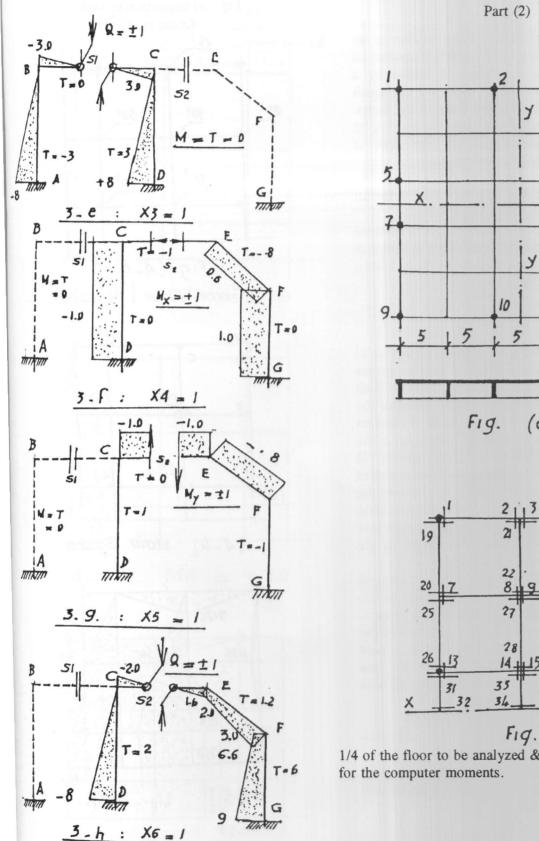
This frame is shown in Figure (3). It consists of two bents, with three fixed supports. It is then necessary to introduce two cut-sections, say at mid-sections in members BC and CE, to get the proper statically determinate main-System (M.S.). There will be six virtual unit equal and opposite loads applied at these cut-sections, as in Figure (3-b), together with the Mo diagram due to main loads.

Figure (3-a) shows the given frame loaded with a normal uniform load over member BC. It also gives the values L1 and r for all members and the twelve sections at which the moments M are to computed.

Figure (3-b) shows the bending moments Mo, and the torsional moments To in the (M.S.). Figures (3-c) to (3-h) show the bending moments M1 and twisting moments T1 due to the virtual loads X1 to X6.

This enables now to get the necessary (DATAVWN3) diagram.





(e)

1/4 of the floor to be analyzed & the numbers of ends

#### Pannelled-Beam Floors

Figure (d) shows a plane and cross section of an example of such floors with 5\*5 pannels supported on 12 columns as shown. The floor is loaded by a uniform load, normal to its plane with p=2 t/m which is assumed to act as concentrated loads at the joints. The floor is symmetrical about axes x-x and y-y as shown. Hence one quarter only of the floor is considered.

The beams have constant depth h, with a value r=EI/GJ=6.25. The elastic Length L1=1.0 for all beams, and the elastic torsional length LT=L1 \*r=6.25. The solution of these beams can be obtained by assuming no torsion resistance in them, as shown in example (4), or considering torsion in these beams, example (5) (see Figure (e))

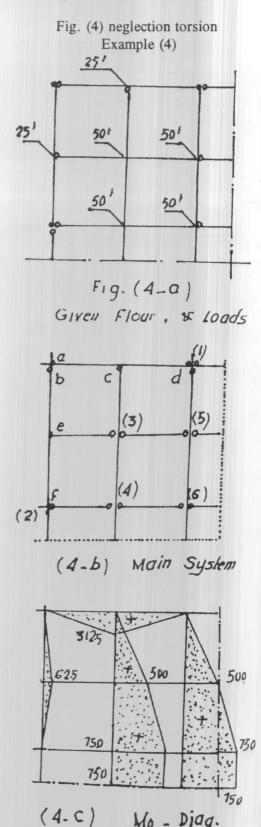
# Example (4)

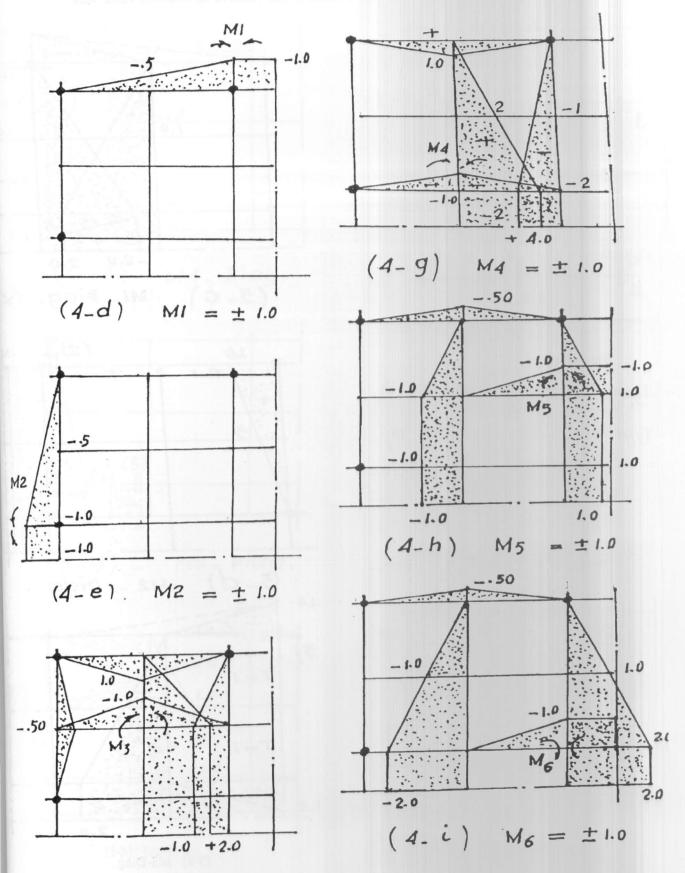
In example (4) the beams are considered unable to resist torsion; T being zero. Hence al external beams will be assumed as hinged at their ends. Thus we shall have (6) six beams with hinged ends. Therefore the floor will need (6) six more unknown redundant moments to be obtained. The hinges introduced at the ends of the external members are shown in (a,b,c,d,e,f); in Figure (4-b) together with the positions of the six unknown moments. Figure (4-c) shows the bending moment Mo for the statically determinate main system. Positive moments are drawn at the bottom side of the horizontal members, and to the right of the vertical members as shown in the Figure.

Figures (4-d) to (4-i) give the bending moment diagrams M (I) for the six redundant moments M (I). The torsional moments MT (I) and MTo will be zero. It is sufficient then to use the (NW) program and the (DATAVW)<sup>1</sup> program to get the solution. It is now possible to obtain the necessary data for the (DATAVWO9) program which is to be (merged) with the (VW)<sup>1</sup> program to get the final program.

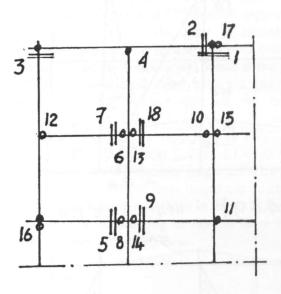
# Example (5)

In this example the beams are considered to resist torsion. The torsional elastic length of all beams will be equal to 6.25 as mentioned before.



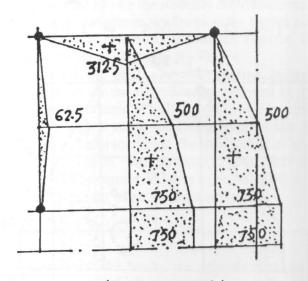


Example (5)
Taking torsion into account.

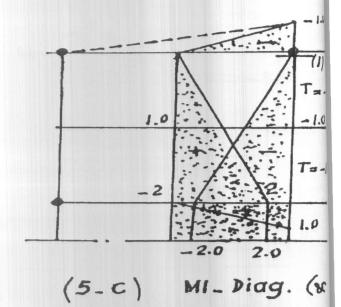


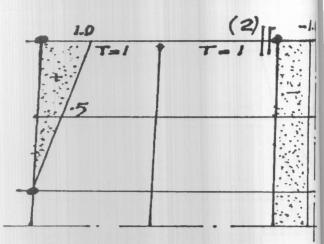
Symbols:

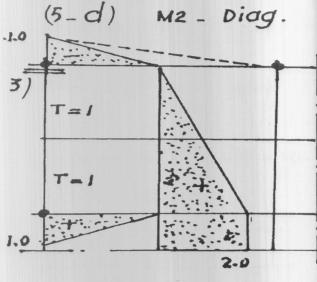
Type(a)  $\xrightarrow{\times}$  Free Rot.  $\phi_{\times}$ Type(b)  $\xrightarrow{\circ}$  Free Rot.  $\phi_{\gamma}$ (5. a). Main System.

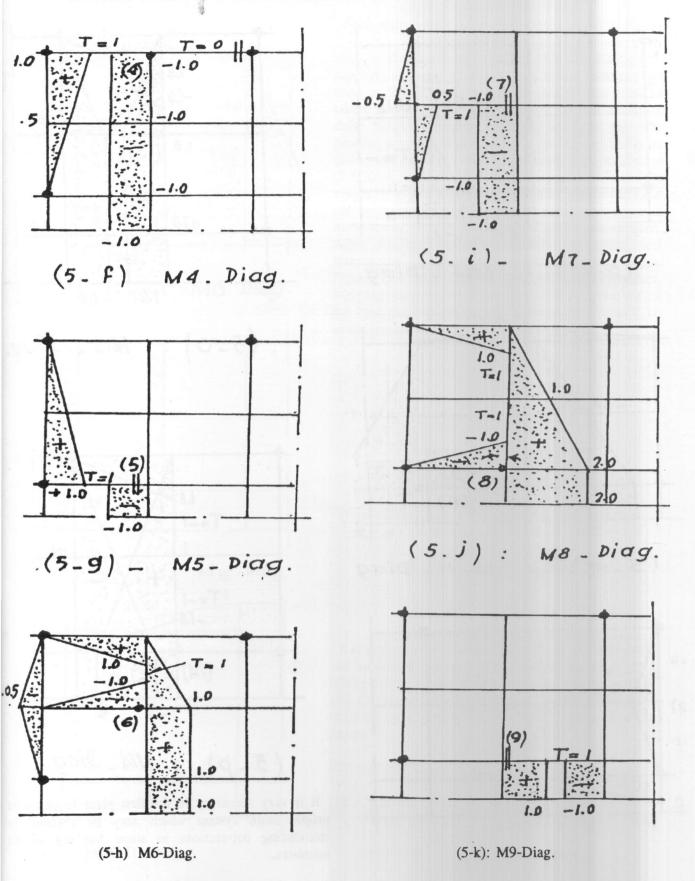


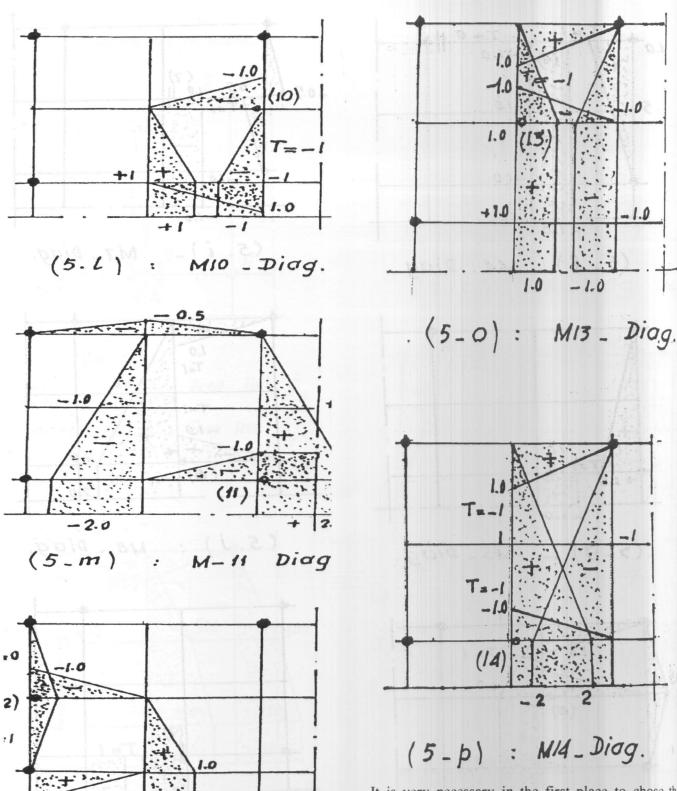
(5-b) No. Diag.
T = 0 in all members.





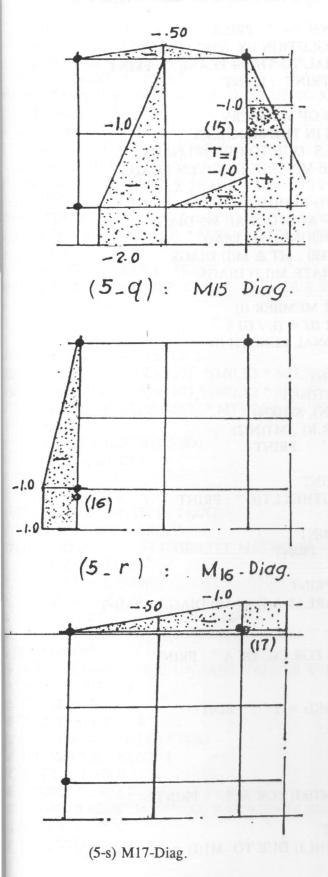




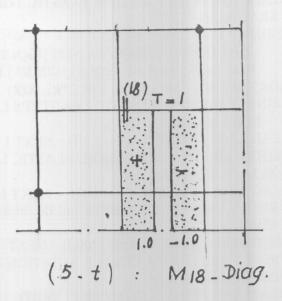


It is very necessary in the first place to chose the proper main system which may be obtained by introducing cut-sections in some but not all the members.

(5-n) M-12 Diag.



Such main system, as one solution is shown in Figure (5-a). No cut-Sections are shown rotation normal to the axes of the members together with suitable type of supports allowing axial rotations in the members are used as shown in Figure (5-a). Much care must be taken in chosing this main system, and in arranging these hinges, whether type (a), or type (b). The bending moment Mo-Diagram is next shown in Figure (5-b). Still here (To) will remain zero. Figures (5-c) to (5-t) show the bending moment diagrams M(I) and the values of T (I) for the (18) redundant values M(I) and the positions of the redundant equal and opposite moments  $M(I) = \pm 1$  applied. at the proper sections. After loading the (DATAVWN) program, and introducing in there the necessary values from. Figures (5-b) to (t-t) we get (DATAVWNST) program. Merging this program with the (VWN) program we get the final diagram producing moments M (I) and T (I) in the pannelled floor.



It is interesting to compare the results of the moments and torsions developing in the floor to see the effect of taking the torsional rigidity of the beams into account, and the variation in these values which can be easily obtained by chaining the torsional rigidity (r) of the beams.

## REFERENCES

[1] Computer Analysis applied to the Method of Virtual Work. by A.F.S. Diwan. Ph.D. Alexandria Engineering Journal Vol. 31. No. 2, April 1992.

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10 PRINT TAB(4) " NAME OF PROGRAM ---- VWN ---- " : PRINT
20 PRINT TAB(4) " THIS PROGRAM GIVES THE SOLUTION BY VW" : PRINT
25 PRINT TAB(2) " FOR FRAMES LOADED NORMAL TO THEIR PLANE " : PRINT
30 PRINT "----" : PRINT : PRINT
40 REM R = NOM. OF REDUNDANT VALUES
50 REM N = NOM. OF MOMENTS AT THE ENDS OF MEMBERS
55 REM N/2 = NOM. OF TORSIONAL MOMENTS IN THE MEMBERS
60 REM MO(I) = BENDING MOMENTS IN THE M.S DUE TO GIVEN LOADS
65 REM MT0(I) = TORSIONAL MOMENTS IN THE M.S DUE TO GIVEN LOADS
70 REM M1(I,J) = BENDING MOMENTS DUE TO REDUNDAT LOADS <math>X(I) = 1
75 REM MT1(I) = TORSIONAL MOMENTS DUE TO REDUNDANT LOADS X(I) = 1
90 REM G GIVES THE POSITION OF THE C.G. OF ADDITIONAL MO DIAG.
100 REM A = AVERAGE MOMENT IN THE ADDITIONAL MO DIAG.
110 REM D(I) = DISPLACEMENT COEFS. FROM MO, MT & M(I) DIAGS.
120 REM S(I,J) = \dots FROM APPROPRIATE M(I,J) DIAGS.
140 REM L(I) = LENGTH OF MEMBER (I)
150 REM L1(I) = ELASTIC LENGTH [L / EI] FOR MEMBER (I)
160 REM R(I) = VALUE [ EI / GJ ] FOR MEMBER (I) = (L / GJ)
170 REM LT(I) = [L1 * R] (I) = ELASTIC TORSIONAL LENGTH (I)
180 READR, N
190 PRINT " R = " , R : PRINT " N = " , N : PRINT
210
        DIM L1(N/2), M0(N), A(N/2), G(N/2), LT(N/2)
220
        DIM D(R), M(R,N), S(R,R), C(R,R), BM(N), MTO(N)
230 DIM TM0( N ), MT1(R,N), Z(R,2*R), X(R), V(R,R), MT(N/2)
240 PRINT TAB(6)" 1 - ELASTIC LENGTHES L1(I) " : PRINT
250
       FOR I = 1 TO (N/2)
260 READ L1(I): PRINT TAB(4) L1(I): NEXT I: PRINT
265 PRINT TAB(6) " 2 -TORSIONAL ELASTIC LENGTHES LT(I) " : PRINT
270 FOR I = 1 TO (N/2)
275 READ LT(I): PRINT TAB(4) LT(I): NEXT I: PRINT
280 PRINT TAB(6)" 3 - MOMENTS MO IN M.S. " : PRINT
290
     FOR I = 1 TO N
300 READ MO(I): PRINT TAB(4) MO(I): NEXT I: PRINT
320 PRINT TAB(6) " 4 - AVERAGE ADDITIONAL AREAS A(I) IN M0 DIAG." : PRINT
330 FOR I = 1 TO N / 2
340 READ A(I): PRINT A(I): NEXT I: PRINT
360 PRINT TAB(6)" 5 - POSITION OF CENTROID G FOR CG. OF A ": PRINT
370 FOR I = 1 \text{ TO N } / 2
380 READ G(I): PRINT G(I): NEXT I: PRINT
400 PRINT TAB(6)" 6 - MOMENTS M(I,J) DUE TO M(I) = 1 ": PRINT
410 FOR I = 1 TO R
420
     FOR J = 1 TO N
430 READ M(I,J): PRINT M(I,J), : NEXT J
    PRINT : NEXT I : PRINT
444 PRINT TAB(6), " 7 - TORSIONLAL MOMENTS MT0(I) FOR M.S.": PRINT
448
     FOR I = 1 TO N/2
452 READ MT0(I): PRINT MT0(I),: NEXT I: PRINT
454 PRINT TAB(6) " 8 - TORSIONAL MOMENIS MT1(I,J) DUE TO M1(I) = 1 " : PRINT
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FOR I = 1 TO R
    FOR J = 1 TO N/2
  READ MT1(I,J): PRINT MT1(I,J) .: NEXT J
  PRINT : NEXT I : PRINT
  PRINT TAB(6) " 9 - DISP. COIFS. D(I) DUE TO MO, MTO & MI, MT1 DIAGS. " : PRINT
  FOR I = 1 TO R
  D(I) = 0
   FOR K = 1 TO (N - 1) STEP 2
  D(I) = D(I) + L1((K+1)/2) * (M0(K)*M(I,K)/3 + M0(K+1) * M(I,(K+1))/3)
  D(I) = D(I) + L1((K+1)/2) * (M0(K)* M(I,(K+1))/6 + M0(K+1)* M(I,K)/6)
  D(I) = D(I) + L1((K+1)/2) * A((K+1)/2) * M(I,(K+1)) * G((K+1)/2)
 D(I) = D(I) + L1((K+1)/2) * A((K+1)/2) * (1-G((K+1)/2)) * M(I,K)
 D(I) = D(I) + LT((K+1)/2) * MTO((K+1)/2) * MT1(I,((K+1)/2))
   NEXT K
   PRINT D(I): NEXT I: PRINT
 PRINT TAB(6)" 10 - DISP. COEFS. S(I,J) DUE TO M(I,J) ": PRINT
  FOR I = 1 TO R
  FOR J = 1 TO R
    S(I,J) = 0
  FOR K = 1 TO (N-1) STEP 2
 S(I,J) = S(I,J) + L1((K+1)/2) *(M(I,K) * M(J,K)/3 + M(I,(K+1)) * M(J,(K+1))/3)
 |S(I,J)| = S(I,J) + L1((K+1)/2) * (M(I,K) * M(J,(K+1))/6 + M(I,(K+1)) * M(J,K)/6)
 S(I,J) = S(I,J) + LT((K+1)/2) * MT1(I,((K+1)/2)) * MT1(J,((K+1)/2))
   NEXT K
   IF J < > I THEN GOTO 800
  NEXT J : NEXT I
  FOR I = 1 TO R
  FOR J = 1 TO R
 PRINT S(I,J), : NEXT J : PRINT
  NEXT I: PRINT: PRINT
 PRINT TAB(6) " 11 - FLEXIBILITY MATRIX V = INVERS OF S " : PRINT
    FOR I = 1 TO R
     FOR J = 1 TO R
       Z(I,J) = S(I,J)
         NEXT J: NEXT I: PRINT
   IF R = 1 THEN V(1,1) = 1 / S(1,1): PRINT V(1,1): GOTO 1000
 GOSUB 10000

    PRINT TAB(6)" 12- REDUNDANT VALUES X (R) " : PRINT

   FOR I = 1 TO R
     X(I) = 0
   FOR K = 1 TO R
     X(I) = X(I) - V(I,K) * D(K)
      NEXT K: NEXT I
  FOR I = 1 TO R
    PRINT TAB(4) X(I) :NEXT I : PRINT
    PRINT TAB(6)" 14 - FINAL B.MOMENTS BM & TORQUES TM ": PRINT
10 FOR I = 1 TO N
      BM(I) = 0
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20

50

```
1230
      FOR J = 1 TO R
       BM(I) = BM(I) + X(J) * M(J,I)
1240
1250
          NEXT J
       BM(I) = BM(I) + M0(I)
1260
       NEXT I
1262
1264 FOR I = 1 TO (N/2)
1266 \text{ MT(I)} = 0
     FOR J = 1 TO R
1268
1270 MT(I) = MT(I) + X(J) * MT1(J,I)
1272
     NEXT J
1274 \text{ MT(I)} = \text{MT(I)} + \text{MT0(I)}
1276
         NEXT I
1278 INPUT "YOU WANT SCREEN (1) OR PRINTER (2), OR END (3)", SP
      ON SP GOTO 1284, 1360, 1500
1282
1284 PRINT : PRINT
1286 FOR I = 1 TO (N-1) STEP 2
1290 PRINT TAB(4) BM(I), BM(I+1)
1310
      NEXT I : PRINT : PRINT
1312 PRINT TAB(6) " TWISTING MOMENTS MT(I) " : PRINT
1314 \text{ FOR I} = 1 \text{ TO N/2}
1316 PRINT TAB(6) MT(I): NEXT I: PRINT
1320 PRINT TAB(6) " THIS IS ALL NOW " : PRINT
1340
        GOTO 1278
1360 FOR I = 1 TO (N-1) STEP 2
1370 LPRINT TAB(4) BM(I), BM(I+1)
1380
     NEXT I : LPRINT
1382 LPRINT TAB(6), "TWISTING MOMENTS MT(I)": LPRINT
1384
         FOR I = 1 TO N/2
1386
      LPRINT TAB(6) MT(I): NEXT I: LPRINT: LPRINT
1400 LPRINT TAB(4) " THAT IS ALL NOW " : LPRINT
1500
        END
10000
         FOR I = 1 TO R
10010
          FOR J = (R+1) TO 2 * R
10020
           Z(I,J) = 0
10030
            NEXT J
10035
         Z(I, R+I) = 1
10040
            NEXT I
10050
         P = 0
10060
           P = P + 1
10070
        C(P,P) = Z(P,P)
10080
         FOR J = 1 \text{ TO } (2 * R)
10090
        Z(P,J) = Z(P,J) / C(P,P)
10100
           NEXT J
10110
        FOR I = (P + 1) TO R
10120
          C(I,P) = Z(I,P)
        FOR J = P TO 2 * R
10130
10140
         Z(I,J) = Z(I,J) - C(I,P) * Z(P,J)
             NEXT J
10150
```

```
NEXT I
       IF P = 1 THEN 10060
    FOR I = 1 TO (P-1)
     C(I,P) = Z(I,P)
   FOR J = P TO (2 * R)
    Z(I,J) = Z(I,J) - C(I,P) * Z(P,J)
      NEXT J
       NEXT I
   IF P = R THEN GOTO 10260
    GOTO 10060
 FOR I = 1 TO R
   FOR J = 1 TO R
   V(I,J) = Z(I,R+J)
 PRINT V(I,J), : NEXT J
    PRINT : NEXT I : PRINT
      RETURN
APPENDIX (2)
PRINT " NAME -----
PRINT " ENTER DATA FOR R , N "
PRINT " ENTER DATA FOR ELASTIC LENGTHES L1(I) "
PRINT " ENTER DATA FOR ELASTIC TORSIONAL LENGTHES LT(I) "
PRINT " ENTER DATA B. MOMENTS MO(I) IN M.S. "
PRINT " ENTER DATA FOR ADDITIONAL AVERAGE ORDINATE IN A(I) "
PRINT " ENTER DATA FOR G: CENTROID OF ADDIT. AREA A(I) "
PRINT "ENTER DATA FOR M1(I,J) DUE TO M(I) = 1"
PRINT " ENTER DATA FOR MTO(I) IN M.S. "
PRINT " ENTER DATA FOR MT1(I,J) DUE TO M(I) "
APPENDIX (3) (DATAVWN1)
PRINT " ENTER DATA FOR R.N"
DATA 3,8
PRINT " ENTER DATA FOR ELASTIC LENGTHES L1(I) "
DATA 6, 3, 6, 6
PRINT " ENTER DATA FOR ELASTIC TORTIONAL LENGTHES LT(I) "
DATA 9, 3, 12, 6
PRINT " ENTER DATA B. MOMENTS MO(I) IN M.S. "
DATA -48, 0, -6, -6, -4.8, -34.8, -18, -54
PRINT " ENTER DATA FOR ADDITIONAL AVERAGE ORDINATE IN A(I) "
DATA 0,4,0,0
PRINT "ENTER DATA FOR G: CENTROID OF ADDIT. AREA A(I) "
DATA 0, .5, 0, 0
PRINT "ENTER DATA FOR M(I,J) DUE TO M(I) = 1"
DATA -1, -1, 0, 0, .6, .6, 1, 1
DATA 0, 0, 1, 1, .8, .8, 0, 0
DATA -8, 0, -2, 2, 1.6, 6.6, 3, 9
```

```
3440 PRINT " ENTER DATA FOR MT0(I) IN M.S. "
3450 DATA 6, 0, -3.6, -30
3480 PRINT " ENTER DATA FOR MT1(I,J) DUE TO M(I) "
3490 DATA 0, -1, -.8, 0
3500 DATA -1, 0, .6, 1
3510 DATA 2, 0, 1.2, 6
```

## MERGE (VWN) RUN

-66.85296 .7879229 -1.545921 8.274521 6.146863 -11.57759 -11.42259 -32.69193

### TWISTING MOMENTS MT (I)

1.545921 .7879229 5.59505 -5.905039

#### THAT IS ALL NOW

#### APPENDIX (4) (DATAVWN2)

```
3000 PRINT " ENTER DATA FOR R, N"
3010 DATA 3, 10
3020 PRINT " ENTER DATA FOR ELASTIC LENGTHES L1(I) "
3030 DATA 4, 2.5, 2, 3, 3
3040 PRINT " ENTER DATA FOR ELASTIC TORSIONAL LENGTHES LT(I) "
3050 DATA 4, 3.75, 4, 4.5, 4.5
3060 PRINT " ENTER DATA B.MOMENTS MO(I) IN M.S. "
3070 DATA -73.5, -13.5, -24.1, -1.6, -2
3080 DATA -2, -1.25, -34.58, -26, -67.5
3120 PRINT " ENTER DATA FOR ADDITIONAL AVERAGE ORDINATE IN A(I) "
3125 DATA 3, 2.083, 1.333, 3.415, 1.333
3160 PRINT " ENTER DATA FOR G: CENTROID OF ADDIT. AREA A(I) "
3170 DATA .5, .5, .5, .5, .5
3240 PRINT "ENTER DATA FOR M(I,J) DUE TO M(I) = 1 "
3250 DATA -1, -1, -.6, -.6, 0, 0, .78, .78, 1, 1
3260 DATA 0, 0, -.8, -.8, -1, -1, -.625, -.625, 0, 0
3270 DATA -9, -3, -6.6, -1.6, -2, 2, 1.25, 7.65, 5, 9
3440 PRINT " ENTER DATA FOR MTO(I) IN M.S. "
3450 DATA 20, 1.2, 0, -1.56, -22.8
3480 PRINT "ENTER DATA FOR MT1(I,J) DUE TO M(I)"
3490 DATA 0, -.8, -1, -.625, 0
3500 DATA 1, .6, 0, -.78, -1
3510 DATA 6, 1.2, 0, 1.56, 6
```

# PRINT: MERGE (VWN) R U N

```
9708 -13.11235

9687 9.713734

$216 12.13865

$244 -25.37938

$594 -66.90292
```

DATA 0, 0, 2, 0, 1.2, 6

#### TWISTING MOMENTS MT (I)

```
4327
85287
0115
11815
14869
IT IS ALL NOW
APPENDIX (5)
               (DATAVWN 3)
PRINT "ENTER DATA FOR R, N"
DATA 6, 12
PRINT " ENTER DATA FOR ELASTIC LENGTHES L1(I) "
DATA 6, 5, 6, 3, 6, 6
PRINT " ENTER DATA FOR ELASTIC TORSIONAL LENGTHES LT(I) "
DATA 9, 6, 9, 3, 12, 6
PRINT " ENTER DATA B.MOMENTS MO(I) IN M.S. "
0, 0, 0, 0, 0, 84-, 0, 9-, 9-, 0, 8- ATA
PRINT " ENTER DATA FOR ADDITIONAL AVERAGE ORDINATE IN A(I) "
DATA 0, 6, 0, 0, 0, 0
PRINT " ENTER DATA FOR G: CENTROID OF ADDIT. AREA A(I) "
DATA 0, .5, 0, 0, 0, 0

    PRINT " ENTER DATA FOR M(I,J) DUE TO M(I) = 1 "

0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1-, 1- DATA
0, 0, 0, 0, 0, 0, 0, 0, 1-, 1-, 0, 0 ,0 ,0 ,0
0, 0, 0, 0, 0, 0, 8, 0, 8, 5, 8-, 0, 8-, 0, 0
1, 1, 6, 6, 0, 0, 1-, 1-, 0, 0, 0, 6, 6, 1, 1
0, 0, 8,-, 8,-, 1-, 1-, 0, 0, 0, 0, 0, 0, 0 OATA
9, E, 6.6, 6.6, 2, 2-, 8-, 0, 0, 0, 0, 0, 6.6, 3, 9
PRINT " ENTER DATA FOR MTO(I) IN M.S. "
DATA 9, 0, -9, 0, 0, 0
PRINT " ENTER DATA FOR MT1(I,J) DUE TO M(I) "
DATA 0, -1, 0, 0, 0, 0
DATA 1, 0, -1, 0, 0, 0
DATA 3, 0, 3, 0, 0, 0
DATA 0, 0, 0, -1, -.8, 0
DATA 0, 0, 1, 0, -.6, -1
```

#### PRINT: MERGE (VWN)

#### RUN

```
-45.04686 2.253228
-.2117224 -.7366571
2.170338
           -40.45751
2.211513
          -.9145194
-3.385756 -7.180797
-6.700591 -11.25464
```

#### TWISTING MOMENTS MT (I)

2117224 2.253228 -2.8581714.423566 2.990141 -3.950552

#### THAT IS ALL NOW

#### APPENDIX (6) (DATAVW09)

```
3000 PRINT " ENTER DATA FOR R, N"
3010 DATA 6, 36
3020 PRINT " ENTER DATA FOR ELASTIC LENGTHES L(I) "
3030 DATA 1,1,.5,1,1,.5,1,1,.5,1,1,1,1,1,5,.5,.5
3040 PRINT " ENTER DATA B.MOMENTS MO(I) IN M.S. "
3050 DATA 0, 312.5, 312.5, 0, 0, 0
3052 DATA 0 , 0 , 0 , 0 , 0 , 0
3054 DATA 0, 0, 0, 0, 0, 0
3056 DATA 0, 62.5, 0, 500, 0, 500
3058 DATA 62.5, 0, 500, 750, 500, 750
3060 DATA 0, 0, 750, 750, 750, 750
3080 PRINT " ENTER DATA FOR ADDITIONAL AVERAGE ORDINATE IN A(I) "
3085 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3087 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3120 PRINT " ENTER DATA FOR G: CENTROID OF ADDIT. AREA A(I) "
3124 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3128 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3160 PRINT "ENTER DATA FOR M(I,J) DUE TO M(I) = 1"
3162 DATA 0, -.5, -.5, -1, -1, -1
3164 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
3166 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
3168 DATA 0, 0, 0, 0, 0, 0
3170 DATA 0,0,0,0,0,0,0,0,0,0,0,0
3172 DATA 0,0,0,0,0,0,0,-.5,0,0,0
3174 DATA -.5, -1, 0, 0, 0, 0, -1, -1, 0, 0, 0, 0
3178 DATA 0, 1, 1, 0, 0, 0, 0, -1, -1, 0, 0, 0
3180 DATA 0, 0, 0, 0, 0, 0, 0, -.5, 0, 2, 0, -1
```

```
DATA -.5, 0, 2, 2, 0, 0, 0, 0, 2, 2, -1, -1
#DATA 0.0,0,0,0,0,0,0,0,0,0,0
        0, -1, -1, 0, 0, 0, 0, 0, 0, 2, 0, -1
% DATA
        0, 0, 2, 4, -1, -2, 0, 0, 4, 4, -2, -2
8 DATA
MDATA 0, -.5, -.5, 0, 0, 0, 0, 0, 0, -1, -1, -1
2DATA 0,0,0,0,0,0,0,0,0,-1,0,1
#DATA 0, 0, -1, -1, 1, 1, 0, 0, -1, -1, 1, 1
       0, -.5, -.5, 0, 0, 0, 0, 0, 0, 0, 0, 0
% DATA
        0,0,0,-1,-1,-1,0,0,0,-1,0,1
98 DATA
       0, 0, -1, -2, 1, 2, 0, 0, -2, -2, 2, 2
00 DATA
#PRINT " ENTER DATA FOR ADDITIONAL DISP. COEFS. U(I) "
50 DATA 0,0,0,0,0,0
MPRINT "ENTER DATA FOR ADDITIONAL DISP. COEFS. T(I)"
10 DATA 0,0,0,0,0,0
```

## MERGE (VW) R U N

194.2499 4.2499 -105.9545 5.9545 -105.9545 337.8889 7.8884 191.2579 1.2579 191.2579 171.5585 1.5585 353.9733 0.9733 353.9733 181.849 26.33734 464.2157 -99.19036 33734 287.1936 6.3273 193.9125 9.19036 -99.19036 1.1936 287.1936

1.8009 531.8009

#### THAT IS ALL NOW

# APPENDIX (7) (DATAVWNST)

```
MO PRINT " NAME ------ DATAVWN "
MO PRINT " ENTER DATA FOR R, N "
MO DATA 18, 36
MO PRINT " ENTER DATA FOR ELASTIC LENGTHES L1(I) "
MO DATA 1, 1, .5, 1, 1, .5, 1, 1, .5
MO DATA 1, 1, 1, 1, 1, 1, .5, .5, .5
MO PRINT " ENTER DATA FOR ELASTIC TORTIONAL LENGTHES LT(I) "
```

```
3045 DATA 6.25, 6.25, 3.125, 6.25, 6.25, 3.125
3050 DATA 6.25, 6.25, 3.125, 6.25, 6.25, 6.25
3055 DATA 6.25, 6.25, 6.25, 3.125, 3.125, 3.125
3060 PRINT " ENTER DATA B.MOMENTS MO(I) IN M.S. "
3070 DATA 0, 312.5, 312.5, 0, 0, 0
3075 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
3080 DATA 0, 62.5, 0, 500, 0, 500, 62.5, 0, 500, 750
3085 DATA 500, 750, 0, 0, 750, 750, 750, 750
3120 PRINT " ENTER DATA FOR ADDITIONAL AVERAGE ORDINATE IN A(I)
3125 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3130 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3160 PRINT " ENTER DATA FOR G: CENTROID OF ADDIT. AREA A(I) "
3170 DATA .5, .5, .5, .5, .5, .5, .5, .5, .5
3180 DATA .5, .5, .5, .5, .5, .5, .5, .5, .5
3240 PRINT "ENTER DATA FOR M1(I,J) DUE TO M(I) = 1"
3250 DATA 0, 0, 0, -1, 0, 0, 0, 0, 0
3255 DATA 0, 0, 0, 0, 0, 0, 1, 0, 0
3258 DATA 0, 0, 0, 1, 0, -1, 0, 0, 1
3261 DATA 2, -1, -2, 0, 0, 2, 2, -2, -2
3265 DATA 0,0,0,0,0,0,0,0,0
3268 DATA 0,0,0,0,0,0,0,0,0
3271 DATA 1, .5, 0, 0, -1, -1, 0, 0, 0
3274 DATA 0, -1, -1, 0, 0, 0, 0, -1, -1
3280 DATA -1, 0, 0, 0, 0, 0, 0, 0, 0
3283 DATA 0, 0, 0, 1, 0, 0, 0, 0, 0
3286 DATA 0, 0, 0, 1, 0, 0, 0, 0, 1
3289 DATA 2 , 0 , 0 , 0 , 0 , 2 , 2 , 0 , 0
3292 DATA
             0,0,0,0,0,0,0,0,0
             0,0,0,0,0,0,0,0,0
3295 DATA
3298 DATA
             1, .5, -1, -1, 0, 0, .5, 0, -1
            -1, 0, 0, 0, 0, -1, -1, 0, 0
3302 DATA
3305 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3308 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3312 DATA 0, -.5, 0, 0, 0, 0, -.5, -1, 0
3315 DATA 0, 0, 0, 0, 0, -1, -1, 0, 0
          0, 1, 0, 0, 0, 0, 0, -1, 0
3320 DATA
3323 DATA
            0,0,0,0,0,0,0,0,0
            0, -.5, 0, 1, 0, 0, -.5, 0, 1
3326 DATA
            1,0,0,0,0,1,1,0,0
3329 DATA
3332 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3335 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3338 DATA 0, -.5, 0, 0, 0, 0, .5, 0, -1
3341 DATA -1, 0, 0, 0, 0, -1, -1, 0, 0
3345 DATA 0, 1, 0, 0, 0, 0, 0, 0, 0
            0,0,0,0,-1,0,0,0,0
3348 DATA
3351 DATA
            0,0,0,1,0,0,0,0,1
            2,0,0,0,0,2,2,0,0
3354 DATA
3360 DATA 0,0,0,0,0,0,0,0,0
```

```
0,0,0,0,0,0,0,0,0
BDATA
MDATA 0,0,0,0,0,0,0,0,0
DATA 0,0,0,0,0,1,1,-1,-1
        0,0,0,0,0,0,0,0.0
DATA
        -1,0,0,0,0,0,1,0,0
5 DATA
8 DATA
        0.0.0.0.0.0.0.0.0.0
        1,0,-1,0,0,1,1,-1,-1
DATA
DATA 0, -.5, -.5, 0, 0, 0, 0, 0, 0
5DATA 0, 0, 0, 0, 0, 0, -1, -1, -1
*DATA 0, 0, 0, -1, 0, 1, 0, 0, -1
|| DATA -2, 1, 2, 0, 0, -2, -2, 2, 2
        0.0.0.0.0.0.0.1.0.0
5 DATA
        0,0,0,1,0,0,0,0,0
8 DATA
        0, .5, 0, 0, 0, 0, .5, 0, 0
IDATA
        1.0.0.0.0.1.1.0.0
4 DATA
1DATA 0, 0, 1, 0, 0, 0, 0, 0, -1
DATA 0.0.0.0.0.0.0.0.0.0
IDATA 0, 0, 0, 1, 0, -1, 0, 0, 1
BDATA 1, -1, -1, 0, 0, 1, 1, -1, -1
      0,0,1,0,0,0,0,0,0
16 DATA
        0,0,0,0,0,-1,0,0,0
DATA
        0,0,0,1,0,-1,0,0,1
12 DATA
        2, -1, -2, 0, 0, 2, 2, -2, -2
5 DATA
*DATA 0. -.5. -.5. 0. 0. 0. 0. 0. 0
∅DATA 0, -1, -1, 0, 0, 0, -1, 0, 0
2DATA 0, 0, 0, -1, 0, 1, 0, 0, -1
4DATA -2, 1, 2, 0, 0, -2, -2, 2, 2
        0.0.0.0.0.0.0.0.0
8 DATA
DATA
        0,0,0,0,0,0,0,0,0
        0, -.5, 0, 0, 0, 0, -.5, -1, 0
2 DATA
        0.0.0.1.-1.0.0.0.0
4 DATA
6DATA 0, -.5, -.5, -1, -1, -1, 0, 0, 0
8DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
        0,0,0,0,0,0,0,0,0
DATA
        0.0.0.0.0.0.0.0.0
6 DATA
        0,0,0,0,0,0,0,0,1
BDATA
        1, -1, -1, 0, 0, 1, 1, -1, -1
DATA
PRINT " ENTER DATA FOR MTO(I) IN M.S. "
DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
PRINT " ENTER DATA FOR MT1(I,J) DUE TO M(I) "
        0,0,0,0,0,0,0,0,0
DATA
DATA
        0,0,-1,0,0,-1,0,0,0
DATA 1, 1, 0, 0, 0, 0, 0, 0, 0
DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
```

0,0,0,0,0,0,0,0,0

DATA

```
3510 DATA
            1,0,0,1,0,0,0,0,0
3515 DATA 1, 0, 0, 0, 0, 0, 0, 0, 0
3520 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
           0,0,0,0,0,0,1,0,0
3525 DATA
            0,0,0,0,0,0,0,0,0
3530 DATA
3535 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3540 DATA 0, 1, 0, 0, 0, 0, 0, 0, 0
          0,0,0,1,0,0,0,0,0
3545 DATA
            0,0,0,0,0,0,0,0,0
3550 DATA
3555 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3560 DATA 0 . 1 . 0 . 0 . 1 . 0 . 0 . 0 . 0
            0,0,0,0,0,0,0,1,0
3565 DATA
            0,0,0,0,0,0,0,0,0
3570 DATA
3575 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3580 DATA 0, 0, 0, 0, 0, -1, 0, 0, 0
          0,0,0,0,0,0,0,0,0
3590 DATA
3595 DATA
            0,0,0,0,0,0,0,0,0
3600 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3605 DATA 0, 0, 0, 1, 0, 0, 0, 0, 0
3610 DATA 0,0,0,0,0,0,0,0,0
            0, -1, 0, 0, 0, 0, 0, 0, 0
3614 DATA
3618 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3621 DATA 0, -1, 0, 0, -1, 0, 0, 0, 0
           0,0,0,0,0,0,0,0,0
3625 DATA
           0,0,0,0,0,1,0,0,0
3630 DATA
3645 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
3650 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
          0,0,0,0,0,0,0,0,0
3655 DATA
           0,0,0,0,0,0,0,0,0
3660 DATA
3665 DATA 0, 0, 0, 0, 1, 0, 0, 0, 0
3670 DATA 0, 0, 0, 0, 0, 0, 0, 0, 0
```

#### MERGE (VWN)

#### RUN

```
47.31787
           105.5733
151.9595
           -90.57729
-68.90285
           -68.90285
-8.323312
           172.8593
159.2712
           211.009
202.0286
           202.0286
-38.99456
           280.9826
248.1845
           373.3987
360.7047
           360.7047
47.25265
           110.5153
-5.736779
           170.0554
-41.51587
           281.5322
```

DIWAN: Computer Analysis of Structures Loaded Normal to Their Planes By the Method of Virtual Work

```
136.486
          -85.67599
156.2897
          211.5266
248.5693
          373.3552
-64.30789
          -64.30789
202,7927
          202,7927
          360.721
360.721
                                    TWISTING MOMENTS MT (I)
47.25256
41.51587
```

21.2681 12,63421

46.72861 32,96283

47.31787

46.38614

-21.67443

-38.99456

-32.7981

12.69398

0

THAT IS ALL NOW

NOTICE:

This paper is open for discusion within are month from its publication.