STUDY OF HEAT REMOVAL AFTER SHUTDOWN IN ET-RR-1

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ABSTRACT

After shutdown, the reactor power does not drop to zero instantaneously, but falls off rapidly according to a negative period eventually determined by the half life of the longest lived delayed neutron group. The fission product radioactive elements represent a major source of heat after shutdown. The governing heat equations describing the thermal head decay were solved using finite difference method with both explicit and implicit schemes. It was found that dropping mass flow rate to one-fourth of its operating value increases the hot spot temperature. For (ET-RR-1) data, the hot spot temperature is still within the safe limit. Using explicit scheme puts a stability condition on the time step without increasing the accuracy of the solution. Using implicit scheme with a reasonable time step is more efficient in solving the problem.

NOMENCLATURE

A	Cross sectional area
t	Time
q'''	Volumetric thermal source strength
C	Specific heat
U	Coolant physical speed
T	Relative temperature
h	Heat transfer coefficient
m	Coolant mass flow rate
Z	Coordinate in axial direction
H	Fuel rod height
K	Thermal conductivity
r/R_e	Fuel rod radial position
P	Cell pitch
μ	Viscosity
α	Thermal diffusivity
ρ	Density
ξ	Numerical speed $\dfrac{\Delta Z}{\Delta t}$

Subscripts

c	Clad or fuel rod center when 2 used with thermal source strength
e	Extrapolated dimension
f	Fuel
i	Initial at zero time
o	Operating value
s	After shutdown
w	Coolant
c	Clad inner surface
co	Clad outer surface
cw	Between clad and coolant
fc	Between fuel and clad

INTRODUCTION

In reactor shutdown, the reactor power does not immediately drop to zero but falls off rapidly according to a negative period, eventually determined by the half-life of the longest-lived delayed neutron group [1,2].

The rate of power degradation after shutdown is of great concern to thermal hydraulic designer since it affects fuel temperature after shutdown [1].

The amount of such power generation depends upon the power level before shutdown, the length of time after shutdown, and time of operation, since these factors determine the amount of fission products present [1]. Such decay heat is essential to provide shutdown cooling (or decay cooling) for the reactor fuel.

The purpose of this study is to provide a numerical solution for the temperature variation in coolant, clad and fuel of the reactor at any radial position inside the core.

The governing heat balance equations were determined with the boundary and initial conditions. Then, the differential equations were changed to the finite difference form using both explicit and implicit schemes.

The stability conditions for the explicit scheme were discussed.

THE PHYSICAL MODEL

To form the heat balance governing equations, the following assumptions were used:

(1) The heat generation inside the core follows the Bessel function of the zeroth order in radial direction and the cosine shape in the axial direction.
(2) The variation of heat generated in fuel rod in the radial direction is neglected.
(3) Temperature is averaged in the radial direction in each component, namely, fuel, clad, and coolant for each channel.
(4) Change of internal energy of gas is neglected.
(5) Uniform properties of clad and fuel are assumed.
(6) Coolant properties are temperature dependent and are evaluated at the average temperature of coolant.

The governing equations using these assumptions are:

$$A_f \rho_f C_f \frac{\partial T_f}{\partial t} = A_f \bar{q}(Z,t_s) - 2\pi r_{ci} h_{fc}(T_f - T_c) \quad (1)$$

$$A_c \rho_c C_c \frac{\partial T_c}{\partial t} = 2\pi r_{ci} h_{fc}(T_f - T_c) - h_{cw} 2\pi r_{ci}(T_c - T_w) \quad (2)$$

$$A_w \rho_w C_w \frac{\partial T_w}{\partial t} = 2\pi r_{co} h_{cw}(T_c - T_w) - \dot{m} C_{pw} \frac{\partial T_w}{\partial Z} \quad (3)$$

The governing equations are three partial differential equations in time with one in space for coolant.

The initial conditions for the time dependence are [2]:

$$T_f(Z,0) = T_{wi} + \bar{q}_c(r,0) A_f \Big[\frac{H_e}{\dot{m}\pi C_p}\Big(\sin\frac{\pi Z}{H_e} + \sin\frac{\pi H}{2H_e}\Big) + \Big(\frac{1}{h_w 2\pi r_{co}} + \frac{\ln(r_{co}/r_{ci})}{2\pi K_c} + \frac{t_g}{K_g \pi (r_{ci} + r_f)} + \frac{1}{8\pi K_f}\Big)\cos\Big(\frac{\pi Z}{H_e}\Big)\Big] \quad (4)$$

$$T_c(Z,0) = T_{wi} + \bar{q}_c(r,0) A_f \Big[\frac{H_e}{\dot{m}\pi C_p}\Big\{\sin\frac{\pi Z}{H_e} + \sin\frac{\pi H}{2H_e}\Big\} + \Big\{\frac{1}{h_\omega 2\pi r_{co}} + 0.5\frac{\ln(r_{co}/r_{ci})}{2\pi K_c}\Big\}\cos\Big(\frac{\pi Z}{H_e}\Big)\Big] \quad (5)$$

$$T_w(Z,0) = T_{wi} + \bar{q}_c(r,0) A_f \left[\frac{H_e}{\dot{m}\pi C_p}\Big(\sin\frac{\pi Z}{H_e} + \sin\frac{\pi H}{2H_e}\Big)\right] \quad (6)$$

The boundary condition for coolant equation is, $T_w(0,t) = T_{wi}$

The coolant, clad, and fuel temperatures are the averaged values over space in the radial direction across the channel.

The initial thermal source strength q''' (r,Z,0) is taken to be a cosine function in Z-direction and a zeroth order Bessel function in radial direction:

$$\bar{q}_c(r,0) = \bar{q}(0,0,0) J_o\left[\frac{2.405 r}{R_e}\right] \quad (7)$$

The thermal source strength at time t_s after shutdown is related to the thermal source strength during operation through the relation [1,2]:

$$\frac{\bar{q}(Z,t_s)}{\bar{q}(Z,0)} = \begin{cases} [0.1(t_s+10)^{-0.2} - 0.087(t_s+2*10^7)^{-0.2}] - [0.1(t_s+t_o+10)^{-0.2} - 0.087(t_s+t_o+2*10^7)^{-0.2}] & t_s < 200\text{sec} \\ 0.095 t_s^{-0.26}[1 - (\frac{t_o}{t_s}+1)^{-0.2}] & t_s > 200\text{sec} \end{cases} \quad (8)$$

h_{fc} = Overall heat transfer coefficient between fuel and clad [2,5]

$$\frac{1}{h_{fc}} = \frac{r_{ci}}{4K_f} + \frac{2r_{ci}t_g}{K_g(r_{ci}+r_f)} + \frac{0.5r_{ci}\ln(\frac{r_{co}}{r_{ci}})}{K_c} \qquad (9)$$

Where:

t_g = gap thickness = $r_{ci} - r_f$

$h_{c\omega}$ = Overall heat transfer coefficient between clad and coolant [3,5].

$$\frac{1}{h_{c\omega}} = \frac{0.5r_{co}\ln(\frac{r_{co}}{r_{ci}})}{K_c} + \frac{1}{h_\omega} \qquad (10)$$

The convective heat transfer coefficient h_w is a function of temperature. It is evaluated using Weisman correlation for flow parallel to rod bundles [2,4].

$$Nu = \left\{0.042(\frac{P}{2r_{co}}) - 0.024\right\} Re^{0.8} Pr^{1/3} \qquad (11)$$

$$Re = \frac{2\dot{m}}{\pi r_{co}\mu_w}$$

$$Pr = \frac{\nu}{\alpha}, \quad NU = \frac{hD_h}{K_w}$$

The properties of water as coolant are evaluated as a function of temperature using a fitting correlation for viscosity and Prandtl number between 10 °C - 340 °C [4].

$$D_h = \frac{4(P^2 - \pi r_{co}^2)}{2\pi r_{co}}$$

According to the assumption of constant fuel and clad properties h_{fc} is constant, while h_{cw} is dependent upon temperature since it is a function of the convective heat transfer coefficient h_w.

METHOD OF SOLUTION

The equations are finite differenced in both time and space using both explicit and implicit schemes [2,4,5].

(a) Explicit Scheme

$$T_f^{n+1}(i) = (1 - \Delta t\, C_2)\, T_f^n(i) + \Delta t\, C_2\, T_c^n(i) + \Delta t\, C_1(Z_i, t_n)$$

$$T_c^{n+1}(i) = [\,1 - \Delta t C_3 + \Delta t C_4(Z_i,t_n)]\, T_c^n(i) + \Delta t C_3 T_f^n(i) + \Delta t\, C_4\, Tw^n(i) \quad 1 < i < N$$

$$T_w^{n+1}(i) = [1 - \Delta t C_5(Z_i,t_n) - \frac{\Delta t C_6}{\Delta Z}]T_w^n(i) + \Delta t C_5(Z_i,t_n)T_c^n(i) + \frac{\Delta t C_6}{\Delta Z}T_w^n(i-1) \quad 1<i<N$$

where:

N = Number of nodes used in the finite difference

$$C_1(Z_i,t_n) = \frac{\bar{q}(Z_i,t_n)}{\rho_f C_f}$$

$$C_2 = \frac{2\pi r_{ci} h_{fc}}{A_f \rho_f C_f}$$

$$C_3 = \frac{2\pi r_{ci} h_{fc}}{A_c \rho_c C_c}$$

$$C_4(Z_i,t_n) = \frac{h_{cw}(Z_i,t_n).2\pi r_{co}}{A_c \rho_c C_c}$$

$$C_5(Z_i,t_n) = \frac{h_{cw}.2\pi r_{co}}{A_w \rho_w C_{pw}}$$

$$C_6 = \frac{\dot{m}}{A_w\, \rho_w}$$

For stability of the explicit scheme the following conditions should be satisfied,

(i) $$\Delta t < \frac{1}{C_2}$$

$$\Delta t < \frac{A_f \rho_f C_f}{2\pi r_{ci} h_{fc}}$$

(ii) $$\Delta t < \frac{1}{C_3 + C_4}$$

$$\Delta t < \frac{1}{\frac{2\pi r_{ci} h_{fc}}{A_c \rho_c C_c} + \frac{2\pi r_{co} h_{cw}}{A_c \rho_c C_c}}$$

$$\Delta t < \frac{A_c \rho_c C_c}{2\pi r_{ci} h_{fc} + 2\pi r_{co} h_{cw}}$$

(iii) $$\Delta t < \frac{\Delta Z}{C_5 \Delta Z + C_6}$$

$$\Delta t < \frac{\Delta Z}{\frac{h_{cw} 2\pi r_{co}}{A_w \rho_w C_{pw}} \Delta Z + \frac{\dot{m} C_{pw}}{A_w \rho_w C_{pw}}}$$

$$\Delta t < \frac{A_w \rho_w C_{pw} \Delta Z}{h_{cw} 2\pi r_{co} \Delta Z + m \times C_{pw}}$$

Since ΔZ is small then the first term in the denominator is small compared to second term.

$$\Delta t < \frac{A_w \rho_w \Delta Z}{\dot{m}}$$

but m. = $A_w \rho_w U_w$

$$\therefore \Delta t < \frac{A_w \rho_w \Delta Z}{A_w \rho_w U_w}$$

$$\therefore U_w < \xi$$

Where ξ is the numerical speed of the explicit scheme and U_w coolant speed.

(b) IMPLICIT SCHEME

(i) $(1 + C_2 \Delta t) T_f^{n+1}(i) - C_2 \Delta t\, T_c^{n+1}(i)$

$$= T_f^n(i) + \Delta t\, C_1(Z_i, t_n)$$

(ii) $-\Delta t C_3 T_f(i) + [1 + \Delta t C_3 + \Delta t C_4(Z_i, t_n)] T_c^{n+1}(i) - C_4 \Delta t$

$$T_w^{n+1}(i) = T_c^n(i) \qquad 1 \le i \le N$$

(iii) $$-\Delta t C_5(Z_i, t_n) T_c^{n+1}(i) + [1 + C_5(Z_i, t_n)\Delta t + C_6 \frac{\Delta t}{\Delta Z}] T_w^{n+1}(i)$$

$$-C_6 \frac{\Delta t}{\Delta Z} T_w^{n+1}(i-1) = T_w^n(i) \quad 1 < i \le N$$

These equations can be put in the form [6], AT = B

Where :

$$T \text{ is a vector } \begin{bmatrix} [T_f] \\ [T_c] \\ [T_w] \end{bmatrix}$$

$$a(i.i) = 1 + C_2 \Delta t \quad i = 1, N$$

$$a(i, i+N) = -C_2 \Delta t \quad i = 1, N$$

$$a(i, i-N) = -C_3 \Delta t \quad i = N+1, 2N$$

$$a(i,i) = 1 + C_3 \Delta t + C_4(Z_i, t_n)\Delta t \quad i = N+1, 2N$$

$$a(i, i+N) = -C_4(Z_i, t_n)\Delta t \quad i = N+1, 2N$$

$$a(2N+1, 2N+1) = 1$$

$$a(i, i-N) = -C_5(Z_i, t_n)\Delta t \quad i = 2N+2, 3N$$

$$a(i,i) = 1 + C_5(Z_i, t_n)\Delta t + C_6 \frac{\Delta t}{\Delta Z} \quad i = 2N+2, 2N$$

$$a(i, i-1) = -C_6 \frac{\Delta t}{\Delta Z} \quad i = 2N+2, 3N$$

$$b(i) = T_f^n(i) + C_1(Z_i, t_{n+1})\Delta t \quad i = 1, N$$

$$b(i) = T_c^n(i) \quad i = N+1, 2N$$

$b(i) = T_w^n(i) \quad i = 2N+1, 3N$

The gauss-seidel iteration technique is used to solve the equation with a relative accuracy 10^{-5} [6].

The matrix A is a dominant diagonal matrix which accelerates the conversion of solution using iteration [2,5,6].

A particular attention is devoted to Inchass research reactor [3] considering its operating conditions and parametric design. It is assumed that the reactor is operated for 48 hours at full power before shutdown [3].

A list of the design parameters is given to establish a basic for comparison with available data [3].

DISCUSSION

Figure (1) shows the fuel, clad and coolant temperatures with time at the hot spot of a central fuel element.

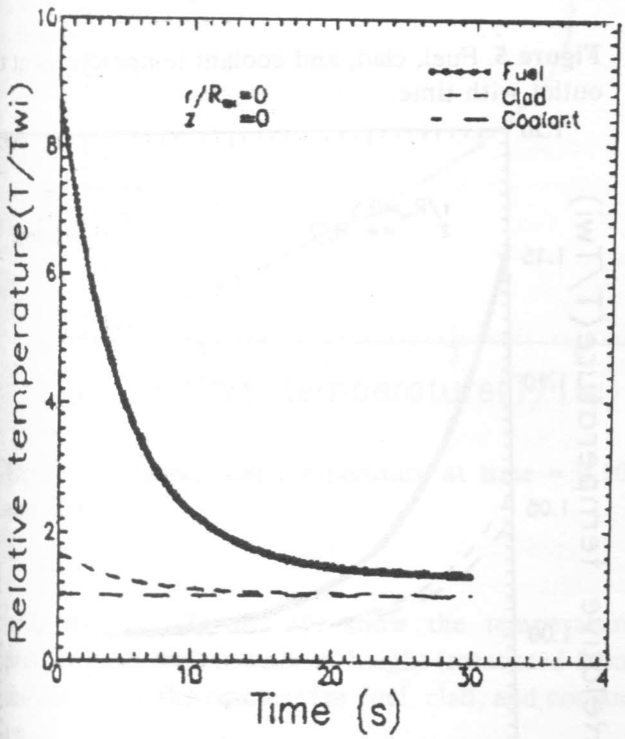

Figure 1. Fuel, clad, and coolant temperatures at the center with time.

The relative temperature cools down from 8 to 2.0 in twelve seconds, provided that coolant continues to circulate with full capacity after shutdown. After twelve seconds the fuel hot spot temperature cools down with a slower rate. It reaches to 1.1 after 30 seconds.

Figure (2) shows the fuel, clad, and coolant temperatures at the hottest point in a fuel rod located at a radial distance r/R_e = 0.25. The temperature reaches to 1.8 after the twelve seconds. The clad temperature does not change much from that of the central fuel element at different times.

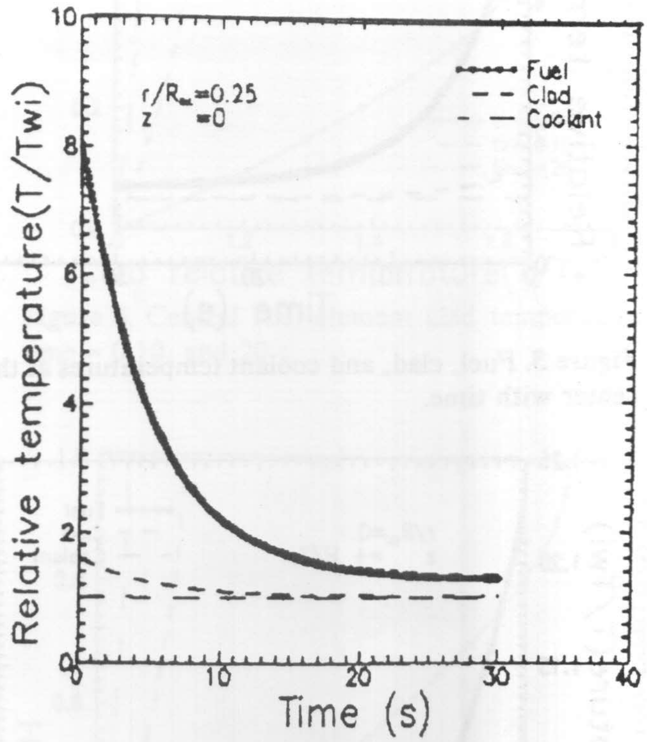

Figure 2. Fuel, clad, and coolant temperatures at the center with time.

It can be noticed that the same criteria applies to coolant temperature variation with time for both the central fuel rod and the one located at r/R_e = 0.25.

Figure (3) shows a lower temperatures for fuel, clad, and coolant for a fuel rod located at r/R_e = 0.5.

For the three figures, the fuel hottest temperature reach to almost the same value after 30 s. This indicates that the fuel hottest temperature reaches to uniform values regardless of the radial position of the fuel rod after 30 second for the assigned mass flow rate (m= 0.39164 Kg/s).

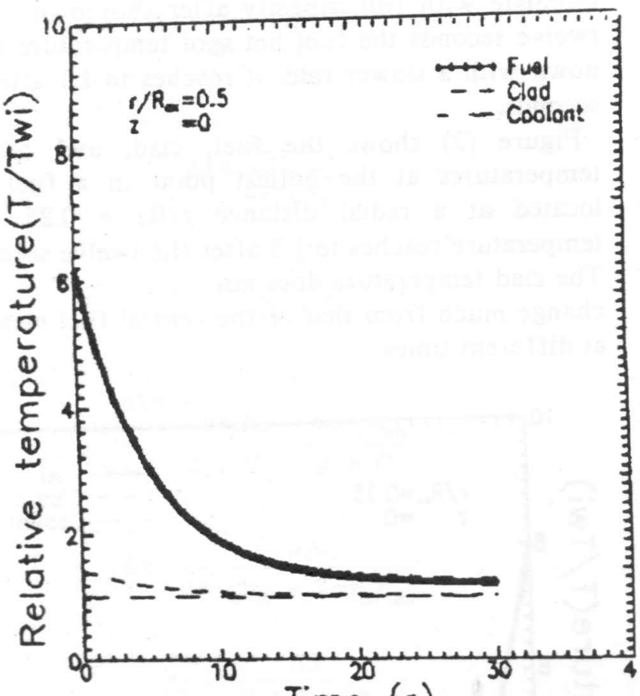

Figure 3. Fuel, clad, and coolant temperatures at the center with time.

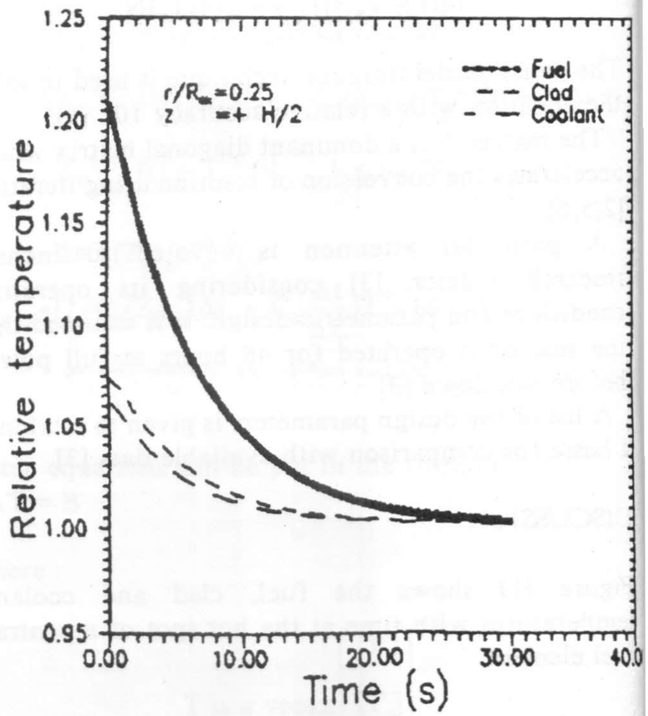

Figure 5. Fuel, clad, and coolant temperatures at the outlet with time.

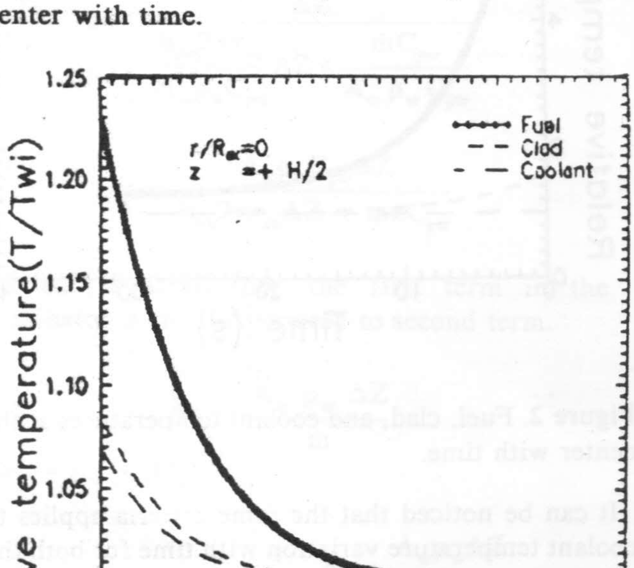

Figure 4.Fuel, clad, and coolant temperatures at the center with time.

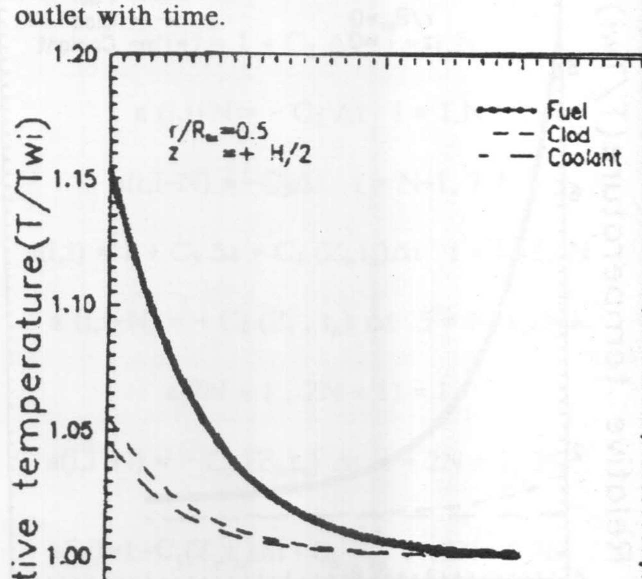

Figure 6. Fuel, clad, and coolant temperatures at the outlet with time.

Figures (4), (5), and (6) give the change of temperature with time at the exit of channels. They show lower temperatures of fuel and clad compared to these at the center of fuel rod but higher temperatures of coolant at the exit of different channels compared to temperatures of coolant at the center of each fuel rod.

Again temperatures drop to equal values after about 30 s.

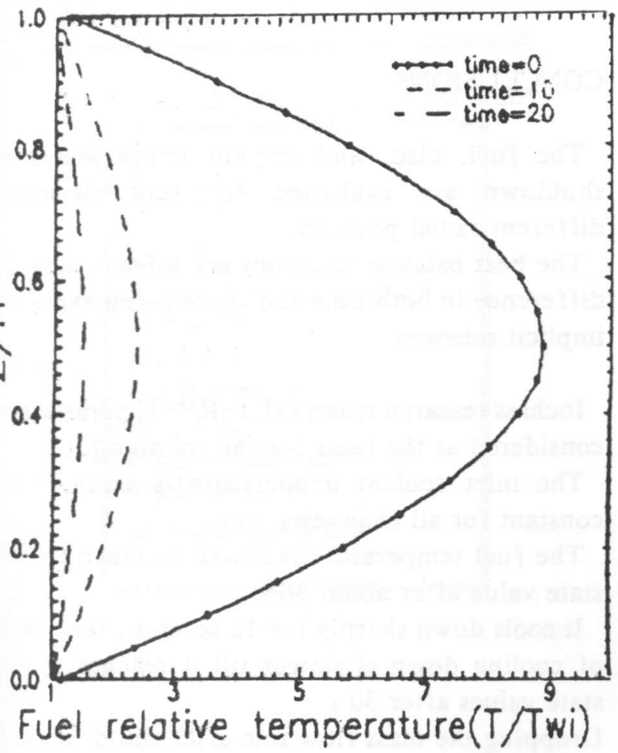

Figure 7. Central fuel temperature at time = 0,10, and 20 s.

Figures (7), (8) and (9) show the temperature profiles with the normalized height (measured from the bottom of the reactor) for fuel, clad, and coolant respectively.

These figures show the initial temperature profiles, and profiles after 10, and 20 seconds.

The maximum fuel and clad temperatures occur at the central height, while the maximum coolant temperature occurs at the exit of the channel.

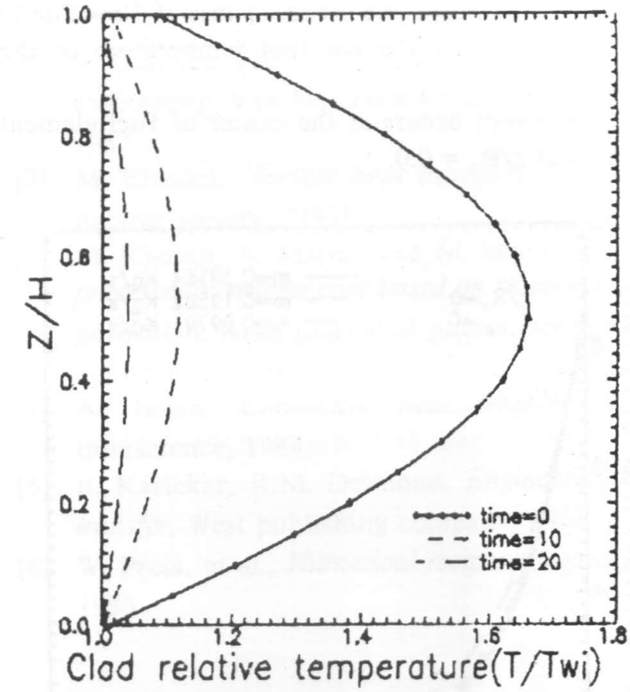

Figure 8. Central fuel element clad temperature at time = 0,10, and 20 s.

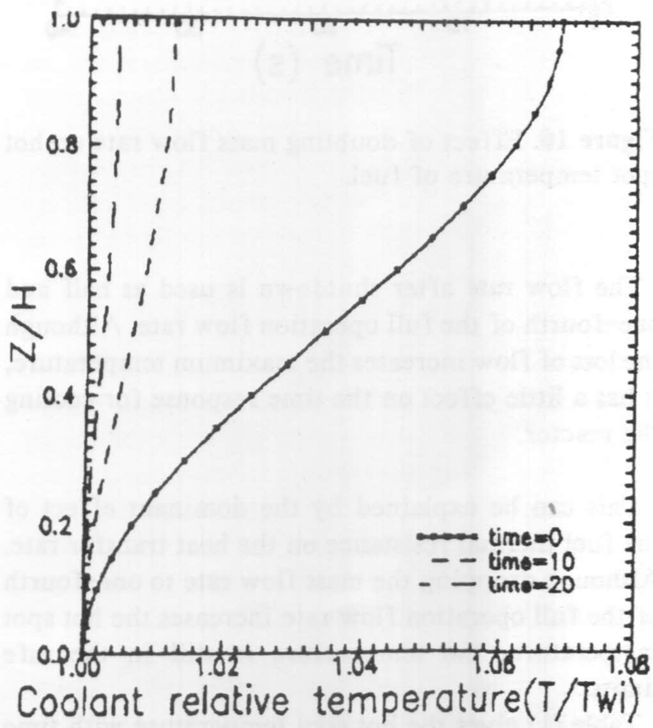

Figure 9. Central fuel element temperature profiles at time = 0,10, and 20 s.

Figure (10) shows the effect of loss of flow due to pumps failure on the hot spot temperature of the reactor.

The hot spot occurs at the center of fuel element located at $r/R_e = 0.0$.

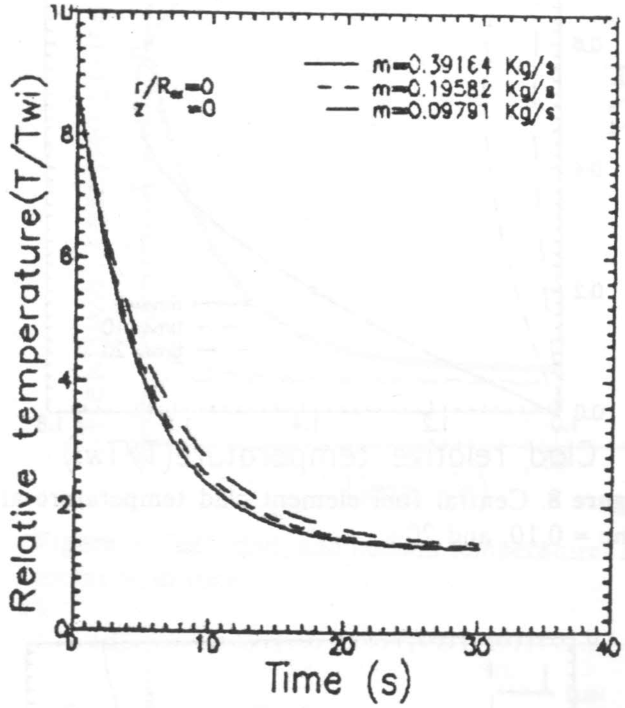

Figure 10. Effect of doubling mass flow rate on hot spot temperature of fuel.

The flow rate after shutdown is used as half and one-fourth of the full operation flow rate. Although the loss of flow increases the maximum temperature, it has a little effect on the time response for cooling the reactor.

This can be explained by the dominant effect of the fuel thermal resistance on the heat transfer rate. Although dropping the mass flow rate to one-fourth of the full operation flow rate increases the hot spot temperature, the temperature is still in the safe limits.

Table (1) gives the hot spot temperature with time using both explicit and implicit schemes. The time step for explicit scheme was chosen to satisfy the stability condition, while the time step for the implicit scheme was chosen about 10 times that of the explicit scheme since the implicit scheme is unconditionally stable.

The comparison shows a very good agreement between explicit and implicit schemes.

The number of nodes is fixed in both schemes. The implicit scheme is more CPU time saving on the computer than the explicit scheme for the same accuracy.

CONCLUSIONS

The fuel, clad, and coolant temperatures after shutdown are evaluated for fuel elements at different radial positions.

The heat balance equations are solved using finite difference in both time and space using explicit and implicit schemes.

Inchass research reactor (ET-RR-1) parameters are considered as the basis for the solution [3].

The inlet coolant temperature is assumed to be constant for all channels.

The fuel temperature is found to reach its steady state value after about 30 s.

It cools down sharply for 12 seconds, then the rate of cooling down is slower till it reaches to steady state values after 30 s.

Dropping the mass flow rate after shutdown to half of its full operation value increases the hot spot temperature, but it does not affect cooling time to steady state.

Dropping the mass flow rate after shutdown to one-fourth of its operating value increases hot spot temperature, but it is found that for (ET-RR-1) data the temperature is still in the safe limits.

Using implicit scheme is found to be more efficient since it is unconditionally stable. The use of explicit scheme to solve the problem forces a stability condition on the chosen time step.

This leads to CPU time consuming program without improving the accuracy of the solution.

A list of the parameters used for the study is given in table (2).

Table 1. Comparison between explicit and implicit schemes

TIME(S)	HOT SPOT TEMPERATURE	
	EXPLICIT	IMPLICIT
0.1	8.695	8.697
0.5	8.114	8.122
1.0	7.453	7.468
1.5	6.858	6.877
2.0	6.311	6.344

Table 2. parameters used for the study

$r_f = 0.005$ m
$r_{co} = 0.0065$ m
$r_{ci} = 0.005$ m
$q'''_{co} = 1.50650 * 10^8 w/m^3$
H = 0.5 m
Clad A L
Coolant water
Fuel UO_2
Pitch = 0.017 m
TWIC = 34 °C
$m^{\cdot}$ = 0.39164 Kg/s

REFERENCES

[1] S. Glasston and A. Sesonske *Nuclear reactor engineering*, Van Nostrand Reinhold Company, 1967.

[2] M. Elwakil, *Nuclear heat transport*, American nuclear society, 21978.

[3] M. Khatab, A. Mariey and M. Shaat, *Thermal processes in reactor core based on shutdown heat generation*, Arab journal of nuclear science and applications, 1984.

[4] A. Bejan, *Convective heat transfer,* Wiley interscience, 1984.

[5] B. Karlekar, R.M. Desmond, *Engineering heat transfer*, West publishing company, 1968.

[6] W. Press, et al., *Numerical recipes*, Cambridge, 1986.